Crosswalk Activity

Process Standards

- ***Problem Solving -*** The process of applying a variety of appropriate strategies based on information provided, referenced, recalled, or developed.
- ***Reasoning and Proof –*** Making and investigating mathematical conjectures. Developing arguments and proofs.
- ***Communication –*** Organizing mathematical thinking coherently and clearly to peers, teachers and others. Using the language of math to express mathematical ideas precisely.
- ***Representation –*** Creating and using multiple representations to organize, record, and communicate mathematical ideas. Using models and interpreting mathematical phenomena.
- ***Connections –*** Recognizing and using connections among math ideas as well as other subjects. Understanding how mathematical ideas interconnect and build on one another.

Strands of Mathematical Proficiency

- ***Conceptual Understanding*** – comprehension of mathematical concepts, operations, and relations
- ***Procedural Fluency*** – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- ***Strategic Competence*** – ability to formulate, represent, and solve mathematical problems
- ***Adaptive Reasoning*** – capacity for logical thought, reflection, explanation, and justification
- ***Productive Disposition*** – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

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strategic competence includes formulating, representing and solving problems.

productive disposition—since that is different from what is explicitly stated in the NCTM Process Standards and is very important for practices.

2

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions,

communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

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"Going Deeper" with the Standards of Mathematical Practice

Standard of Practice	Letter	How  demonstrates the Standard!
1. Make sense of problems and persevere in solving them.		
2. Reason abstractly and quantitatively.		
3. Construct viable arguments and critique the reasoning of others.		
4. Model with mathematics.		
5. Use appropriate tools strategically.		
6. Attend to precision.		
7. Look for and make use of structure.		
8. Look for and express regularity in repeated reasoning.		

"Going Deeper" with the Standards of Mathematical Practice

Standard of Practice	Letter	How  demonstrates the Standard!
1. Make sense of problems and persevere in solving them.	F Math Problem	Raj took an aspirin and went right back to solving the problem; demonstrating his perseverance.
2. Reason abstractly and quantitatively.	H Superman	They are taking a situation in a movie and represented symbolically through physics. They did pause during the manipulation of the math to probe the accuracy of the theories being presented.
3. Construct viable arguments and critique the reasoning of others.	G Loop vs. String	There was a case presented for both series as well as counterexamples for both. They responded to the validity of each explanation. Leonard listed to both sides and made a decision based on the facts.
4. Model with mathematics.	E Do the Math	Sheldon took the "real world" situation of the odds of two people from different backgrounds being compatible and wanted to do the math of the probability. He took a complicated situation and applied what he has comfortable with to solve the problem.
5. Use appropriate tools strategically.	C PowerPoint	Sheldon used a PowerPoint to convey why he should go to Switzerland over Penny.
6. Attend to precision.	B Meet Hawking	Sheldon failed to attend to precision before giving the final copy to somebody else.
7. Look for and make use of structure.	A Friendship Alogorithm	Sheldon took a complicated thing like friendship and broke it down into single objects in a flow chart. Howard noticed the infinite loop so he provided Sheldon with a way to step back and shift his perspective in order to solve the problem.
8. Look for and express regularity in repeated reasoning.	D Best Number	Sheldon took the number 73 and broke it down in several way but found repeated calculations. He evaluated the reasonableness of all of his calculations to prove that it was the best number.

Evidence of the Math Practice Standards

MP	Teacher	Student
1		
2		
3		
4		
5		
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7		
8		

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K-12 Standards for Mathema l Practice Classroom Guide

Mathematics Practices		Student Illustrations:	Teacher Illustrations:
Overarching habits of mind of a productive math thinker	1. Make sense of problems and persevere in solving them	•	•
	6. Attend to precision	•	•
Reasoning and Explaining	2. Reason abstractly and quantitatively	•	•
	3. Construct viable arguments and critique the reasoning of others	•	•

Adapted from: Elementary Mathematics Specialists & Teacher Leaders Project 2012 Engaging in the Mathematical Practices (Look—fors)

Each practice may not be evident during every lesson.

K-12 Standards for Mathem[illegible] Practice Classroom Guide

Mathematics Practices		Student Illustrations:	Teacher Illustrations:
Modeling and Using Tools	**4. Model with mathematics**	•	•
	5. Use appropriate tools strategically	•	•
Seeing structure and generalizing	**7. Look for and make use of structure**	•	•
	8. Look for and express regularity in repeated reasoning	•	•

Adapted from: Elementary Mathematics Specialists & Teacher Leaders Project 2012 Engaging in the Mathematical Practices (Look—fors)

Each practice may not be evident during every lesson.

K-12 Standards for Mathema l Practice Classroom Guide

Mathematics Practices		Student Illustrations:	Teacher Illustrations:
Overarching habits of mind of a productive math thinker	**1. Make sense of problems and persevere in solving them**	• Understand the meaning of the problem and look for entry points to its solution • Analyze information (givens, constrains, relationships, goals) • Make conjectures and plan a solution pathway • Monitor and evaluate the progress and change course as necessary • Check answers to problems and ask, "Does this make sense?"	• Involve students in rich problem---based tasks that encourage them to persevere in order to reach a solution • Provide opportunities for students to solve problems that have multiple solutions • Encourage students to represent their thinking while problem solving
	6. Attend to precision	• Communicate precisely using clear definitions • State the meaning of symbols, carefully specifying units of measure, and providing accurate labels • Calculate accurately and efficiently, expressing numerical answers with a degree of precision • Provide carefully formulated explanations • Label accurately when measuring and graphing	• Emphasize the importance of precise communication by encouraging students to focus on clarity of the definitions, notation, and vocabulary used to convey their reasoning • Encourage accuracy and efficiency in computation and problem---based solutions, expressing numerical answers, data, and/or measurements with a degree of precision appropriate for the context of the problem
Reasoning and Explaining	**2. Reason abstractly and quantitatively**	• Make sense of quantities and relationships in problem situations • Represent abstract situations symbolically and understand the meaning of quantities • Create a coherent representation of the problem at hand • Consider the units involved • Flexibly use properties of operations	• Facilitate opportunities for students to discuss or use representations to make sense of quantities and their relationships • Encourage the flexible use of properties of operations, objects, and solution strategies when solving problems • Provide opportunities for students to decontextualize (abstract a situation) and/or contextualize (identify referents for symbols involved) the mathematics they are learning
	3. Construct viable arguments and critique the reasoning of others	• Use definitions and previously established causes/effects (results) in constructing arguments • Make conjectures and use counterexamples to build a logical progression of statements to explore and support ideas • Communicate and defend mathematical reasoning using objects, drawings, diagrams, and/or actions • Listen to or read the arguments of others • Decide if the arguments of others make sense and ask probing questions to clarify or improve the arguments	• Provide and orchestrate opportunities for students to listen to the solution strategies of others, discuss alternative solutions, and defend their ideas • Ask higher---order questions which encourage students to defend their ideas • Provide prompts that encourage students to think critically about the mathematics they are learning

Adapted from: Elementary Mathematics Specialists & Teacher Leaders Project 2012 Engaging in the Mathematical Practices (Look—fors)

Each practice may not be evident during every lesson.

K-12 Standards for Mathematical Practice Classroom Guide

Mathematics Practices		Student Illustrations:	Teacher Illustrations:
Modeling and Using Tools	**4. Model with mathematics**	• Apply prior knowledge to solve real world problems • Identify important quantities and map their relationships using such tools as diagrams, two---way tables, graphs, flowcharts, and/or formulas • Use assumptions and approximations to make a problem simpler • Check to see if an answer makes sense within the context of a situation and change a model when necessary	• Use mathematical models appropriate for the focus of the lesson • Encourage student use of developmentally and content---appropriate mathematical models (e.g., variables, equations, coordinate grids) • Remind students that a mathematical model used to represent a problem's solution is 'a work in progress,' and may be revised as needed
	5. Use appropriate tools strategically	• Make sound decisions about the use of specific tools (Examples might include: calculator, concrete models, digital technologies, pencil/paper, ruler, compass, protractor) • Use technological tools to visualize the results of assumptions, explore consequences, and compare predications with data • Identify relevant external math resources (digital content on a website) and use them to pose or solve problems • Use technological tools to explore and deepen understanding of concepts	• Use appropriate physical and/or digital tools to represent, explore and deepen student understanding • Help students make sound decisions concerning the use of specific tools appropriate for the grade level and content focus of the lesson • Provide access to materials, models, tools and/or technology---based resources that assist students in making conjectures necessary for solving problems
Seeing structure and generalizing	**7. Look for and make use of structure**	• Look for patterns or structure, recognizing that quantities can be represented in different ways • Recognize the significance in concepts and models and use the patterns or structure for solving related problems • View complicated quantities both as single objects or compositions of several objects and use operations to make sense of problems	• Engage students in discussions emphasizing relationships between particular topics within a content domain or across content domains • Recognize that they quantitative relationships modeled by operations and their properties remain important regardless of the operational focus of a lesson • Provide activities in which students demonstrate their flexibility in representing mathematics in a number of ways • e.g., 76 = (7 x 10) + 6; discussing types of quadrilaterals, etc.
	8. Look for and express regularity in repeated reasoning	• Notice repeated calculations and look for general methods and shortcuts • Continually evaluate the reasonableness of intermediate results (comparing estimates), while attending to details, and make generalizations based on findings	• Engage students in discussion related to repeated reasoning that may occur in a problem's solution • Draw attention to the prerequisite steps necessary to consider when solving a problem • Urge students to continually evaluate the reasonableness of their results

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Targets for Claims 2, 3 and 4

Claim 2:

Target A: Apply mathematics to solve well-posed problems in pure mathematics and those arising in everyday life, society, and the workplace.

Target B: Select and use appropriate tools strategically.

Target C: Interpret results in the context of a situation.

Target D: Identify important quantities in a practical situation and map their relationships (e.g. using diagrams, two-way tables, graphs, flowcharts, or formulas).

Claim 3:

Target A: Test propositions or conjectures with specific examples.

Target B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.

Target C: State logical assumptions being used.

Target D: Use the technique of breaking an argument into cases.

Target E: Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument—explain what it is.

Target F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions.

Target G: At later grades, determine conditions under which an argument does and does not apply.

Claim 4:

Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.

Target B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.

Target C: State logical assumptions being used.

Target D: Interpret results in the context of a situation.

Target E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.

Target F: Identify important quantities in a practical situation and map their relationships (e.g. using diagrams, two-way tables, graphs, flowcharts, or formulas).

Target G: Identify, analyze and synthesize relevant external resources to pose or solve problems.

Smarter Balanced Math Claims

Claim #1 Concepts & Procedures	Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.
Claim #2 Problem Solving	Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
Claim #3 Communication Reasoning	Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Claim #4 Modeling and Data Analysis	Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

Assessment Targets: Targets provide more detail about the range of content and Depth of Knowledge levels. In claim 1, targets correspond to cluster headings from NVACS.

*Claim 1 directly connects to domains and clusters of the grade level standards
*Claim 2 connects with Math Practices 1, 5, 7 & 8
*Claim 3 connects with Math Practices 3 & 6
*Claim 4 connects with Math Practices 2, 4 & 5

Mathematical Practices

1.	Make sense of problems and persevere in solving them
2.	Reason abstractly and quantitatively
3.	Construct viable arguments and critique the reasoning of others
4.	Model with mathematics
5.	Use appropriate tools strategically
6.	Attend to precision
7.	Look for and make use of structure
8.	Look for and express regularity in repeated reasoning

Prepared by the Department of Assessment, WCSD
Printed for A&S Meeting, 14 Nov 2014

MATH K–8 INSTRUCTIONAL PRACTICE GUIDE

Name: Date: Observer Name:

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8

CORE ACTION 3: Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson.[4]

INDICATORS[5 6] / NOTE EVIDENCE OBSERVED OR GATHERED FOR EACH INDICATOR / RATING

4- Teacher provides many opportunities, and most students take them.
3- Teacher provides many opportunities, and some students take them; or teacher provides some opportunities and most students take them.
2- Teacher provides some opportunities, and some students take them.
1- Teacher provides few or no opportunities, or few or very few students take the opportunities provided.

Indicator	Evidence	Rating
A. The **teacher** provides opportunities for all students to work with and practice grade-level problems and exercises. **Students** work with and practice grade-level problems and exercises.		4 3 2 1 ☐ NOT OBSERVED
B. The teacher cultivates reasoning and problem solving by allowing students to productively struggle. Students persevere in solving problems in the face of difficulty.		4 3 2 1 ☐ NOT OBSERVED
C. The teacher poses questions and problems that prompt students to explain their thinking about the content of the lesson. **Students** share their thinking about the content of the lesson beyond just stating answers.		4 3 2 1 ☐ NOT OBSERVED
D. The **teacher** creates the conditions for student conversations where students are encouraged to talk about each other's thinking. **Students** talk and ask questions about each other's thinking, in order to clarify or improve their own mathematical understanding.		4 3 2 1 ☐ NOT OBSERVED
E. The **teacher** connects and develops students' informal language and mathematical ideas to precise mathematical language and ideas. **Students** use increasingly precise mathematical language and ideas.		4 3 2 1 ☐ NOT OBSERVED

If any uncorrected mathematical errors are made during the context of the lesson (instruction, materials, or classroom displays), note them here.

4. There is not a one-to-one correspondence between the indicators for this Core Action and the Standards for Mathematical Practice. These indicators represent the Standards for Mathematical Practice that are most easily observed during instruction.
5. Some portions adapted from 'Looking for Standards in the Mathematics Classroom' 5x8 card published by the Strategic Education Research Partnership (http://math.serpmedia.org/5x8card/).
6. Some or most of the indicators and student behaviors should be observable in every lesson, though not all will be evident in all lessons. For more information on teaching practices, see NCTM's publication Principles to Actions: Ensuring Mathematical Success for All for eight Mathematics Teaching Practices listed under the principle of Teaching and Learning (http://www.nctm.org/principlestoactions).

Indicator	Evidence
	referenced during the summary. • The teacher's use of turn and talk without a whole group discussion of the key idea, risks that not all students walk away with the same understanding of the mathematics of the lesson.
Core Action 3: Provide all students opportunities to exhibit mathematical practices while engaging with the content of the lesson.	
A. The teacher provides opportunities for all students to work with and practice grade-level problems and exercises. Students work with and practice grade-level problems and exercises.	• All students are completing grade-level exercises/problems. • The students are given opportunities to write algebraic expressions consistent with the expectations of 6.EE.A.2. They also work on generating equivalent expressions using the properties of operations (6.EE.A.3) and some students exhibit partial understanding of the skills articulated in 6.EE.A.4 showing that substituting a particular value into different forms of an expression yields the same value if the expressions are equivalent.
B. The teacher cultivates reasoning and problem solving by allowing students to productively struggle. Students persevere in solving problems in the face of difficulty.	• In an interaction with a student at ~9:14, the teacher encourages the student to "make an expression, just put it on the paper and check it" which encourages risk taking and perseverance. • At ~19:36, a student seems to be struggling. The teacher's response helped the student and her table get the correct answer rather than encouraging perseverance. • At ~28:15, the teacher calls on a specific student to "explain what I would do to simplify." The student responds, "It's hard because it's, like, variables." The teacher then moves on to call on a stronger student to answer the question.
C. The teacher poses questions and problems that prompt students to explain their thinking about the content of the lesson. Students share their thinking about the content of the lesson beyond just stating answers.	• In the first part of the lesson, the toothpick problem was presented which encouraged the students to share their developing thinking. At ~3:27, the teacher says, "Please, talk to each other." • At ~16:40, teacher asks, "How do you know?" and "Give us an example of how [2 times n and n times n] are different?" At ~ 25:50, a student explains how to use two different values for the variable in order to show that two expressions are equivalent. • In small groups, the teacher encourages many students explain their thinking (e.g., ~17:28 "Does that makes sense? Can you explain it back to me? ... Let's have Amber explain it to [you] one more time." ~ 17:44: "Let's talk to the group.")

Indicator	Evidence
	• Throughout the rest of the lesson the teacher asks a lot of yes/no questions like: "Does this make sense? Can you see this? Do you have the right expression?" These did not invite students to share their developing thinking and there was little evidence of students responding by sharing their thinking about mathematical ideas.
D. The teacher creates the conditions for student conversations where students are encouraged to talk about each other's thinking. Students talk and ask questions about each other's thinking, in order to clarify or improve their own mathematical understanding.	• At ~ 5:27, the teacher directs students to have conversations about their work as they create their descriptions and tables. Students seem to be asking questions of each other at their tables. • At ~7:45, the teacher says, "You started at 5, and, Denise, it looks like you were maybe starting at 1. Is that OK? Why is it OK?" • "At ~16:13, the teacher asks a table of students whether #2 is equal to the target expression. She points out that students at the table have different answers and says, "Which is right? I don't know." She walks away from the table and students use an example of substitution to justify their reasoning. • At ~19:47, the teacher asks a student whether a given expression is equivalent to the target expression. When the student says no, the teacher follows up by asking, "Why not?" Two students discuss how they can use collecting like terms to prove the equivalence, with some support from the teacher. • At ~ 21:50, there are two students excitedly discussing their solution methods. In contrast, when individual students are sharing their answers with the whole group, there isn't a lot of interaction with the student presenting. • At ~ 11:45, a student poses a question to clarify his understanding of the expression another student wrote 2s + 2 ("Don't you have to use the number of toothpicks in it?"). The student presenting her work responds but doesn't seem to understand her question, so the teacher jumps in to clarify.
E. The teacher connects and develops students' informal language and mathematical ideas to precise mathematical language and ideas. Students use increasingly precise mathematical language and ideas.	• The teacher makes an effort to use precise language to refer to the properties and algebraic terms (e.g., ~6:09, ~13:08). • When the teacher describes the task at ~4:42, she says, "I'm going to ask you to create an algebraic expression, complete the table and write a description in words of the pattern." In this direction, she is not precise about the fact that all of these representations show the number of toothpicks in a given shape. This lack of precision in defining variables and the meaning of

Indicator	Evidence
	expressions is reflected in the way that students talk about their work. • At ~12:58, a student uses the precise language for expressions/equations when she suggests that what is written in the header should be "algebraic expressions or equations." • At ~18:20, the teacher connects the distributive property to the game Angry Birds, "So, remember if we distribute this Angry Bird to this pig, that's Angry Bird times pig... So, this Angry Bird to this Angry Bird, Angry Bird times Angry Bird."

If any uncorrected mathematical errors are made during the context of the lesson (instruction, materials, or classroom displays), note them here:

- There are issues of mathematical correctness and precision. At ~13:58, the teacher says "s is for shape" instead of precisely stating the meaning of the variable (e.g., "s represents the stage number").