Claim 4: Modeling and Data Analysis

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Claim 4: Modeling and Data Analysis

"Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems."

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Smarter Balanced Cognitive Rigor Matrix

Depth of Thinking (Webb) + Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	- Recall conversions, terms, facts			
Understand	-Evaluate an expression -Locate points on a grid or number on number line -Solve a one-step problem -Represent math relationships in words, pictures, or symbols	- Specify, explain relationships -Make basic inferences or logical predictions from data/observations -Use models /diagrams to explain concepts -Make and explain estimates	-Use concepts to solve non-routine problems -Use supporting evidence to justify conjectures, generalize, or connect ideas -Explain reasoning when more than one response is possible -Explain phenomena in terms of concepts	-Relate mathematical concepts to other content areas, other domains -Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	-Follow simple procedures -Calculate, measure, apply a rule (e.g., rounding) -Apply algorithm or formula -Solve linear equations -Make conversions	-Select a procedure and perform it -Solve routine problem applying multiple concepts or decision points -Retrieve information to solve a problem -Translate between representations	-Design investigation for a specific purpose or research question - Use reasoning, planning, and supporting evidence -Translate between problem & symbolic notation when not a direct translation	-Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	-Retrieve information from a table or graph to answer a question -Identify a pattern/trend	-Categorize data, figures -Organize, order data -Select appropriate graph and organize & display data -Interpret data from a simple graph -Extend a pattern	-Compare information within or across data sets or texts -Analyze and draw conclusions from data, citing evidence -Generalize a pattern -Interpret data from complex graph	-Analyze multiple sources of evidence or data sets
Evaluate			-Cite evidence and develop a logical argument -Compare/contrast solution methods -Verify reasonableness	-Apply understanding in a novel way, provide argument or justification for the new application
Create	- Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	-Generate conjectures or hypotheses based on observations or prior knowledge and experience	-Develop an alternative solution -Synthesize information within one data set	-Synthesize information across multiple sources or data sets -Design a model to inform and solve a practical or abstract situation

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Rationale for Claim 4

- In the real world, problems do not come neatly ”packaged.” Real-world problems are complex and often contain insufficient or superfluous data.
- Assessment tasks will involve *formulating* a problem that is tractable using mathematics; that is, formulating a model. This will usually involve making assumptions and simplifications.
- Students will need to select from the data at hand or estimate data that are missing. (Such tasks are therefore distinct from the well-formulated problem-solving tasks described in Claim 2.)
- Students will identify variables in a situation and construct relationships between them. Once students have formulated the problem, they will tackle it (often in a decontextualized manner) before interpreting their results and then checking the results for reasonableness.

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Targets for Claim 4

Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace. (DOK 2, 3)

Target B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem. (DOK 2, 3, 4)

Target C: State logical assumptions being used. (DOK 1, 2)

Target D: Interpret results in the context of a situation. (DOK 2, 3)

Target E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon. (DOK 3, 4)

Target F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)

Target G*: Identify, analyze and synthesize relevant external resources to pose or solve problems. (DOK 3, 4)

*Assessed in Performance Tasks only

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Claim 4 Clusters and Standards

Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	HS
3.OA.A	4.OA.A	5.NBT.B	6.RP.A	7.RP.A	8.EE.3	N-Q.A
3.OA.D	4.NF.B	5.NF.A	6.NS.A	7.NS.A	8.EE.4	A-SSE.B
3.MD.A	4.MD.A*	5.NF.B	6.NS.C	7.EE.B	8.EE.B	A-CED.A
3.MD.C	4.MD.B*	5.MD.A*	6.EE.B	7.G.A*	8.EE.C	A-REI.A
3.MD.D*	4.MD.C*	5.MD.B*	6.EE.C	7.G.B*	8.F.B*	A-REI.B
		5.MD.C	6.G.A*	7.SP.A*	8.G.B	F-IF.B
		5.G.A*	6.SP.A*	7.SP.B*	8.G.C*	F-IF.C
			6.SP.B*	7.SP.C*	8.SP.A*	F-BF.A
						S-ID.A
						S-ID.B
						S-IC.1
						S-IC.B
						A-REI.C
						F-LE.A
						F-LE.B
						F-TF.5
						G-GMD.3
						G-MG

*Denotes additional and supporting clusters

Essential Properties of Claim 4 Items/Tasks

- A key feature of items and tasks in Claim 4 is that the student is confronted with a contextualized, or "real world" situation and must decide which information is relevant and how to represent it.
- As some of the examples provided illustrate, "real world" situations do not necessarily mean questions that a student might really face; it means that mathematical problems are embedded in a practical, application context.
- In this way, items and tasks in Claim 4 differ from those in Claim 2, because while the goal is clear, the problems themselves are not yet fully formulated (well-posed) in mathematical terms."

Excerpted from Smarter Balanced Math Content Specifications p. 72.

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Essential Properties of Claim 4 Items/Tasks

- Items/tasks in Claim 4 assess student expertise in choosing appropriate content and using it effectively in formulating models of the situations presented and making appropriate inferences from them.
- Claim 4 items and tasks should sample across the content domains, with many of these involving more than one domain.
- Items and tasks of this sort require students to apply mathematical concepts at a significantly deeper level of understanding of mathematical content than is expected by Claim 1.

Excerpted from Smarter Balanced Math Content Specifications p. 73.

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Essential Properties of Claim 4 Items/Tasks

- Claim 4 items will be included in the computer-adaptive portion of the summative assessment (CAT) and also embedded in performance tasks.
  - For the (CAT), Claim 4 will be assessed using a combination of
    - Multiple-Choice, single correct response ; Multiple Choice, multiple correct response; Hot Text; Equation/Numeric; Drag and Drop, Hot Spot, and Graphing; Matching Tables T/F and Y/N variations; and Fill-in Table
  - The Performance Tasks may also include Short Text responses will
- To preserve the focus and coherence of the standards, Claim 4 items/tasks must draw clearly on knowledge and skills articulated in the progression of standards up to and including that grade.

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Essential Properties of Claim 4 Items/Tasks

- The intent is that each of the targets should not lead to a separate item/task, but will provide evidence for several of the assessment targets defined for Claim 4. It is in *using* content from different areas, including work studied in earlier grades, that students demonstrate their problem-solving proficiency.
- Another important distinction between Claim 1 specification tables is that the evidence required of students to satisfy Claim 4 centers around specific statements of the ***mathematical practices*** (MP) contained in the CCSSM. Though not exclusive, MP2, MP4, and MP5 are particularly relevant for Claim 4 items.

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MP2: Reason abstractly and quantitatively.

Mathematically proficient students:

- make sense of quantities and their relationships in problem situations.
- bring two complementary abilities to bear on problems involving quantitative relationships:
  - *Decontextualize* (abstract a given situation and represent it symbolically; and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents) and
  - *Contextualize* (pause as needed during the manipulation process in order to probe into the referents for the symbols involved).
- use quantitative reasoning that entails creating a coherent representation of the problem at hand, considering the units involved, and attending to the meaning of quantities (not just how to compute them).
- know and flexibly use different properties of operations and objects.

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MP4: Model with mathematics.

Mathematically proficient students:

- apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
  - In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
  - By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
- identify important quantities in a practical situation.
- map relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- analyze those relationships mathematically to draw conclusions.
- interpret their mathematical results in the context of the situation.
- reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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MP5: Use appropriate tools strategically.

Mathematically proficient students:

- consider available tools when solving a mathematical problem. (Tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software.)
- are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
- detect possible errors by using estimations and other mathematical knowledge.
- know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.
- identify relevant mathematical resources and use them to pose or solve problems.
- use technological tools to explore and deepen their understanding of concepts.

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Distinguishing Between Claim 4 and Claims 1 & 2

- In early grades when equations are still new to students, an important distinction between Claim 2 and Claim 4 is requiring a model that would lead to a problem's solution.
- In Claim 2, problems are well posed, while in Claim 4, they may have extraneous or missing information.
- In Claims 1 and 2, measurements of objects or figures can be accurately determined. In Claim 4, modeling is used to make approximations.
- In Claim 1, data analysis is straightforward and procedural. In Claim 4, the analysis should be tied to some useful purpose in the real world.

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The Mathematics Assessment

Sample Claim 4 Items

Task Model 1: Target A

Apply mathematics to solve problems arising in everyday life, society, and the workplace. (DOK 2, 3)

Problems used to assess this target for Claim 4 should not be completely formulated (as they are for the same target in Claim 2), and require students to extract relevant information from within the problem and find missing information through research or the use of reasoned estimates.

Grade 5 Example of Task Model 1/Target A

56806

Mary, Sally, and Erin competed in a three-part race.

- Mary's swim time was 0.10 hour faster than Erin's run time.
- Sally's finish time was 0.12 hour faster than Mary's finish time.
- Erin finished the race in 2.72 hours.

Drag the numbers into the boxes to complete the missing times for each girl.

0 1 2 3 4 5 6 7 8 9

Delete

Event	Mary's Times (hr)	Sally's Times (hr)	Erin's Times (hr)
Swim	□.□□	0.73	0.54
Bike	1.67	□.□□	1.28
Run	1.38	1.36	□.□□

Rubric: (3 points) The student is able to complete all parts of the table correctly. Each part is independently scored as 1 point.
Mary's swim time: 0.80
Sally's bike time: 1.64
Erin's run time: 0.90

Response Type: Drag and Drop

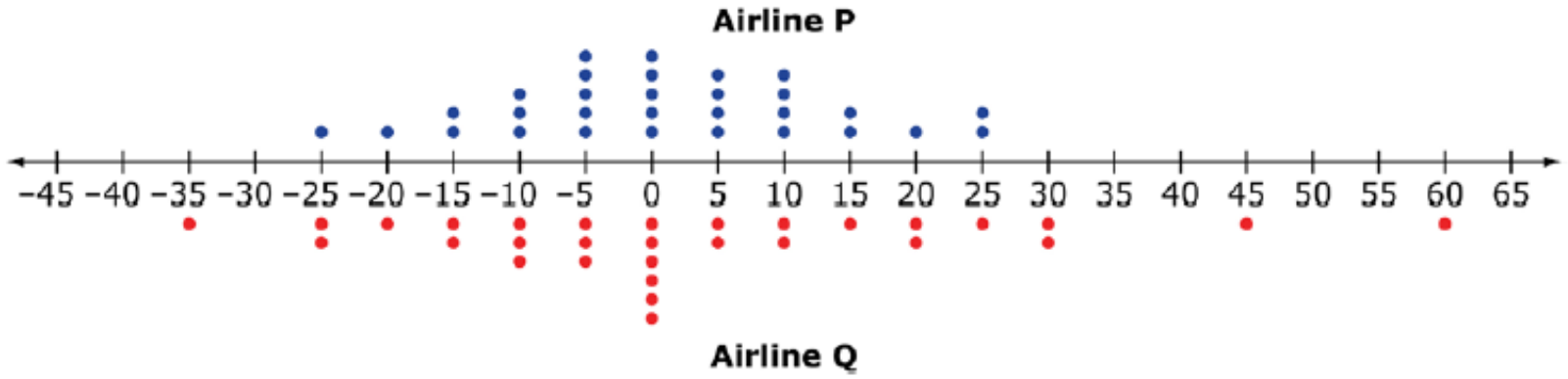
Task Model 2: Target B

Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem. (DOK 2, 3, 4)

At the secondary level, these chains should typically take a successful student 10 minutes to complete. Times will be somewhat shorter for younger students, but still give them time to think and explain. *For a minority of these tasks*, subtasks may be constructed to facilitate entry and assess students' progress toward expertise. Even for such "apprentice tasks," part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes.

HS Example Item for Task Model 2/Target B

The dot plots below compare the number of minutes 30 flights made by two airlines arrived before or after their scheduled arrival times.



- Negative numbers represent the minutes the flight arrived before its scheduled time.
- Positive numbers represent the minutes the flight arrived after its scheduled time.
- Zero indicates the flight arrived at its scheduled time.

Assuming you want to arrive as close to the scheduled time as possible, from which airline should you buy your ticket? Use the ideas of center and spread to justify your choice.

Exemplar: I would buy the ticket from Airline P. Both airlines are likely to have an on-time arrival since they both have median values at 0. However, Airline Q has a much greater range in arrival times. Airline Q could arrive anywhere from 35 minutes early to 60 minutes late. For Airline P, the flights arrived within 10 minutes on either side of the scheduled arrival time about 2/3 of the time, and for Airline Q, that number was only about 1/2. For these reasons, I think Airline P is the better choice.

Task Model 3: Target C

State logical assumptions being used. (DOK 1, 2)

Tasks used to assess this target ask students to use stated assumptions, definitions, and previously established results in developing their reasoning. In some cases, the task may require students to provide missing information by researching or providing a reasoned estimate.

Grade 5 Example of Task Model 3/Target C

Gina is going to a party. She wants to bake 3 cookies for each person at the party. What other information does Gina need to decide the total number of cookies to bake?

A. the number of people going to the party
B. the number of cookies the recipe makes
C. how long the party is going to last
D. how long to bake the cookies

Rubric: (1 point) The student identifies the relevant variable needed (e.g., A).

Task Model 4: Target D

Interpret results in the context of a situation. (DOK 2, 3)

Tasks used to assess this target should ask students to link their answer(s) back to the problem's context. (See Claim 2, Target C for further explication.)

Example Grade 4 Item for Task Model 4/Target D

A group of 137 students and 15 adults go to a museum. The students and adults have to take the elevator up to the 6th floor.

- The elevator can hold a maximum of 12 people.
- At least one adult must ride with each group of students on the elevator.

Part A

What is the **fewest** number of elevator trips it will take to get all of the students and adults to the 6th floor?

Enter your response in the first response box.

Part B

What is the **fewest** number of people on the final elevator trip?

Enter your response in the second response box.

Rubric:

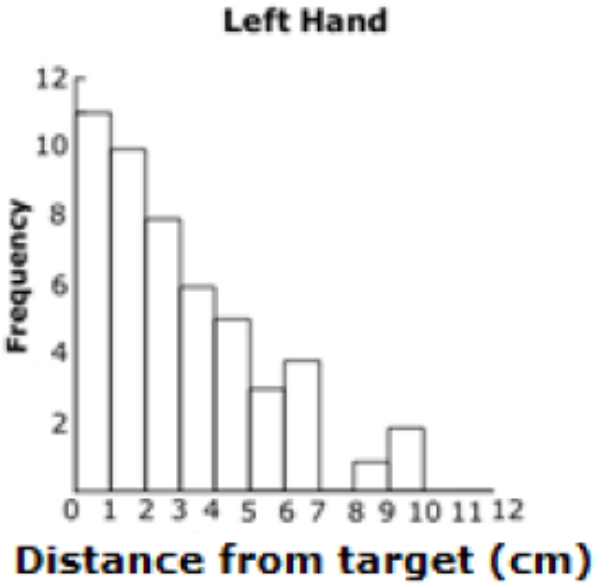
(2 points) The student correctly finds the minimum number of trips (13) and the total number of people on the last elevator (8).

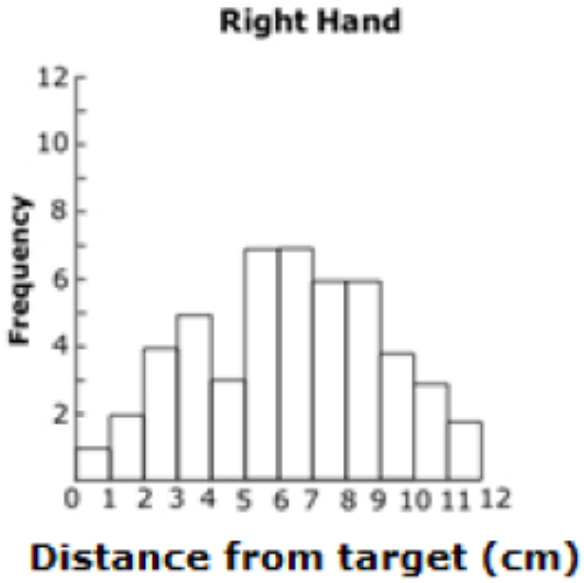
(1 point) The student correctly finds the minimum number of trips OR the total number of people on the last elevator (13 or 8).

(0 points) Any other response

High School Item for Task Model 4/Target D

Lisa was throwing a dart at a target. She threw 50 times with her left hand and 50 times with her right hand. The histograms show the distance Lisa missed the target by each time.





What do the median distances say about Lisa's ability to throw a dart with her right hand compared to her left hand?

High School Item for Target D (continued)

A. Lisa throws better with her left hand because the median for her left hand is greater than the median for her right hand.

B. Lisa throws better with her right hand because the median for her left hand is less than the median for her right hand.

C. Lisa throws better with her left hand because the median for her left hand is less than the median for her right hand.

D. Lisa throws better with her right hand because the median for her left hand is greater than the median for her right hand .

Rubric: (1 point) The student selects the correct option (e.g., C).

Response Type: Multiple choice, single correct response

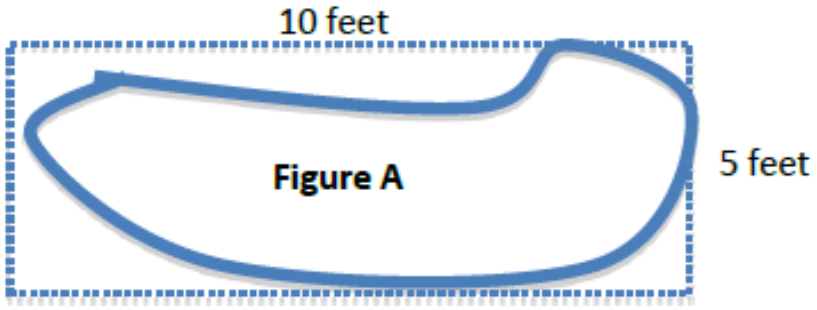
Task Model 5: Target E

Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon. (DOK 3, 4)

Tasks used to assess this target ask students to investigate the efficacy of existing models (e.g., develop a way to analyze the claim that a child's height at age 2 doubled equals his/her adult height) and suggest improvements using their own or provided data. Other tasks for this target will ask students to develop a model for a particular phenomenon (e.g., analyze the rate of global ice melt over the past several decades and predict what this rate might be in the future). Longer constructed-response items and extended performance tasks should be used to assess this target.

Example Grade 4 Item for Task Model 5/Target E

Liam uses a rectangle with length 10 feet and width 5 feet to estimate the area of Figure A.



Which statement correctly describes the **true area** of Figure A?

The true area of Figure A is –

A. less than 25 square feet.
B. greater than 75 square feet.
C. is between 25 square feet and 50 square feet.
D. is between 50 square feet and 75 square feet.

Rubric: (1 point) The student correctly describes the area (e.g., C).

Response Type: Multiple Choice, single correct response

HS Example Item for Task Model 5/Target E

Maia deposits $5500 in a bank account. The money earns interest annually, and the interest is deposited back into her account.

Maia uses an online calculator to determine the amount of money she would have in the bank at the end of each year. Follow these steps to use the calculator.

- Select a number of years in which Maia will have her money deposited in the bank.
- Select "Find Amount."
- The amount of money that Maia will have in her account at the end of the year, up to 12 years, will appear in the table.

You may use the calculator as many times as you need to solve the following problem.

Years	Money in Bank

Enter an equation that models the amount of money, y, Maia would have in the bank at the end of t years.

Rubric: (1 point) The student is able to determine an equation to fit the situation [e.g., $y=5500(1.03)^t$].

Task Model 6: Target F

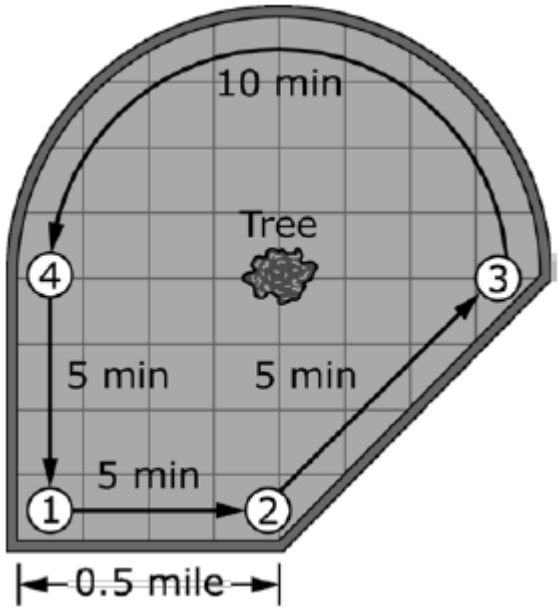
Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)

Unlike Claim 2 where this target might appear as a separate target of assessment (see Claim 2, Target D), it will be embedded in a larger context for items/tasks in Claim 4. The mapping of relationships should be part of the problem posing and solving related to Claim 4 Targets A, B, E, and G.

Grade 8 Example Item for Task Model 6/Target F

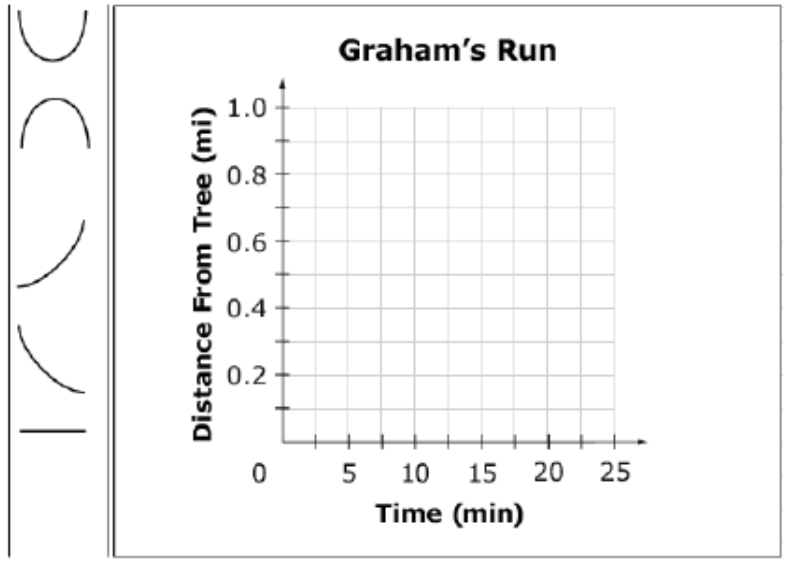
Graham has a running route that follows a fence surrounding a field. The diagram shows four locations along the fence. It also shows the time it takes Graham to run from one location to the next.

- Graham starts his run at location 1, which is about 0.7 mile from a tree in the field.
- His speed varies during his entire run but is constant from one location to the next.
- Graham finishes his run back at location 1.



Use the diagram to construct a graph of Graham's distance from the tree as a function of time. Construct the graph by dragging pieces of the graph to the appropriate location on the coordinate grid.

- Each piece represents a 5-minute interval.
- The pieces of the graph may be used more than once.



Grade 8 Example Item for Target F (continued)

Exemplar:

Rubric:
(2 points) The student constructs the correct graph (see exemplar).
(2 points) The student constructs an incorrect continuous graph that contains the points (0, 0.7) and (25, 0.7).

Response Type: Drag and Drop

Grade 5 Example Item for Task Model 6/Target F

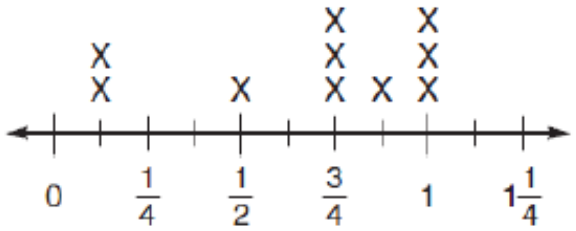
Example Stem (M): Use the 10 data values shown to complete a line plot.

$\frac{7}{8}$, $\frac{3}{4}$, 1, $\frac{3}{4}$, 1, 1, $\frac{1}{8}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{8}$

Complete the line plot that represents the data by clicking above each tick mark to make an X appear.

Interaction: A line plot with fractional measures will be given (see exemplar).

Exemplar:



Rubric: (1 point) The student correctly completes a line plot that displays all 10 data points with no incorrect or missing points.

Response Type: Hot Spot

HS Example Item for Task Model 6/Target F

A technical college offers the following 12 subjects for a 3-year degree, where the length of each class is one year.

Class Code	Class Name
M1	Mechanics Level 1
M2	Mechanics Level 2
E1	Electronics Level 1
E2	Electronics Level 2
B1	Business Studies Level 1
B2	Business Studies Level 2
B3	Business Studies Level 3
C1	Computer Systems Level 1
C2	Computer Systems Level 2
C3	Computer Systems Level 3
T1	Technology & Information Level 1
T2	Technology & Information Level 2

Each student will take 4 classes per year, thus completing 12 subjects in 3 years.

HS Example Item for Task Target F (continued)

A student can only take a class at a higher level if the student has completed the lower level(s) of the same class in a previous year. For example, you can only take Business Studies Level 3 after completing Business Studies Levels 1 and 2.

In addition, Electronics Level 1 can only be taken after completing Mechanics Level 1, and Electronics Level 2 can only be taken after completing Mechanics Level 2.

Decide which classes should be offered for which year, by completing the following table. Enter the class codes in the table.

	Class 1	Class 2	Class 3	Class 4
Year 1				
Year 2				
Year 3				

Exemplar:

	Class 1	Class 2	Class 3	Class 4
Year 1	M1	B1	C1	T1
Year 2	E1	M2	B2	C2
Year 3	C3	E2	T2	B3

Rubric:
(1 point) The student correctly completes the chart as shown in the exemplar.

Response Type: Fill-in Table

Item Quality Criteria

1a. Does the item provide evidence to support the intended claim? Does the item closely align to the claim, target and primary Common Core standard (including cluster level)?

1b. Is this the most appropriate item type to gather evidence to support the target and standard?

1c. Is the item mathematically correct, including its use of precise mathematical language?

1d. Is the item worth asking?

1e. Does the item appear to be accessible to all students? If not, could the item be revised to be made more accessible and still measure the target and standard?

1f. Do the answer choices or rubrics capture the essence of the target and standard?

1g. Is the item/task developmentally appropriate?

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Accessibility and Accommodations

Accessibility and Accommodations Considerations for Claim 4

- Many students with disabilities can analyze and create increasingly complex models of real world phenomena but have difficulty communicating their knowledge and skills in these areas. Students should be able to express their knowledge and skills through the use of assistive technology to construct shapes or develop explanations via speech to text.
- For English learners, the assessment should allow explanations that include diagrams, tables, graphic representations, and other mathematical representations in addition to text.

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Questions

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Reflect on Guiding Questions

- What do educators need to do to support student learning?
- What do educators need to do to ensure that students are prepared for the Smarter Balanced assessments?

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