**Unit 8: Investigation 6 (2 Days)**

**SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE AND THE QUADRATIC FORMULA**

**CCSS: A-REI 4, A-SSE 3b, F-IF 8a**

**Overview**

Students will learn how to solve quadratic equations by completing the square and using the quadratic formula.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Solve a quadratic equation that cannot be factored by completing the square and by using the quadratic formula.

**Assessment Tools: How Will They Show What They Know**

* + **Exit Slip 8.6** asks students to show that they know how to use both methods.
	+ **Journal Entry** asks students to explain which method they prefer to use.

**Launch Notes**

Start by looking at a quadratic equation that cannot be solved by factoring. Students will have seen examples in Investigation 8.7.1 (questions 2g and 3f). These should peak their curiosity about other solution methods. Explain that this investigation will introduce them to two solution methods, completing the square and using a formula.

**Teaching Strategies**

I. Use **Activity 8.6.1** to introduce the technique of completing the square to put a quadratic function in standard form into vertex form. The activity starts with a concrete model (algebra tiles) and progresses to functions expressed symbolically. Once the function is in vertex form the vertex can be easily identified and the techniques of Investigation 4 may be used to find the x-intercepts, if there are any. Emphasize that finding the *x*-intercepts of the function f(*x*) is equivalent to solving the equation f(*x*) = 0 for *x*.

**Differentiated Instruction (For Learners Needing More Help)**

Use concrete algebra blocks as well as the pictoral representations shown in Activity 8.8.1 to show completing the square. Encourage students to use either the blocks or pictures as long as necessary. You may also want to restrict the problems given to these students to those equations with *a* = 1 and where *b* is an even number.

Show students how the completing the square method is another way of transforming a quadratic function from standard form to vertex form. Provide students with functions in standard form and ask that they use the method of completing the square to convert the function to vertex form. Students should recognize the distinction between a function and an equation. When completing the square on a function, students will add and subtract the same number to the function. For example, in the expression below, 9 is added and subtracted from the same side of the expression.

$$f\left(x\right)=x^{2}+6x-5=x^{2}+6x+9-5-9=(x+3)^{2}-14$$

II. Introduce the quadratic formula as a method to solve the quadratic equation in standard form$ ax^{2}+bx+c=0$. Express the solutions in the form:

$$x=-\frac{b}{2a}\pm \frac{\sqrt{b^{2}-4ac}}{2a}$$

This form of the quadratic formula reinforces the connection between the x-intercepts and the axis of symmetry. Remind students that the x-intercepts of the parabola $y=ax^{2}+bx+c$ are the solutions of the equation $ax^{2}+bx+c=0$, and these solutions are symmetric to the line:
$$x=-\frac{b}{2a}$$

It should make sense that if there are *x* intercepts they should lie at equal distances to the left and to the right of the line of symmetry: that explains the ± part of the formula.

**Differentiated Instruction (Enrichment)**

**Activity 8.6.2** is designed to help students see the connection between completing the square and the quadratic formula. This may be assigned as enrichment or used for whole class discussion in an honors class.

All students should understand that the quadratic formula results from the completing the square method, even if they are unable to provide a proof, and that the formula provides students a shortcut for finding solutions of a quadratic equation in standard form. **Activity 8.6.3** provides students with practice using the formula and has them explore the relationship between the discriminant $b^{2}-4ac$ and the number of real solutions.

**Differentiated Instruction (For Learners Needing More Help)**

Many students have difficulty substituting into the quadratic formula. Here are two tips that may help.

1. Immediately below the equation you are trying to solve, write the values of *a, b,* and *c*.

For example for $x^{2}-3x+10=0 $ write

 *a =* 1*, b =* –3, *c* = 10.

Then write the formula $x=-\frac{b}{2a}\pm \frac{\sqrt{b^{2}-4ac}}{2a}$ and substitute.

2. When substituting for $b^{2}$ always put the number in parentheses, for example $(-3)^{2}$, and remember that the square of a negative number is positive!

**Group Activity**

Form groups of three. Each student should write a quadratic equation in standard form. Pass papers to the left. Each student then uses the formula to solve the given equation. Pass papers to the left again. This time each student checks the previous student’s work by graphing the associated quadratic function. Check first that the number of solutions is correct (0, 1 or 2) and then that if there are *x*-intercepts they agree with the results obtained by the formula.

III. Students should continue to solve real world problems involving parabolas and quadratic functions. **Activity 8.6.4** introduces students to the Golden Ratio, which can be derived from a non-factorable quadratic equation. You may want to show a video on the golden rectangle such as <http://www.youtube.com/watch?v=fmaVqkR0ZXg>.

One example is a problem that uses a parabola to model the arch of a bridge. The following activity asks students to determine the equation of a parabola given the parabola’s vertex and x-intercepts. The equation is then used to solve problems. The problem below is similar to a problem from *Math Trails*, which can be found at: [http://olc.spsd.sk.ca/DE/math20/math20trailolc/documents/quad\_function.pdf](http://olc.spsd.sk.ca/DE/math20/math20trailolc/documents/quad_function.pdf%22%20%5Ct%20%22_blank).

**Question:** The following bridge was built around 1950. Will the arch’s dimensions allow new taller tractor trailers to pass through? The bridge is pictured below:



You are given the total distance from the bottom of the arch on the left side to the bottom of the arch on the right side. The 4 foot median is centered below directly below the arch, and there are 14 foot sidewalks on the outsides of each lane. The lanes, median, and sidewalks are pictured below.

**Journal Entry**

Which method for solving a quadratic equation do you prefer, completing the square or using the formula? Why?

**Resources and Materials**

* Activity 8.6.1 Completing the Square
* Activity 8.6.2 Proving the Quadratic Formula
* Activity 8.6.3 Using the Quadratic Formula
* Activity 8.6.4 Golden Rectangles
* Exit Slip 8.6
* Projector for video clips