Claim 2: Problem Solving

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Claim 2: Problem-Solving

"Students can solve a range of complex, well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies."

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Smarter Balanced Cognitive Rigor Matrix

Depth of Thinking (Webb) + Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	- Recall conversions, terms, facts			
Understand	-Evaluate an expression -Locate points on a grid or number on number line -Solve a one-step problem -Represent math relationships in words, pictures, or symbols	- Specify, explain relationships -Make basic inferences or logical predictions from data/observations -Use models /diagrams to explain concepts -Make and explain estimates	-Use concepts to solve non-routine problems -Use supporting evidence to justify conjectures, generalize, or connect ideas -Explain reasoning when more than one response is possible -Explain phenomena in terms of concepts	-Relate mathematical concepts to other content areas, other domains -Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	-Follow simple procedures -Calculate, measure, apply a rule (e.g., rounding) -Apply algorithm or formula -Solve linear equations -Make conversions	-Select a procedure and perform it -Solve routine problem applying multiple concepts or decision points -Retrieve information to solve a problem -Translate between representations	-Design investigation for a specific purpose or research question - Use reasoning, planning, and supporting evidence -Translate between problem & symbolic notation when not a direct translation	-Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	-Retrieve information from a table or graph to answer a question -Identify a pattern/trend	-Categorize data, figures -Organize, order data -Select appropriate graph and organize & display data -Interpret data from a simple graph -Extend a pattern	-Compare information within or across data sets or texts -Analyze and draw conclusions from data, citing evidence -Generalize a pattern -Interpret data from complex graph	-Analyze multiple sources of evidence or data sets
Evaluate			-Cite evidence and develop a logical argument -Compare/contrast solution methods -Verify reasonableness	-Apply understanding in a novel way, provide argument or justification for the new application
Create	- Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	-Generate conjectures or hypotheses based on observations or prior knowledge and experience	-Develop an alternative solution -Synthesize information within one data set	-Synthesize information across multiple sources or data sets -Design a model to inform and solve a practical or abstract situation

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Rationale for Claim 2

- Assessment items and tasks focused on Claim 2 include problems in pure mathematics and problems set in context.
- Problems are presented as items and tasks that are well-posed (that is, problem formulation is not necessary) and for which a solution path is not immediately obvious.
- These problems require students to construct their own solution pathway rather than follow a provided one. Such problems will therefore be less structured than items and tasks presented under Claim 1. Students will need to select appropriate conceptual and physical tools to use.

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Targets for Claim 2

Target A: Apply mathematics to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace. (DOK 2, 3)

Target B: Select and use appropriate tools strategically. (DOK 1, 2)

Target C: Interpret results in the context of a situation. (DOK 2)

Target D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)

Claim 2 Content Standards by Grade Level						
Grade 3	**Grade 4**	**Grade 5**	**Grade 6**	**Grade 7**	**Grade 8**	**High School**
3.OA.A	4.OA.A	5.NBT.B	6.RP.A	7.RP.A	8.EE.B	N-Q.A
3.OA.D	4.NBT.B	5.NF.A	6.NS.A	7.NS.A	8.EE.C	A-SSE.A
3.NBT.A*	4.NF.A	5.NF.B	6.NS.C	7.EE.A	8.F.A	A-SSE.B
3.NF.A	4.NF.B	5.MD.A*	6.EE.A	7.EE.B	8.F.B*	A-CED.A
3.MD.A	4.NF.C	5.MD.C	6.EE.B	7.G.A*	8.G.A	A-REI.2
3.MD.B*	4.MD.A*	5.G.A*	6.EE.C	7.G.B*	8.G.B	A-REI.B
3.MD.C	4.MD.C*		6.G.A*		8.G.C*	A-REI.C
3.MD.D*						A-REI.D
						F-IF.A
						F-IF.B
						F-IF.C
						F-BF.A
						G-SRT.C
						S-ID.C
						S-CP.A

* Denotes additional and supporting clusters

Essential Properties of Claim 2 Items/Tasks

- For the computer-adaptive (CAT) portion of the summative assessment, Claim 2 will be assessed using a combination of :
  - multiple choice, single correct response; multiple choice, multiple correct response; matching tables; equation/numeric; graphing; and fill-in table items/tasks that focus on making sense of problems and using perseverance in solving them.
- To preserve the focus and coherence of the standards as a whole, Claim 2 items/tasks must draw clearly on knowledge and skills articulated in the progression of standards up to and including that grade.

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Essential Properties of Claim 2 Items/Tasks

- Each of the targets should not lead to a separate item/task
- Items and tasks should provide evidence for several of the assessment targets defined for Claim 2.
- It is in *using* content from different areas, including work studied in earlier grades, that students demonstrate their problem-solving proficiency.

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Mathematical Practices

- The evidence required of students to satisfy Claim 2 centers around specific statements of the ***mathematical practices*** (MP) contained in the CCSSM. Though not exclusive, MP1, MP5, MP7, and MP8 are particularly relevant for Claim 2 items.

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MP1: Make sense of problems and persevere in solving them.

Mathematically proficient students:

- explain to themselves the meaning of a problem and look for entry points to its solution.
- analyze givens, constraints, relationships, and goals
- make conjectures about the form and meaning of the solution attempt.
- plan a solution pathway rather than simply jumping into a solution.
- consider analogous problems and try special cases and simpler forms of insight into the solutions.
- monitor and evaluate their progress and change course if necessary.
- transform algebraic expressions or change the viewing window on their graphing calculator to get information.
- explain correspondences between equations, verbal descriptions, tables, and graphs.
- draw diagrams of important features and relationships, graph data, and search for regularity or trends.
- use concrete objects or pictures to help conceptualize and solve a problem.
- check their answers to problems using a different method**.**
- ask themselves, “Does this make sense?”
- understand the approaches of others in solving complex problems and identify correspondences between approaches.

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MP5: Use appropriate tools strategically.

Mathematically proficient students:

- consider available tools when solving a mathematical problem. (Tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software.)
- are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
- detect possible errors by using estimations and other mathematical knowledge.

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MP7: Look for and make use of structure.

Mathematically proficient students:

- look closely to discern a pattern or structure.
- Young students might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have.
- Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for the distributive property.
- In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.
- step back for an overview and can shift perspective.
- see complicated things, such as some algebraic expressions, as single objects or composed of several objects.

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MP8: Look for and express regularity in repeated reasoning.

Mathematically proficient students:

- notice if calculations are repeated.
- look for both general methods and shortcuts.
- Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations and conclude they have a repeated decimal.
- Middle school students might abstract the equation (y-2)/(x-1)=3 by paying attention to the calculation of slope as they repeatedly check whether the points are on the line through (1, 2) with a slope 3.
- maintain oversight of the process of solving a problem, while attending to the details.
- continually evaluate the reasonableness of intermediate results.

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Claim 1 Targets versus Claims 2, 3 and 4 Targets

- Claim 1 targets are grade specific.
- For Claims 2,3 and 4 the same targets are included across all grades

Distinguishing Between Claims 2 and 4

- In early grades when equations are still new to students, an important distinction between Claim 2 and Claim 4 is requiring a model that would lead to a problem's solution.
- In Claim 2 problems are well posed, while in Claim 4 they may have extraneous or missing information.
- In Claim 2, measurements of objects or figures can be accurately determined. In Claim 4, modeling is used to make approximations.

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The Mathematics Assessment

Sample Claim 2 Items

Task Model 1: Target A

Apply mathematics to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace.

Under claim 2, the problems should be completely formulated, and students should be asked to find a solution path from among their readily available tools.

- Mathematical information is presented in a table or graph or extracted from a context.

Grade 4 Example of Task Model 1/Target A

Tina and Marco play a number game. Tina gives Marco a number and he does three computations.

- He multiplies the number by 2.
- He adds 7 to the answer.
- Then, he subtracts 2 from that answer.

What number should Tina give Marco so that the final answer is 37?

Exemplar: 16

Rubric: (1 point) The student enters the correct number.

Grade 8 Example Item for Task Model 1/Target A

Two sides of a right triangle have lengths $\sqrt{10}$ units and $\sqrt{6}$ units. There are two possible lengths for the third side.

Enter the longest possible side length, in units, for the third side of this triangle.

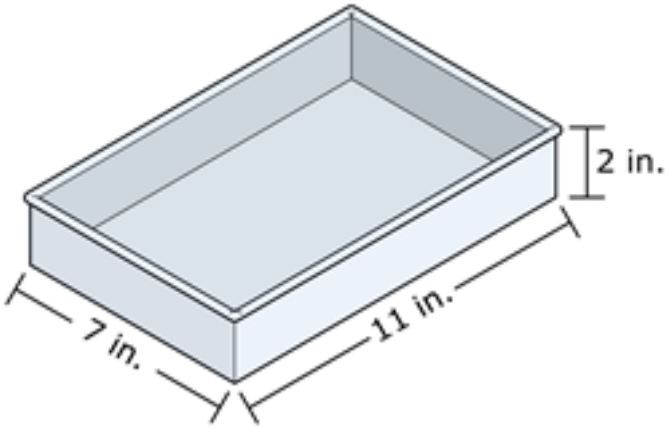
Exemplar: 4

(1 point) The student correctly enters the longest side of the triangle.

Response Type: Equation/numeric

Example High School Item for Task Model 1/Target A

Hannah makes 6 cups of cake batter. She pours and levels all the batter into a rectangular cake pan with a length of 11 inches, a width of 7 inches, and a depth of 2 inches.



One cubic inch is approximately equal to 0.069 cup.

What is the **depth** of the batter in the pan when it is completely poured into the pan? Round your answer to the nearest $\frac{1}{8}$ inch.

Exemplar: $1\frac{1}{8}$

Rubric: (1 point) Student correctly determines the depth.

Task Model 2: Target B

Select and use appropriate tools strategically. (DOK 1, 2)

Tasks used to assess this target should allow students to find and choose tools; for example using a protractor in physical space.

- Mathematical information is presented in a table or graph or extracted from a context.
- The student is asked to solve a problem that requires strategic use of tools or formulas.

Example Grade 5 Item for Task Model 2/Target B

Shelbi builds a platform with wooden boards. Each board is 100 millimeters wide.

Shelbi places the boards side by side to build the platform.

The platform has a total width of 12 meters. Enter the **fewest** number of boards that Shelbi needs to build the platform.

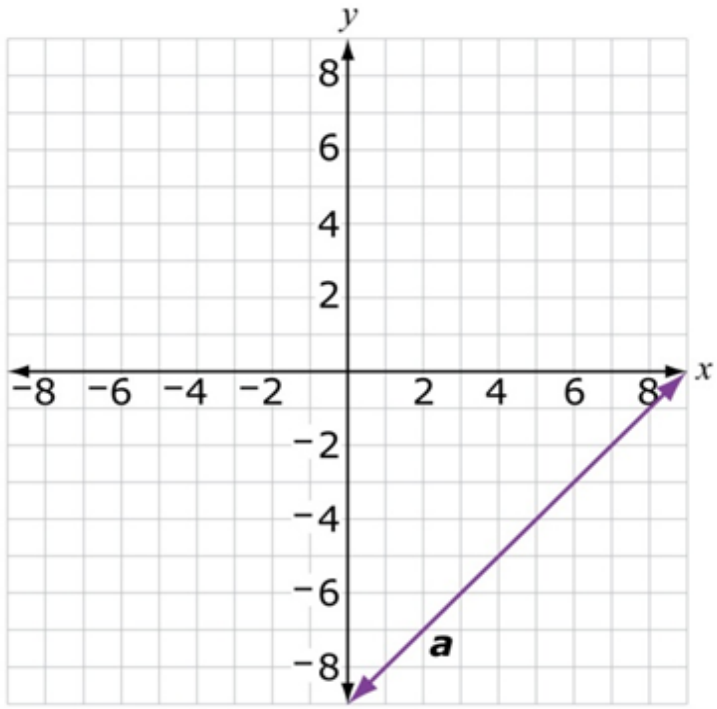
Exemplar: 120

Rubric: (1 point) Student enters the correct number.

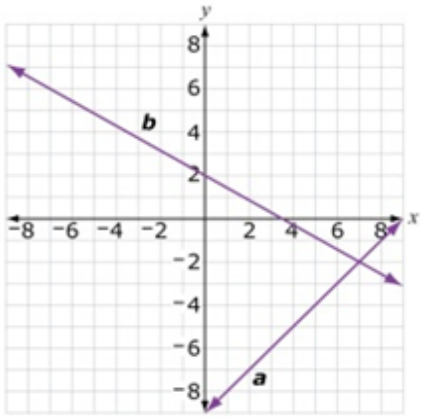
Grade 8 Example Item for Task Model 2/Target B

Line *a* is shown on the graph below. Use the Add Arrow tool to construct line *b* on the graph so that:

- Line *a* and line *b* represent a system of linear equations with a solution of (7, -2).
- The slope of line *b* is greater than -1 and less than 0.
- The *y*-intercept of line *b* is positive.



Exemplar:



Rubric: (1 point) The student is able to construct a line that meets the requirements.

Task Model 3: Target C

Interpret results in the context of a situation. (DOK 2)

Tasks used to assess this target should ask students to link their answer(s) back to the problem's context. In early grades, this might include a judgment by the student of whether to express an answer to a division problem using a remainder or not based on the problem's context. In later grades, this might include a rationalization for the domain of a function being limited to positive integers based on a problem's context (e.g., understanding that the number of buses required for a given situation cannot be 32½, or that the negative values for the independent variable in a quadratic function modeling a basketball shot have no meaning in this context).

Grade 5 Example of Task Model 3/Target C

Carl feeds his dog $2\frac{1}{2}$ cups of dog food every day. Each bag contains 64 cups of dog food. What is the maximum number of days that Carl can feed his dog exactly $2\frac{1}{2}$ cups of dog food from one full bag?

Exemplar: 25

Rubric: (1 point) The student is able to determine the maximum number of servings from one bag of food and interpret the remainder as not being enough for another serving.

Grade 7 Example Item for Task Model 3/Target C

This table shows the monthly account changes in Sara's account balance. For example, the account change of +$38 means that Sara's balance was $38 more at the end of the last day of January than at the beginning of first day in January.

Month	Account Change
January	+$38
February	−$30
March	−$19
April	+$49

Determine whether each statement about Sara's bank account balance is true based on the table. Select True or False for each statement.

Statement	True	False
Sara has less money in her account at the end of February than at the end of any other month.		
Sara's account balance is the same at the end of April as it is at the end of January.		
Sara has more money in her account at the end of April than she had at the **beginning** of January.		

Key: FTT

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Task Model 4: Target D

Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)

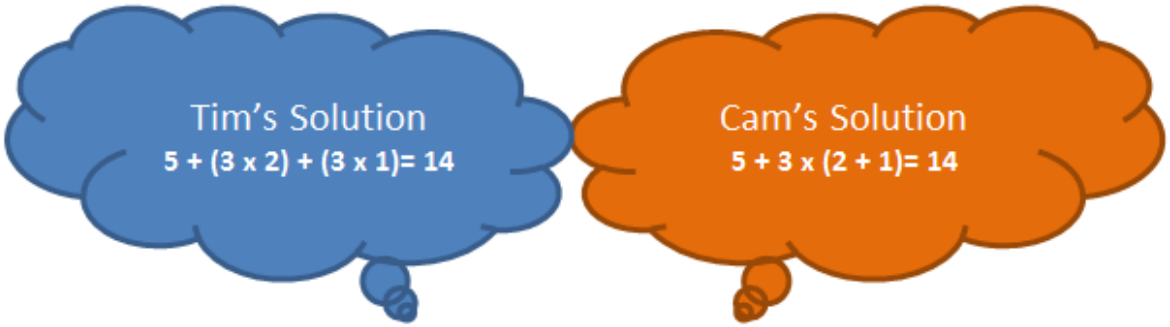
For Claim 2 tasks, this may be a separate target of assessment explicitly asking students to use one or more potential mappings to understand the relationship between quantities. In some cases, item stems might suggest ways of mapping relationships to scaffold a problem for Claim 2 evidence.

Example Grade 4 Item for Task Model 4/Target D

Tim and Cam made posters for art class. They each used 1 poster board, 3 markers, and 3 feet of ribbon. The table shows the cost of their supplies.

Supplies	Cost
1 Poster board	$5
3 Markers	$2 + $2 + $2
3 feet Ribbon	$1 + $1 + $1

They each figured out how much the supplies cost in a different way.



Which of these explains why they got the same result?

A. $(3 \times 2) + (3 \times 1) = 3 \times (2 + 1)$*

B. $5 + (3 \times 2) = 3 \times (2 + 1)$

C. $(3 \times 2) = 3 \times (2 + 1)$

D. $3 \times (5 + 1 + 1) = 5 + 3 \times (2 + 1)$

Example Grade 7 Item for Task Model 4/Target D

(Source: Adapted from Illustrative Mathematics, Grade 7.EE)

The students in Mr. Sanchez's class are converting distances measured in miles (m) to kilometers (k). Abby and Renato use the following methods to convert miles to kilometers.

- Abby takes the number of miles, doubles it, then subtracts 20% of the result.
- Renato first divides the number of miles by 5, then multiplies the result by 8.

Which equation correctly shows why both their methods produce the same result?

A. $2m - 0.20 = \frac{m}{5} \bullet 8$

B. $2m - 0.20(2m) = \frac{m}{5} \bullet 8$

C. $2m - 2.20m = \frac{m}{5} + 8\left(\frac{m}{5}\right)$

D. $0.20(2m) - 2m = \frac{m}{5} + 8\left(\frac{m}{5}\right)$

High School Item for Task Model 4/Target D

Consider triangle *ABC*, where angle *C* is a right angle.

Drag all measures shown for angle *A* into the correct column.

Angle *A*	cos *A* < sin *A*	cos *A* = sin *A*	cos *A* > sin *A*
10°			
65°			
45°			
30°			
85°			
70°			

Rubric: (1 point) The student correctly classifies all angles (e.g., see below).

Angle *A*	cos *A* < sin *A*	cos *A* = sin *A*	cos *A* > sin *A*
10°	65°	45°	30°
65°	85°		10°
45°	70°		
30°			
85°			
70°			

Response Type: Drag and Drop

Item Quality Criteria

1a. Does the item provide evidence to support the intended claim? Does the item closely align to the claim, target and primary Common Core standard (including cluster level)?

1b. Is this the most appropriate item type to gather evidence to support the target and standard?

1c. Is the item mathematically correct, including its use of precise mathematical language?

1d. Is the item worth asking?

1e. Does the item appear to be accessible to all students? If not, could the item be revised to be made more accessible and still measure the target and standard?

1f. Do the answer choices or rubrics capture the essence of the target and standard?

1g. Is the item/task developmentally appropriate?

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Accessibility and Accommodations

Accessibility and Accommodations Considerations for Claim 2

- Math is a visual discipline and Claim 2 items can become very reading intensive. Item writers should be cognizant of the needs of students with reading difficulties, English learners, and students with visual impairments.
- Avoid vocabulary that is likely to be unfamiliar to many students.
- Avoid complex grammatical syntax (refer to the ideas in the language complexity training).
- Use graphics only when necessary and keep graphics simple and straightforward.

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Questions

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Reflect on Guiding Questions

- What do educators need to do to support student learning?
- What do educators need to do to ensure that students are prepared for the Smarter Balanced assessments?

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