**Unit 8: Investigation 4 (4 Days)**

**QUADRATIC FUNCTIONS IN FACTORED FORM**

***CCSS: A-APR 1, F-IF 4, F-IF 7a, F-BF-3***

**Overview**

Students will discover that quadratic functions can be expressed in factored form and will model real world situations using quadratic functions in factored form. Using the graph of a quadratic function in factored form, students will find the vertex of the parabola to solve a variety of problems. They will learn how to multiply polynomials and apply this skill to convert quadratic functions in factored form to quadratic functions in standard form.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Graph and find the vertex of quadratic functions in factored form.
* Use the zero product property to find the intercepts of a quadratic function in factored form.
* Multiply combinations of monomials, binomials, and trinomials.
* Convert quadratic functions in factored form to standard form.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 8.4.1** asks students to calculate the *x*-intercepts and the vertex of a quadratic function.
* **Journal Entry 1** asks students to compare and contrast the three forms of a quadratic function.
* **Exit Slip 8.4.2** asks students to show steps as they multiply polynomials.
* **Journal Entry 2** asks students to explain how they determine the signs of terms in the product of two binomials.

**Launch Notes**

This investigation provides a great opportunity to engage students by appealing to their business sense. Facilitate a discussion about the cost of a sweatshirt. Discuss who determines the price of a consumer product and the factors that lead to setting the price (supply, demand, expenses, etc.). Give students an opportunity to debate a price that they would set for a new sweatshirt design they are trying to sell.

**Closure Notes**

The investigation should close with students feeling comfortable being able to expand a variety of different combinations of monomials, binomials, and trinomials. **See Exit Slip 8.4.2** This skill will be applied to real world examples as we work with quadratic functions in factored form and convert them to standard form. We will reverse the process in the next investigation, which will prove even more useful.

**Teaching Strategies**

1. **Activity 8.4.1** **Functions in Factored Form** introduces students to the factored form of a quadratic function. Using graphs and tables they observe that the *x*-intercepts of the quadratic function coincide with the *x*-intercepts of the linear factors. This leads to a statement of the Zero Product Property, which is used to solve equations where the product of two or more factors is zero. Students are also led to conjecture that the *x*-coordinate of the vertex of a parabola is the average of the two *x*-intercepts if they exist.

In **Activity 8.4.2 Finding the Maximum Profit**, students model the income generated by the sale of sweatshirts when the number of sweatshirts sold is a function of the decrease in price. In this example, when the price of a sweatshirt is $20, 30 sweatshirts are sold. Each time the sweatshirt price decreases by $1 an additional 5 sweatshirts are sold. What decrease in price (and resulting sweatshirt price) will maximize the revenue?

In this exercise, *x* represents the amount the price of a sweatshirt is dropped from its original price of $20. The number of shirts sold is 30 + 5*x*, the price of each shirt is 20 – *x*, so the income is (30 + 5*x*)(20 – *x*). Have students create a table and a graph of this function by evaluating values of *x* that result in prices from 0$ to 30$. Students should notice that the graph of this function is a parabola opening down with vertex at (7, 845). Dropping the price by $7 generates the maximum income of $845. Also, have students identify the *x*-intercepts and *y*-intercept from the graph. Point out the connection between the binomial (20 – *x*) and the second *x*-intercept. Model for students how to set the binomial equal to zero and solve for *x*, which leads them to an *x*-intercept. Challenge students to articulate why this procedure makes sense.

Have students work with a partner to compare the symbolic function to its graph. They should look at the first binomial (30 + 5*x*) and discuss how to use the symbolic function to identify the *x*-intercept. Some pairs of students or the entire class may need some guidance or additional examples to recognize that the *x* value of the vertex is the average of the two *x*-intercepts. Students may need to be reminded how to calculate the *y*-value of the vertex once they identify the *x*-value of the vertex, since their experience finding vertices in Investigation 2 was with functions in standard form. Close with **Exit Slip 8.4.1,** which asks students to identify the *x-*intercepts and vertex of a quadratic function in factored form. Use student responses from Exit Slip 8.4.1 to establish heterogeneous groups for the Group Activity.

**Group Activity**

Put students in heterogeneous groups of three. Give each group **Activity 8.4.3 Password**. Students will work to match quadratic equations in various forms to their *y*-intercept, *x*-intercepts, and vertex.

1. **Activity 8.4.3 Password** helps students understand the advantages of each form of the quadratic function. The factored form most easily reveals the *x-*intercepts; the standard form the *y*-intercept, and the vertex form the vertex.

**Differentiated Instruction (Enrichment)**

Challenge students to write a quadratic equation in factored form when given two *x*-intercepts and a *y*-intercept or when given the vertex and one *x*-intercept. See **Activity 8.4.4 Writing Quadratic Equations in Factored Form**.

**Differentiated Instruction (For Learners Needing More Help)**

Have struggling learners list sequential steps for calculating the intercepts and vertex of a quadratic function in factored form.

The following YouTube video shows how to graph a quadratic function in factored form. <http://www.youtube.com/watch?v=spFkuWt5Fh4>

**Journal Prompt 1**

You have seen three forms of quadratic functions: standard, vertex, and factored. Which do you think is the most useful? Why?

1. **Activity 8.4.5 Multiplying Polynomials** provides students an opportunity to practice multiplying polynomials. This activity may be assigned as classwork or for homework. Motivate the need to multiply binomials by posing the question: “How can we rewrite a quadratic equation in factored form so that it is in standard form?” Point out that this will entail multiplying two polynomials to obtain a new polynomial. Like integers, polynomials may be added, subtracted, and multiplied. Just as performing these operations on two integers always results in another integer, performing these operations on two polynomials results in another polynomial. The set of polynomials is said to be “closed” under the operations of addition, subtraction, and multiplication.

**Differentiated Instruction (Enrichment)**

Ask students who are interested in abstract questions to think about division. Is the set of integers closed under division? What about the set of polynomials? Ask students to explain their reasoning. (Neither set is closed.)

Provide students with multiple strategies for multiplying monomials, binomials, and trinomials. To help students construct a geometric understanding of multiplying monomials and binomials, present students with algebra tiles or area models as a method for expanding products. These models provide a visual understanding of the intermediate products and help students to learn the multiplication formulas. For example, to multiply (2*x*)(*x* + 7) have students calculate the inside areas and calculate the total area in the following diagram:



To multiply (*x* + 2)(*x* + 5) have students calculate the inside areas and calculate the total

area of this rectangle.



Now, show students how to model the same two multiplications using algebra tiles. For a quick explanation of algebra tiles, you can show the following video: <http://media.mivu.org/mvu_pd/a4a/homework/multiplication_multiplication_intro.html>

You may want to click here for a PowerPoint that reviews how to use Algebra Tiles: <http://www.ufgop.org/browse/aHR0cDovL3d3dy50cmlhbmdsZWhpZ2hmaXZlLm9yZy9wZGYvMDA1X2FsZ2VicmFfdGlsZXMucGRm>

Students can either model the two polynomial multiplications using concrete algebra tiles or virtual ones. Two excellent algebra tile apps are on the National Library of Virtual Manipulatives site at: <http://nlvm.usu.edu/en/nav/category_g_4_t_2.html> and at NCTM’s Illuminations site at: <http://illuminations.nctm.org/ActivityDetail.aspx?ID=216>.

Instructions for using the NCTM activity are included as a file to be used with **Activity 8.4.5 Multiplying Polynomials.**

**Differentiated Instruction (For Learners Needing More Help)**

Encourage students to continue to use algebra tiles or to draw area models for all the problems in **Activity 8.4.5**.

**Differentiated Instruction (For Enrichment)**

Expand on **Activity 8.4.5** by asking students to find products of three or more binomials: e.g. (*x* + 4)(2*x* – 3)(*x* – 5). Also ask students to find squares and cubes of binomials such as $(x-2)^{2}$ and $(x-2)^{3}$ and look for patterns.

In addition, show students how the distributive property is used when multiplying polynomials. Students are familiar with using the distributive property to multiply monomials by binomials and monomials by trinomials. Show them that they can extend this concept to multiply two binomials or multiply a binomial by a trinomial. For example, to multiply (2*x* + 6)(3*x* – 4), distribute the first binomial (2*x* + 6) to both terms in the second binomial. This results in the following expression:

(2*x* + 6)(3*x* – 4) = (2*x* + 6)(3*x*) – (2*x* + 6)(4) = 6*x*2 + 18*x* – 8*x* – 24

= 6*x*2 + 10*x* – 24

Encourage students to use whatever method they feel comfortable with: concrete materials (algebra tiles), illustrations (area models), or the more formal method of showing application of the distributive property. Avoid teaching “tricks” like FOIL which are often applied mindlessly. FOIL is a particular algorithm that only works when multiplying a binomial times a binomial.

You may introduce students to Wolfram Alpha, [www.wolframalpha.com](http://www.wolframalpha.com), a free online computer algebra system that expands, simplifies, and factors algebraic expressions. This resource allows students to expand products of binomials and could be used a resource to check their work.

1. Review the three forms of the quadratic function that have been introduced in this unit: standard form $f\left(x\right)=ax^{2}+bx+c$ (Investigation 1), vertex form $f\left(x\right)=a(x-h)^{2}+k$ (Investigation 2) and factored form $f\left(x\right)=a(x-p)(x-q)$ (Investigation 4). Discuss with the class what we know already about changing a function from one form to another. Students should recall that if they have a quadratic equation in standard form, they can find the *x*-coordinate of the vertex using the formula $x=-\frac{b}{2a}$. Then they may substitute this value into the function to find the *y*-coordinate of the vertex and then write the function in vertex form. Similarly, if the function is in factored form they can find the line of symmetry (and the *x*-coordinate) of the vertex by taking the average of the two *x*-intercepts, *p* and *q.* Once they have the vertex, only a few steps are required to write the function in vertex form.

Now that we know how to multiply binomials we can expand our ability to change forms. We can take a function in factored form and expand the expression to produce a function in standard form. Likewise when the function is in vertex form we can expand the binomial square to get the function in standard form. **Activity 8.4.6 Standard Form for Quadratic Functions** focuses on these skills.

In Investigations 5 & 6, we will examine the ways in which quadratic functions in standard form can be changed to quadratic functions in factored and vertex form. This chart summarizes the six ways that the form of a quadratic function may be changed.

|  |  |  |
| --- | --- | --- |
| **To change a function** | **Here’s what to do** | **See****Investigation** |
| **From** | **To** |
| Standard form | Vertex form | Find the vertex using *x* = –*b/(2a*) | Investigation 2 |
| Factored form | Vertex form | Find the average of the *x-*intercepts | Investigation 4 |
| Factored form | Standard form | Multiply binomials | Investigation 4 |
| Vertex form | Standard form | Expand square of a binomial | Investigation 4 |
| Standard form | Factored form | Factor a quadratic trinomial | Investigation 5 |
| Complete the square or use the quadratic formula | Investigation 6 |
| Vertex form | Factored form | First find the standard form then find the factored form | Investigations 4, 5, and 6 |

**Group Activity**

Students work in groups of four. Each group has four slips of paper with the following types of multiplication problems: (1) monomial by binomial, (2) monomial by trinomial, (3) binomial by binomial, and (4) binomial by trinomial. Each student draws a slip of paper and makes up a problem in the given category. Pass the papers to the right. Students perform the multiplication problem on the slip given to them. Pass the papers to the right again and check the previous student’s work.

**Differentiated Instruction (For Learners Needing More Help)**

Modify the group of activity by placing students in groups of 3 and eliminating the last option (binomial by trinomial).

Use **Exit Slip 8.4.2** to assess students’ abilities to multiply polynomials.

**Resources and Materials**

* **Activity 8.4.1** Functions in Factored Form
* **Activity 8.4.2** Finding the Maximum Profit
* **Activity 8.4.3** Password
* **Activity 8.4.4** Writing Quadratic Functions in Factored Form
* **Activity 8.4.5** Multiplying Polynomials
* **Activity 8.4.6** Standard Form for Quadratic Functions
* **Exit Slip 8.4.1** Quadratic Functions
* **Exit Slip 8.4.2** Multiplying Polynomials
* **Algebra Tiles Instructions for Investigation 4**
* Algebra Tiles
* Student Journals
* Graphing Calculators