Smarter Balanced Assessment System: Connecting the Mathematics Claims to Classroom Instruction Grades 6-12

Connecticut State Department of Education
Fall 2014

Food for Thought

Learning Targets

- I understand the types of learning activities and tasks that will support student mastery of the Standards for Mathematics Content and develop student expertise with the Standards for Mathematical Practice.
- I understand how my classroom instruction supports student learning.
- I understand how classroom activities and tasks align with the summative assessment.

Success Criteria

- I can explain how the Connecticut Core Standards (CCS) for Mathematics connect to the Smarter Balanced Claims.

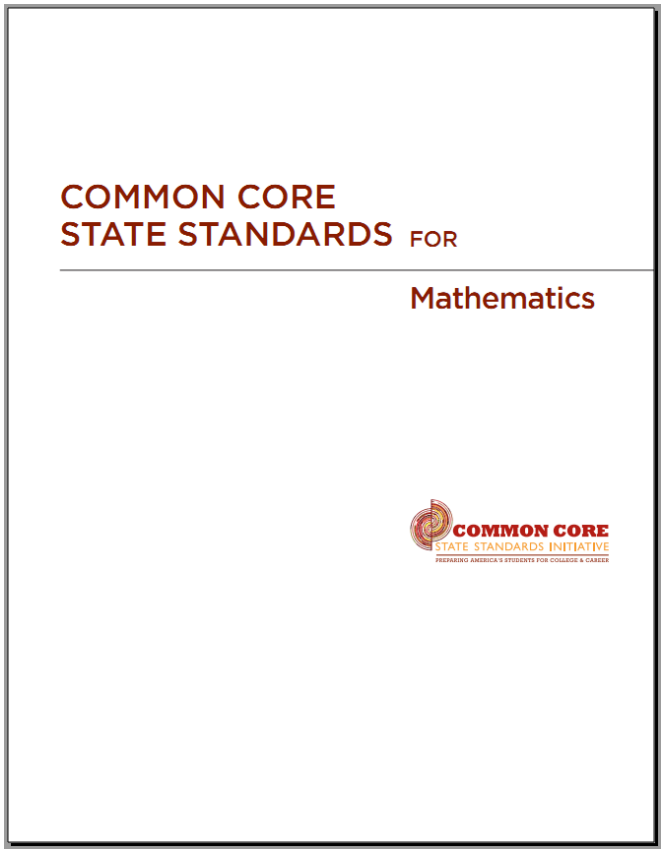
- I can meet the scope of the Content Standards and Practice Standards through best practices and by incorporating a variety of strategies and activities in my instruction.

Before We Begin…

The CCS Require Three Shifts in Mathematics

Handout 1

- **Focus** strongly where the standards focus
- **Coherence**: **Think** across grades and **link** to major topics within grades Handout
- **Rigor**: In major topics, pursue **conceptual understanding**, procedural skill and **fluency**, and **application** with equal intensity



Key Areas of Focus in Mathematics

Grade	Focus Areas in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding
K–2	Addition and subtraction - concepts, skills, and problem solving and place value
3–5	Multiplication and division of whole numbers and fractions – concepts, skills, and problem solving
6	Ratios and proportional reasoning; early expressions and equations
7	Ratios and proportional reasoning; arithmetic of rational numbers
8	Linear algebra and linear functions

Key Areas of Focus in High School Mathematics

Conceptual Category	Focus Clusters
Numbers and Quantity	Real numbers and Quantities
Algebra	Seeing Structure in Expressions, Arithmetic with Polynomial and Rational Expressions, Creating Equations, Reasoning with Equations and Inequalities
Functions	Interpreting Functions, Building Functions, and Linear, Quadratic and Exponential Models
Geometry	Similarity, Right Triangles and Trigonometry
Statistics and Probability	Interpreting Categorical and Quantitative Data and Making Inferences and Drawing Conclusions

Shift #3: Rigor

Required Fluencies for Grades K-6

Grade	Standard	Required Fluency
K	K.OA.5	Add/subtract within 5
1	1.OA.6	Add/subtract within 10
2	2.OA.2 2.NBT.5	Add/subtract within 20 (know single-digit sums from memory) Add/subtract within 100
3	3.OA.7 3.NBT.2	Multiply/divide within 100 (know single-digit products from memory) Add/subtract within 1000
4	4.NBT.4	Add/subtract within 1,000,000
5	5.NBT.5	Multi-digit multiplication
6	6.NS.2,3	Multi-digit division Multi-digit decimal operations

The Standards for Mathematical Content

Ratios and Proportional Relationships 7.RP

Analyze proportional relationships and use them to solve real-world and mathematical problems.

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $^{1/2}/_{1/4}$ miles per hour, equivalently 2 miles per hour.*
2. Recognize and represent proportional relationships between quantities.
   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
   c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*
   d. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.
3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

The 8 Standards for Mathematical Practice

Hand out 2

Habits of Mind of a Productive Mathematical thinker

MP1 Make sense of problems and persevere in solving them.

MP6 Attend to precision

Reasoning & Explaining

MP2 Reason abstractly and quantitatively.

MP3 Construct

Modeling & Using Tools

MP4 Model with Mathematics

MP5 Use appropriate tools strategically

Seeing Structure & Generalizing

MP7 Look for and make use of structure

MP8 Look for and express regularity in repeated reasoning

Reminders…

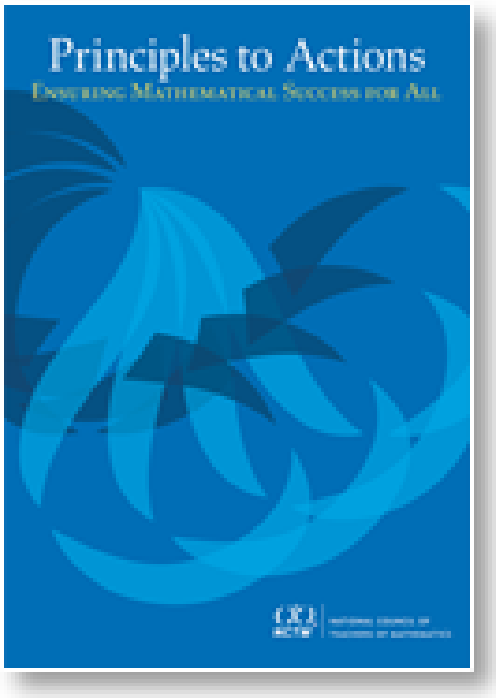
Keep in mind that the CCS for Mathematics:

- are **NOT** discrete skills to be taught in isolation.
- define both content and practices that should be included in instruction.

Cloning, drilling and killing is **not** an effective instructional practice.

High quality instruction  **Improved student achievement**

NCTM Principles to Actions: Ensuring Mathematical Success for All



The primary purpose of Principles to Actions is to fill the gap between the adoption of rigorous standards and the enactment of practices, policies, programs, and actions required for successful implementation of those standards.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM

Guiding Principles for School Mathematics

1. Teaching and Learning
2. Access and Equity
3. Curriculum
4. Tools and Technology
5. Assessment
6. Professionalism

→ **Essential Elements of Effective Mathematics Programs**

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM

Eight Mathematics Teaching Practices

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations
4. Facilitate meaningful mathematical discourse
5. Pose purposeful questions
6. Build procedural fluency from conceptual understanding
7. Support productive struggle in learning mathematics
8. Elicit and use evidence of student thinking

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM, p.10

The Mathematics Claims

Claims for the Mathematics Summative Assessment

Overall Claim for Grades 3-8

Students can demonstrate **progress toward** college and career readiness in mathematics.

Overall Claim for Grade 11

Students can demonstrate college and career readiness in mathematics.

Claim #1 – Concepts and Procedures	"Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency."
Claim #2 – Problem Solving	"Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies."
Claim #3 - Communicating Reasoning	"Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others."
Claim #4 - Modeling and Data Analysis	"Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems."

From the Smarter Balanced Mathematics Content Specifications

Smarter Balanced Assessment Consortium

Overall Claim and Mathematics Teaching Practices

Overall Claim	Mathematics Teaching Practice
Grades 3-8: • Students can demonstrate **progress toward** college and career readiness in mathematics. **Grade 11:** • Students can demonstrate college and career readiness in mathematics.	Establishing Mathematics Goals to Focus Learning Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses goals to guide instructional decisions. Elicit and use evidence of student thinking Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning

Teacher and Student Actions

Establish mathematics goals to focus learning Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit. Identifying how the goals fit within a mathematics learning progression. Discussing and referring to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning. Using the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction.	Engaging in discussions of the mathematical purpose and goals related to their current work in the mathematics classroom (e.g., What are we learning? Why are we learning it?) Using the learning goals to stay focused on their progress in improving their understanding of mathematics content and proficiency in using mathematical practices. Connecting their current work with the mathematics that they studied previously and seeing where the mathematics is going. Assessing and monitoring their own understanding and progress toward the mathematics learning goals.

NCTM. (2014). Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: NCTM p. 16

Teacher and Student Actions

Elicit and use evidence of student thinking Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Identifying what counts as evidence of student progress toward mathematics learning goals. Eliciting and gathering evidence of student understanding at strategic points during instruction. Interpreting student thinking to assess mathematical understanding, reasoning, and methods. Making in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend. Reflecting on evidence of student learning to inform the planning of next instructional steps.	Revealing their mathematical understanding, reasoning, and methods in written work and classroom discourse. Reflecting on mistakes and misconceptions to improve their mathematical understanding. Asking questions, responding to, and giving suggestions to support the learning of their classmates. Assessing and monitoring their own progress toward mathematics learning goals and identifying areas in which they need to improve.

NCTM. (2014). Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: NCTM p. 56

Claim 1
Concepts and Procedures

Claim 1: Concepts and Procedures

"Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency."

Rationale for Claim 1

- This claim addresses procedural skills and the conceptual understanding on which developing skills depend. It is important to assess how aware students are of how concepts link together and why mathematical procedures work the way they do.
- Central to understanding this claim is making the connection to these elements of the mathematical practices as stated in the CCS for Mathematics.
  - **MP5. Use appropriate tools strategically.**
  - **MP6. Attend to precision.**
  - **MP7. Look for and make use of structure.**
  - **MP8. Look for and express regularity in repeated reasoning.**

Smarter Balanced Assessment Targets

- Provide more detail about the range of content and Depth of Knowledge levels.

- Intended to support the development of high-quality items and tasks that contribute evidence to the claims.

Smarter Balanced Cognitive Rigor Matrix

Handout 4

Depth of Thinking (Webb) + Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	- Recall conversions, terms, facts			
Understand	-Evaluate an expression -Locate points on a grid or number on number line -Solve a one-step problem -Represent math relationships in words, pictures, or symbols	- Specify, explain relationships -Make basic inferences or logical predictions from data/observations -Use models /diagrams to explain concepts -Make and explain estimates	-Use concepts to solve non-routine problems -Use supporting evidence to justify conjectures, generalize, or connect ideas -Explain reasoning when more than one response is possible -Explain phenomena in terms of concepts	-Relate mathematical concepts to other content areas, other domains -Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	-Follow simple procedures -Calculate, measure, apply a rule (e.g., rounding) -Apply algorithm or formula -Solve linear equations -Make conversions	-Select a procedure and perform it -Solve routine problem applying multiple concepts or decision points -Retrieve information to solve a problem -Translate between representations	-Design investigation for a specific purpose or research question - Use reasoning, planning, and supporting evidence -Translate between problem & symbolic notation when not a direct translation	-Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	-Retrieve information from a table or graph to answer a question -Identify a pattern/trend	-Categorize data, figures -Organize, order data -Select appropriate graph and organize & display data -Interpret data from a simple graph -Extend a pattern	-Compare information within or across data sets or texts -Analyze and draw conclusions from data, citing evidence -Generalize a pattern -Interpret data from complex graph	-Analyze multiple sources of evidence or data sets
Evaluate			-Cite evidence and develop a logical argument -Compare/contrast solution methods -Verify reasonableness	-Apply understanding in a novel way, provide argument or justification for the new application
Create	- Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	-Generate conjectures or hypotheses based on observations or prior knowledge and experience	-Develop an alternative solution -Synthesize information within one data set	-Synthesize information across multiple sources or data sets -Design a model to inform and solve a practical or abstract situation

Claim 1 Assessment Targets

Grade 7 SUMMATIVE ASSESSMENT TARGETS
Providing Evidence Supporting Claim #1

Claim #1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content for this claim may be drawn from any of the Grade 7 clusters represented below, with a much greater proportion drawn from clusters designated "m" (major) and the remainder drawn from clusters designated "a/s" (additional/supporting) – with these items fleshing out the major work of the grade. Sampling of Claim #1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims #2, #3, and #4.[18] Grade level content emphases are summarized in Appendix A and CAT sampling proportions for Claim 1 are given in Appendix B.

Ratios and Proportional Relationships (7.RP)

Target A [m]: Analyze proportional relationships and use them to solve real-world and mathematical problems. (DOK 1, 2)

Tasks for this target will require students to identify and represent proportional relationships in various formats (tables, graphs, equations, diagrams, verbal descriptions) and interpret specific values in context. (See 7.G Target E for possible context.) Other tasks will require students to compute unit rates, including those associated with ratios of fractions.

Multistep problems involving ratio and percent will be assessed by tasks in Claims 2 and 4.

From the Smarter Balanced Mathematics Content Specifications

Claim 1 and the Mathematics Teaching Practices

Claim 1	Mathematics Teaching Practices
Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency	**Use and connect mathematical representations** Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving. **Build procedural fluency from conceptual understanding** Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

Teacher and Student Actions

Use and connect mathematical representations Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Selecting tasks that allow students to decide which representations to use in making sense of the problems. Allocating substantial instructional time for students to use, discuss, and make connections among representations. Introducing forms of representations that can be useful to students. Asking students to make math drawings or use other visual supports to explain and justify their reasoning. Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation. Designing ways to elicit and assess students' abilities to use representations meaningfully to solve problems.	Using multiple forms of representations to make sense of and understand mathematics. Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations. Making choices about which forms of representations to use as tools for solving problems. Sketching diagrams to make sense of problem situations. Contextualizing mathematical ideas by connecting them to real-world situations. Considering the advantages or suitability of using various representations when solving problems.

NCTM. (2014). Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: NCTM p. 29

Teacher and Student Actions

Build procedural fluency from conceptual understanding Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Providing students with opportunities to use their own reasoning strategies and methods for solving problems. Asking students to discuss and explain why the procedures that they are using work to solve particular problems. Connecting student-generated strategies and methods to more efficient procedures as appropriate.	Making sure that they understand and can explain the mathematical basis for the procedures that they are using. Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems. Determining whether specific approaches generalize to a broad class of problems.

NCTM. (2014). Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: NCTM p.47-48

Middle School Activity 1

Illustrative Mathematics

8.NS Comparing Rational and Irrational Numbers

Activity 1

Handout 5a

For each pair of numbers, decide which is larger without using a calculator. Explain your choices.

a. π^2 or 9

b. $\sqrt{50}$ or $\sqrt{51}$

c. $\sqrt{50}$ or 8

d. -2π or -6

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Bringing It Back to the Standards

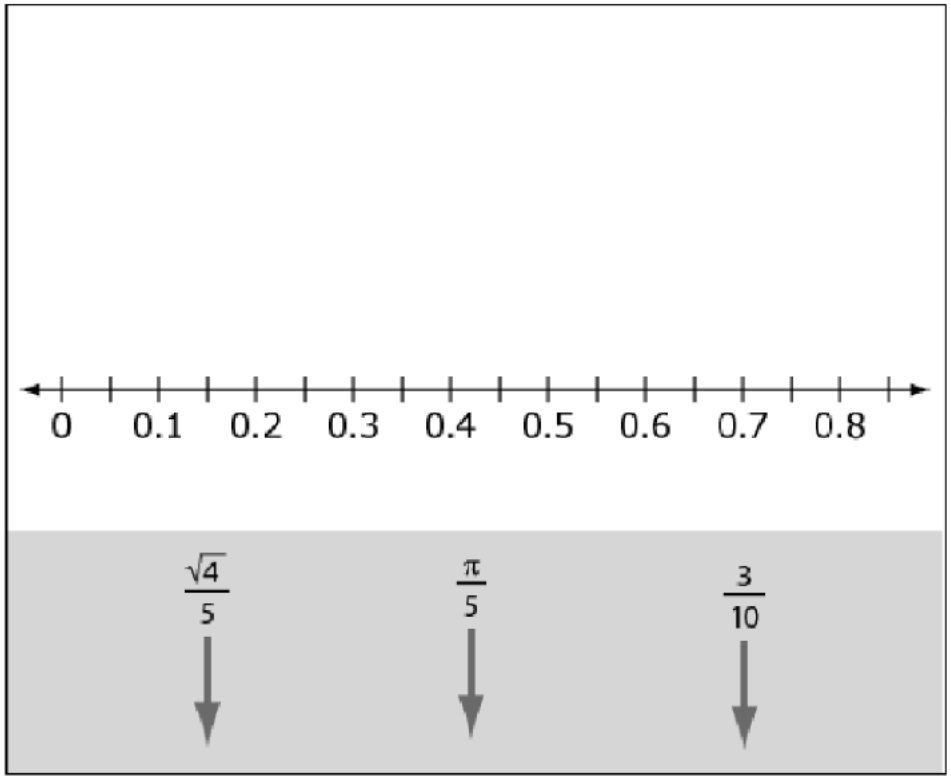
Using the "Bringing it Back to the Standards and the Teaching Practices" graphic organizer, identify which standards and teaching practices were addressed in Activity 1:

- Standards for Mathematical Content
- Standards for Mathematical Practice
- Mathematics Teaching Practices (including teacher/student actions)

Bringing it Back to the Assessment

1860

Drag each number to its correct position on the number line.



From the Smarter Balanced Grade 8 Practice Test

High School Activity 1

A-SSE Graphs of Quadratic Functions

Activity 1

Handout 5b

a. Graph these equations on your graphing calculator at the same time. What happens? Why?

$$y_1 = (x-3)(x+1)$$
$$y_2 = x^2-2x-3$$
$$y_3 = (x-1)^2-4$$
$$y_4 = x^2-2x+1$$

b. Below are the first three equations from the previous problem.

$$y_1 = (x-3)(x+1)$$
$$y_2 = x^2-2x-3$$
$$y_3 = (x-1)^2-4$$

These three equations all describe the same function. What are the coordinates of the following points on the graph of the function? From which equation is each point most easily determined? Explain.

i. vertex: _____

ii. y-intercept: _____

iii. x-intercept(s): _____

c. Make up an equation for a quadratic function whose graph satisfies the given condition. Use whatever form is most convenient.

i. Has a vertex at $(-2,-5)$.

ii. Has a y-intercept at $(0,6)$

iii. Has x-intercepts at $(3,0)$ and $(5,0)$

iv. Has x-intercepts at the origin and $(4,0)$

v. Goes through the points $(4,2)$ and $(1,2)$

Bringing It Back to the Standards

Using the "Bringing it Back to the Standards and the Teaching Practices" graphic organizer, identify which standards and teaching practices were addressed in Activity 1:

- Standards for Mathematical Content
- Standards for Mathematical Practice
- Mathematics Teaching Practices (including teacher/student actions)

Bringing it Back to the Assessment

1915

Determine whether each expression is equivalent to $(x^3 + 8)$. Select Yes or No for each expression.

	Yes	No
$(x + 8)^3$	☐	☐
$(x - 2)(x^2 + 2x + 4)$	☐	☐
$(x + 2)(x^2 - 2x + 4)$	☐	☐

From the Smarter Balanced Grade 11 Practice Test

Claim 2
Problem Solving

Claim 2: Problem-Solving

"Students can solve a range of complex, well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies."

Rationale for Claim 2

- Assessment items and tasks focused on Claim 2 include problems in pure mathematics and problems set in context.
- Problems are presented as items and tasks that are well-posed (that is, problem formulation is not necessary) and for which a solution path is not immediately obvious.
- These problems require students to construct their own solution pathway rather than follow a provided one. Such problems will therefore be less structured than items and tasks presented under Claim 1. Students will need to select appropriate conceptual and physical tools to use.

Rationale for Claim 2

- The evidence required of students to satisfy Claim 2 centers around specific statements of the ***mathematical practices*** (MP) contained in the CCSSM.

- Though not exclusive, MP1, MP5, MP7, and MP8 are particularly relevant for Claim 2 items.

Claim 2 Assessment Targets

Target A: Apply mathematics to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace. (DOK 2, 3)

Target B: Select and use appropriate tools strategically. (DOK 1, 2)

Target C: Interpret results in the context of a situation. (DOK 2)

Target D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)

Claim 2 Content Standards by Grade Level						
Grade 3	**Grade 4**	**Grade 5**	**Grade 6**	**Grade 7**	**Grade 8**	**High School**
3.OA.A	4.OA.A	5.NBT.B	6.RP.A	7.RP.A	8.EE.B	N-Q.A
3.OA.D	4.NBT.B	5.NF.A	6.NS.A	7.NS.A	8.EE.C	A-SSE.A
3.NBT.A*	4.NF.A	5.NF.B	6.NS.C	7.EE.A	8.F.A	A-SSE.B
3.NF.A	4.NF.B	5.MD.A*	6.EE.A	7.EE.B	8.F.B*	A-CED.A
3.MD.A	4.NF.C	5.MD.C	6.EE.B	7.G.A*	8.G.A	A-REI.2
3.MD.B*	4.MD.A*	5.G.A*	6.EE.C	7.G.B*	8.G.B	A-REI.B
3.MD.C	4.MD.C*		6.G.A*		8.G.C*	A-REI.C
3.MD.D*						A-REI.D
						F-IF.A
						F-IF.B
						F-IF.C
						F-BF.A
						G-SRT.C
						S-ID.C
						S-CP.A

* Denotes additional and supporting clusters

Claim 2 and the Mathematics Teaching Practices

Claim 2	Mathematics Teaching Practices
Students can solve a range of complex, well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.	Implement tasks that promote reasoning and problem solving Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and that allow for multiple entry points and varied solution strategies.

Teacher and Student Actions

Implement tasks that promote reasoning and problem solving Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Motivating students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding. Selecting tasks that provide multiple entry points through the use of varied tools and representations. Posing tasks on a regular basis that require a high level of cognitive demand. Supporting students in exploring tasks without taking over student thinking. Encouraging students to use varied approaches and strategies to make sense of and solve tasks.	Persevering in exploring and reasoning through tasks. Taking responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas. Using tools and representations as needed to support their thinking and problem solving. Accepting and expecting that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another.

NCTM. (2014). Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: NCTM p. 24

Middle School Activity 2

6.EE Seven to the What!

Activity 2

Handout 7a

a. What is the last digit of 7^{2011}? Explain.

b. What are the last *two* digits of 7^{2011}? Explain.

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Bringing It Back to the Standards

Using the "Bringing it Back to the Standards and the Teaching Practices" graphic organizer, identify which standards and teaching practices were addressed in Activity 2:

- Standards for Mathematical Content
- Standards for Mathematical Practice
- Mathematics Teaching Practices (including teacher/student actions)

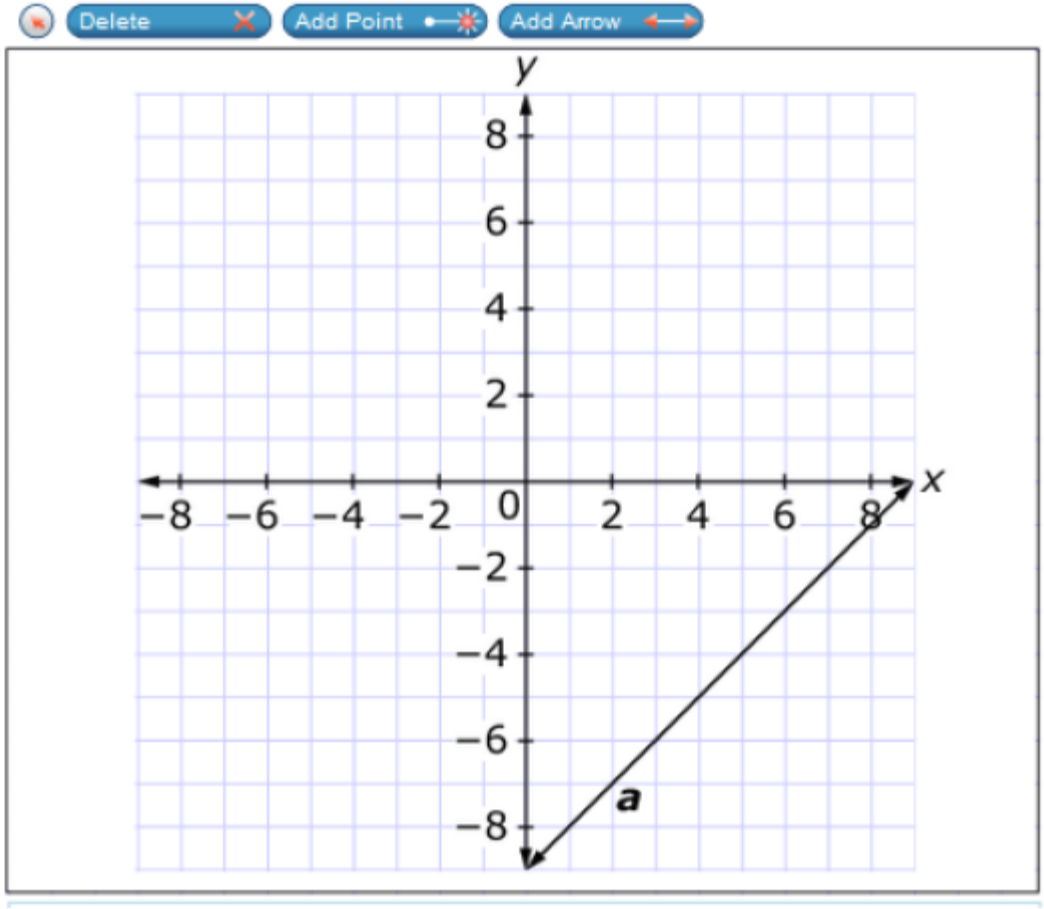
Bringing it Back to the Assessment

1834

Line *a* is shown on the graph.

Use the Add Arrow tool to construct line *b* on the graph so that:

- Line *a* and line *b* represent a system of linear equations with a solution of (7, −2).
- The slope of line *b* is greater than −1 and less than 0.
- The *y*-intercept of line *b* is positive.



From the Smarter Balanced Grade 8 Practice Test

High School Activity 2

S-CP Alex, Mel, and Chelsea Play a Game

Activity 2

Alex, Mel, and Chelsea play a game that has 6 rounds. In each round there is a single winner, and the outcomes of the rounds are independent. For each round the probability that Alex wins is $\frac{1}{2}$, and Mel is twice as likely to win as Chelsea. What is the probability that Alex wins three rounds, Mel wins two rounds, and Chelsea wins one round?

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Bringing It Back to the Standards

Using the "Bringing it Back to the Standards and the Teaching Practices" graphic organizer, identify which standards and teaching practices were addressed in Activity 2:

- Standards for Mathematical Content
- Standards for Mathematical Practice
- Mathematics Teaching Practice (including teacher/student actions)

Bringing it Back to the Assessment

At a local fair, the price of admission includes the opportunity for a person to spin a wheel for free ride tickets.

- Each spin of the wheel is a random event.
- The result from each spin of the wheel is independent of the results of previous spins.
- Each spin of the wheel awards tickets according to the probabilities shown below.

Spin the Wheel

1 ticket	35%
2 tickets	25%
3 tickets	20%
5 tickets	15%
10 tickets	5%

Let X be the number of tickets a person wins based on 2 spins. There are 13 possible values for X.

Some values of X are more common than others. For example, winning only 2 tickets in 2 spins is a somewhat common occurrence with probability 0.1225. It means the person wins 1 ticket on the first spin and 1 ticket on the second spin ($0.35 \cdot 0.35$). A list of the possible values of X and the corresponding probabilities for most values of X is shown below.

Fill in the three missing probability values in the table.

X	Probability
2	0.1225
3	0.1750
4	
5	0.1000
6	0.1450
7	0.0750
8	0.0600
10	
11	0.0350
12	0.0250
13	
15	0.0150
20	0.0025

Claim 3
Communicating Reasoning

Claim 3: Communicating Reasoning

"Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others."

Rationale for Claim 3

- Claim 3 refers to a recurring theme in the CCSS for Mathematics content and practice standards—the ability to construct and present a clear, logical, convincing argument.
  - For older students, this may take the form of a rigorous, deductive proof based on clearly stated axioms.
  - For younger students, this will involve more informal justifications .
- Assessment tasks that address this claim will typically present a claim or a proposed solution to a problem and will ask students to provide an example, a justification, an explanation, or a counterexample.

Rationale for Claim 3

- The evidence required of students to satisfy Claim 3 centers around specific statements of the ***mathematical practices*** (MP) contained in the CCSSM.
- Though not exclusive, MP3 and MP6 are particularly relevant for Claim 3 items.

Targets for Claim 3

Target A: Test propositions or conjectures with specific examples. (DOK 2).

Target B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. (DOK 3, 4).

Target C: State logical assumptions being used. (DOK 2, 3)

Target D: Use the technique of breaking an argument into cases. (DOK 2, 3)

Target E: Distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in the argument—explain what it is. (DOK 2, 3, 4)

Target F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (DOK 2, 3)

Target G: At later grades, determine conditions under which an argument *does* and *does not* apply. (DOK 3, 4)

Claim 3 Standards by Grade Level

Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	HS
3.OA.B	4.OA.3	5.NBT.2	6.RP.A	7.RP.2	8.EE.1	N-RN.A
3.NF.A	4.NBT.A	5.NBT.6	6.RP.3	7.NS.A	8.EE.5	N-RN.B
3.NF.1	4.NBT.5	5.NBT.7	6.NS.A	7.NS.1	8.EE.6	N-RN.3
3.NF.2	4.NBT.6	5.NF.1	6.NS.1	7.NS.2	8.EE.7a	A-SSE.2
3.NF.3	4.NF.A	5.NF.2	6.NS.C	7.EE.1	8.EE.7b	A-APR.1
3.MD.A	4.NF.1	5.NF.B	6.NS.5	7.EE.2	8.EE.8a	A-REI.A
3.MD.7	4.NF.2	5.NF.3	6.NS.6		8.F.1	A-REI.1
	4.NF.3a	5.NF.4	6.NS.7		8.F.2	A-REI.2
	4.NF.3b	5.NF.7a	6.EE.A		8.F.3	A-REI.10
	4.NF.3c	5.NF.7b	6.EE.3		8.G.1	A-REI.11
	4.NF.4a	5.MD.C	6.EE.4		8.G.2	F-IF.1
	4.NF.4b	5.MD.5a	6.EE.B		8.G.4	F-IF.5
	4.NF.C	5.MD.5b	6.EE.6		8.G.5	F-IF.9
	4.NF.7	5.G.B*	6.EE.9		8.G.6	G-CO.C
		5.G.4*			8.G.8	G-CO.9
						G-CO.10
						G-CO.11
						A-APR.B
						A-APR.4
						A-APR.6
						A-REI.C
						F-BF.3
						F-BF.4a
						F-TF.1
						F-TF.2
						F-TF.8
						G-CO.A
						G-CO.B
						G.SRT.A
						G.SRT.B

Claim 3 and the Mathematics Teaching Practices

Claim 3	Mathematics Teaching Practices
Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.	**Facilitate meaningful mathematical discourse** Effective teaching of mathematics facilitates discourse among students in order to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments. **Pose purposeful questions** Effective teaching of mathematics uses purposeful questions to assess and advance student reasoning and sense making about important mathematical ideas and relationships.

Teacher and Student Actions

Facilitate meaningful mathematical discourse Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations. Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion. Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches. Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.	Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse. Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments. Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others. Identifying how different approaches to solving a task are the same and how they are different.

NCTM. (2014). Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: NCTM p. 35

Teacher and Student Actions

Pose purposeful questions Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking. Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification. Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion. Allowing sufficient wait time so that more students can formulate and offer responses.	Expecting to be asked to explain, clarify, and elaborate on their thinking. Thinking carefully about how to present their responses to questions clearly, without rushing to respond quickly. Reflecting on and justifying their reasoning, not simply providing answers. Listening to, commenting on, and questioning the contributions of their classmates.

NCTM. (2014). Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: NCTM p. 41

Middle School Activity 3

7.RP Dueling Candidates

Activity 3

Joel and Marisa are running for president at their middle school (grades 6-8). After the votes are in, Joel and Marisa are each convinced that they have won the election:

- Joel argues that he has won a larger percentage of the overall vote than Marisa so he should be the new president.
- Marisa argues that she has won a larger percentage than Joel of the 6th grade vote and the 7th grade vote. Since the majority of the grades voted for her, she should be the new president.

Is it possible that both Joel and Marisa are making accurate claims? Explain.

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Bringing It Back to the Standards

Using the "Bringing it Back to the Standards and the Teaching Practices" graphic organizer, identify which standards and teaching practices were addressed in Activity 2:

- Standards for Mathematical Content
- Standards for Mathematical Practice
- Mathematics Teaching Practice (including teacher/student actions)

Bringing it Back to the Assessment

1857

Look at the equation.

$\frac{2}{3} \times \frac{\square}{\square} = n$

Sarah claims that for any fraction multiplied by $\frac{2}{3}$, n will be less than $\frac{2}{3}$.

To convince Sarah that this statement is only sometimes true:

Part A: Drag one number into each box so the product, n, is less than $\frac{2}{3}$.

Part B: Drag one number into each box so the product, n, is **not** less than $\frac{2}{3}$.

1
2
3
4
5
6
7
8
9

Delete

Part A: **Product n is less than $\frac{2}{3}$**

$\frac{2}{3} \times \frac{\square}{\square} = n$

Part B: **Product n is not less than $\frac{2}{3}$**

$\frac{2}{3} \times \frac{\square}{\square} = n$

From the Smarter Balanced Grade 6 Practice Test

High School Activity 3

A-SSE Taxes and Sales

Activity 3

Judy is working at a retail store over summer break. A customer buys a $50 shirt that is on sale for 20% off. Judy computes the discount, then adds sales tax of 10%, and tells the customer how much he owes. The customer insists that Judy first add the sales tax and then apply the discount. He is convinced that this way he will save more money because the discount amount will be larger.

a. Is the customer right?

b. Does your answer to part (a) depend on the numbers used or would it work for any percentage discount and any sales tax percentage? Find a convincing argument using algebraic expressions and/or diagrams for this more general scenario.

http://s3.amazonaws.com/illustrativemathematics/illustration_pdfs/000/000/677/original/illustrative_mathematics_677.pdf?1390750155

Bringing It Back to the Standards

Using the "Bringing it Back to the Standards and the Teaching Practices" graphic organizer, identify which standards and teaching practices were addressed in Activity 2:

- Standards for Mathematical Content
- Standards for Mathematical Practice
- Mathematics Teaching Practice (including teacher/student actions)

Bringing it Back to the Assessment

2029

The radius of sphere *Y* is twice the radius of sphere *X*. A student claims that the volume of sphere *Y* must be exactly twice the volume of sphere *X*.

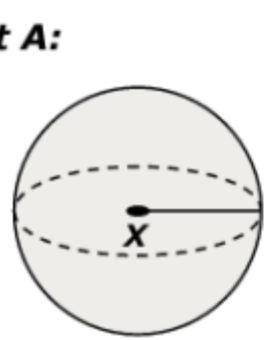
Part A: Drag numbers into the boxes to create one example to evaluate the student's claim.

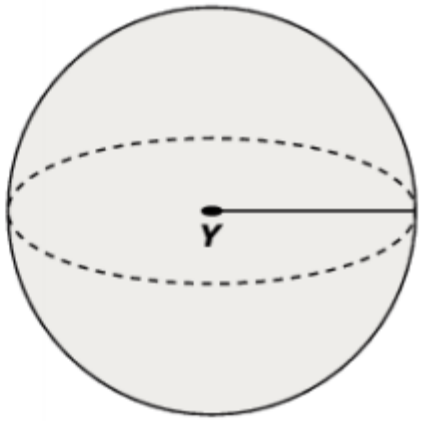
Part B: Decide whether the student's claim is true, false, or cannot be determined. Select the correct option.

0 1 2 3 4 5 6 7 8 9

Delete

Part A:





Radius = ☐ in

Volume = $\frac{4}{3}\pi$ ☐ in^3

Radius = ☐ in

Volume = $\frac{4}{3}\pi$ ☐ in^3

Part B:

True False Cannot be determined

From the Smarter Balanced Grade 11 Practice Test

Claim 4
Modeling and Data Analysis

Claim 4: Modeling and Data Analysis

"Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems."

Rationale for Claim 4

- In the real world, problems do not come neatly ”packaged.” Real-world problems are complex and often contain insufficient or superfluous data.
- Assessment tasks will involve *formulating* a problem that is tractable using mathematics; that is, formulating a model. This will usually involve making assumptions and simplifications.
- Students will need to select from the data at hand or estimate data that are missing. (Such tasks are therefore distinct from the well-formulated problem-solving tasks described in Claim 2.)
- Students will identify variables in a situation and construct relationships between them. Once students have formulated the problem, they will tackle it (often in a decontextualized manner) before interpreting their results and then checking the results for reasonableness.

Rationale for Claim 4

- The evidence required of students to satisfy Claim 4 centers around specific statements of the ***mathematical practices*** (MP) contained in the CCSSM.

- Though not exclusive, MP2, MP4, and MP5 are particularly relevant for Claim 4 items.

Targets for Claim 4

Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace. (DOK 2, 3)

Target B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem. (DOK 2, 3, 4)

Target C: State logical assumptions being used. (DOK 1, 2)

Target D: Interpret results in the context of a situation. (DOK 2, 3)

Target E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon. (DOK 3, 4)

Target F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)

Target G*: Identify, analyze and synthesize relevant external resources to pose or solve problems. (DOK 3, 4)

*Assessed in Performance Tasks only

Claim 4 Clusters and Standards

Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	HS
3.OA.A	4.OA.A	5.NBT.B	6.RP.A	7.RP.A	8.EE.3	N-Q.A
3.OA.D	4.NF.B	5.NF.A	6.NS.A	7.NS.A	8.EE.4	A-SSE.B
3.MD.A	4.MD.A*	5.NF.B	6.NS.C	7.EE.B	8.EE.B	A-CED.A
3.MD.C	4.MD.B*	5.MD.A*	6.EE.B	7.G.A*	8.EE.C	A-REI.A
3.MD.D*	4.MD.C*	5.MD.B*	6.EE.C	7.G.B*	8.F.B*	A-REI.B
		5.MD.C	6.G.A*	7.SP.A*	8.G.B	F-IF.B
		5.G.A*	6.SP.A*	7.SP.B*	8.G.C*	F-IF.C
			6.SP.B*	7.SP.C*	8.SP.A*	F-BF.A
						S-ID.A
						S-ID.B
						S-IC.1
						S-IC.B
						A-REI.C
						F-LE.A
						F-LE.B
						F-TF.5
						G-GMD.3
						G-MG

*Denotes additional and supporting clusters

Claim 4 and the Mathematics Teaching Practices

Claim 4	Mathematics Teaching Practices
Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.	**Support Productive Struggle in Learning Mathematics** Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Teacher and Student Actions

Support productive struggle in learning mathematics Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle. Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them. Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles. Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle. Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks. Persevering in solving problems and realizing that is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up. Helping one another without telling their classmates what the answer is or how to solve the problem.

NCTM. (2014). Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: NCTM p. 52

Middle School Activity 4

Understanding Proportional Relationships

Activity 4

Handout 9a

Bringing It Back to the Standards

Using the “Bringing it Back to the Standards and the Teaching Practices” graphic organizer, identify which standards and teaching practices were addressed in Activity 2:

- Standards for Mathematical Content
- Standards for Mathematical Practice
- Mathematics Teaching Practice (including teacher/student actions)

Bringing it Back to the Assessment

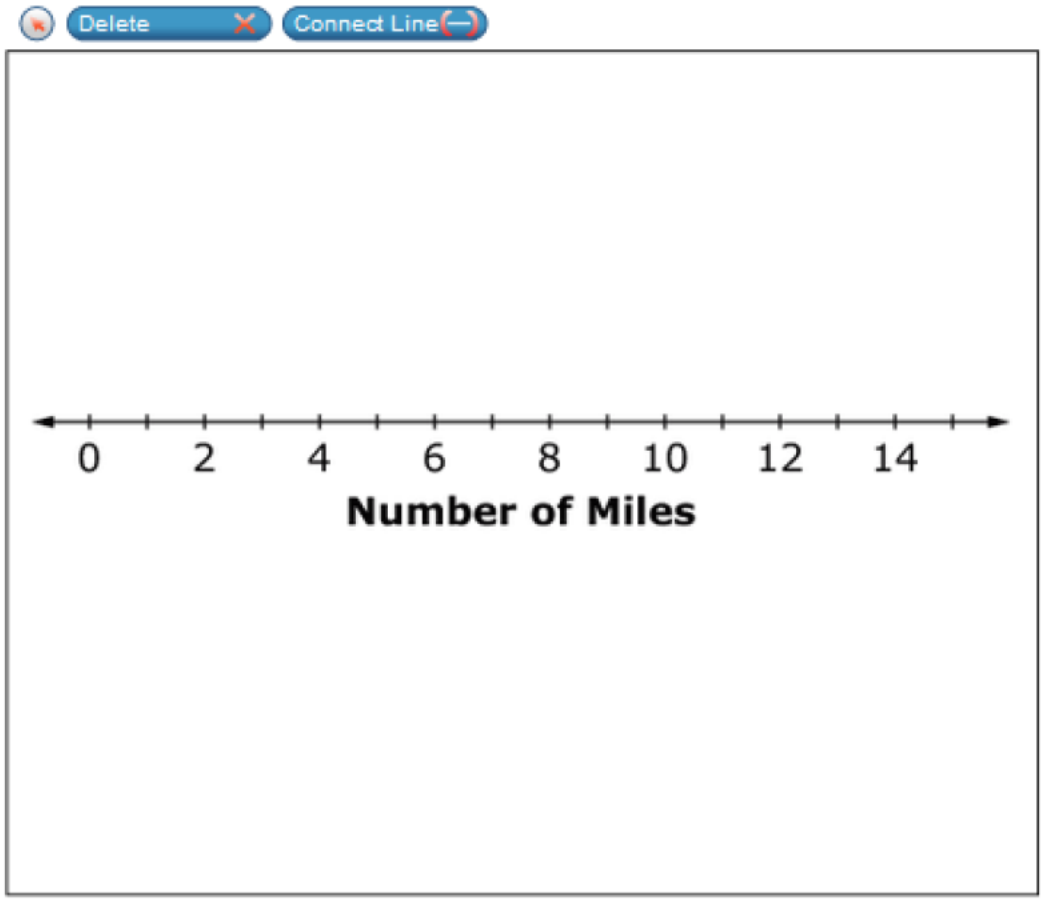
1798

A boat takes 3 hours to reach an island 15 miles away.

The boat travels:

- at least 1 mile but no more than 6 miles during the first hour
- at least 2 miles during the second hour
- exactly 5 miles during the third hour

Use the Connect Line tool to show the range of miles the boat could have traveled during the **second** hour, given the conditions above.



From the Smarter Balanced Grade 6 Practice Test

High School Activity 4

A-SSE Course of Antibiotics

Activity 4

Susan has an ear infection. The doctor prescribes a course of antibiotics. Susan is told to take 250 mg doses of the antibiotic regularly every 12 hours for 10 days.

Susan is curious and wants to know how much of the drug will be in her body over the course of the 10 days. She does some research online and finds out that at the end of 12 hours, about 4% of the drug is still in the body.

a. What quantity of the drug is in the body right after the first dose, the second dose, the third dose, the fourth dose?

b. When will the total amount of the antibiotic in Susan's body be the highest? What is that amount?

c. Answer Susan's original question: Describe how much of the drug will be in her body at various points over the course of the 10 days.

http://s3.amazonaws.com/illustrativemathematics/illustration_pdfs/000/000/805/original/illustrative_mathematics_805.pdf?1390750113

Bringing It Back to the Standards

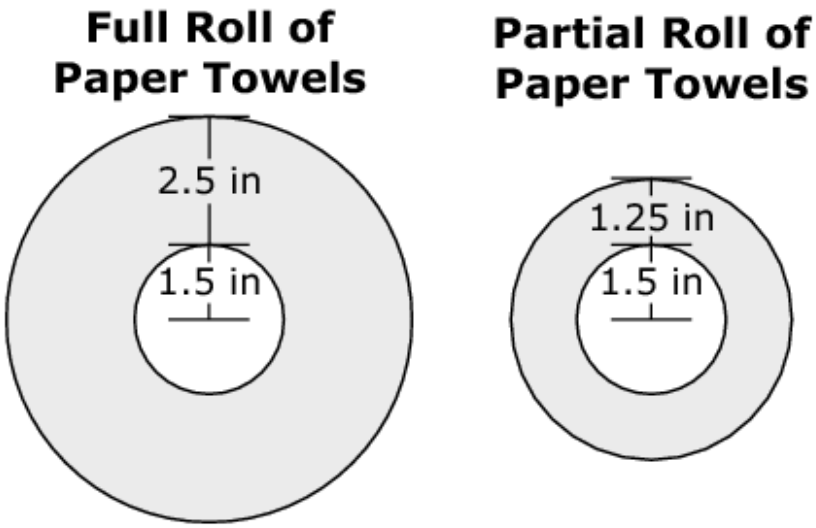
Using the "Bringing it Back to the Standards and the Teaching Practices" graphic organizer, identify which standards and teaching practices were addressed in Activity 2:

- Standards for Mathematical Content
- Standards for Mathematical Practice
- Mathematics Teaching Practice (including teacher/student actions)

Bringing it Back to the Assessment

2032

The diagram shows the end view of a roll of paper towels when it is full and the end view of the roll after some of the paper towels have been used.



When the full roll of paper towels is unrolled, it has a length of 528 inches of paper towels of uniform width and thickness. Enter the length, in inches, of the paper towels remaining on the partial roll.

From the Smarter Balanced Grade 11 Practice Test

Supporting Students Who Will Take the Smarter Balanced Assessments

Support for Students

- Providing high quality instruction throughout the year.
- Provide opportunities for students to engage with the Smarter Balanced Practice Tests and Training Tests.
  - Practice Tests are available by grade level
  - Training Tests are available by grade-band
- Available on the Smarter Balanced Web site

  http://www.smarterbalanced.org/practice-test/

Purpose of the Practice Tests

- Allows teachers, students, parents, and other interested parties to experience a full grade-level assessment
- Provides an opportunity for students to become familiar with the keyboard commands and embedded universal tools, designated supports, and accommodations
- Provides an opportunity for students who use assistive technology to test the functionality prior to testing

Practice Tests

- Grade-level scoring guides are available for download
- Important Limitations
  - The Practice Tests do not encompass the full range of content that students will encounter on the operational assessments in 2015, and should not be used to guide instructional decisions.

Training Tests

- The Training Tests are designed to provide students and teachers with opportunities to quickly familiarize themselves with the software and navigational tools that they will use on the Smarter Balanced Assessments.
- The Training Tests are organized by grade bands (grades 3 to 5, 6 to 8, and high school) and each test contains 14-15 questions.
- The questions were selected to provide students with an opportunity to practice a range of question types.
- The Training Tests do not contain performance tasks. Similar to the Practice Test, the Training Tests include all embedded universal tools, designated supports, and accommodations.

How Does the Smarter Balanced Assessment System Support Instruction?

The Smarter Balanced Assessment System

The Smarter Balanced Assessment System includes:

- Digital Library resources to help educators implement the *formative assessment process* to improve teaching and learning
- *Interim assessments* to check student progress throughout the year and help teachers plan and improve instruction (Note: the interim assessments are not intended to be used for accountability purposes)
- Year-end *summative assessments* that are used for accountability purposes

Questions

Success Criteria

- I can explain how the Connecticut Core Standards (CCS) for Mathematics connect to the Smarter Balanced Claims.

- I can meet the scope of the Content Standards and Practice Standards through best practices and by incorporating a variety of strategies and activities in my instruction.

Resources

- CT Core Standards: http://ctcorestandards.org/
- Connecticut Dream Team 2014 Math Resources: http://ctcorestandards.org/?page_id=3771
- Achieve the Core: http://achievethecore.org/
- Illustrative Mathematics: https://www.illustrativemathematics.org/
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Additional resources are included in participant folder

Resources

- Smarter Balanced Assessment Consortium: http://www.smarterbalanced.org/

- Link to CSDE Student Assessment Smarter Balanced page: http://www.sde.ct.gov/sde/cwp/view.asp?a=2748&q=334488