**Compound Interest**

1. $10,000 is invested in a certificate of deposit (CD) that pays interest at the rate of 3% per year compounded annually. This means that every year the interest is computed and then added to the principal. This is shown in the table below. The first three years have been filled in.

|  |  |  |  |
| --- | --- | --- | --- |
| **Column 1** | **Column 2** | **Column 3** | **Column 4** |
| **Year** | **Principal at the beginning of the year**  (same as principal at the end of the previous year) | **Interest for the year**  (= amount in Column 2 times 0.03) | **Principal at the end of the year**  (= amount in column 2 + amount in column 3) |
| 1 | $10,000.00 | $300.00 | $10,300.00 |
| 2 | $10,300.00 | $309.00 | $10,609.00 |
| 3 | $10,609.00 | $318.27 | $10,927.27 |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  |  |
| 8 |  |  |  |
| 9 |  |  |  |
| 10 |  |  |  |

1. Complete the table above. (Round answers to the nearest cent.)
2. Describe any patterns you see in the table.
3. Do the amounts in Column 4 appear to be growing linearly or exponentially? How can you tell?
4. The amount each year can be considered a percent increase of 3%. You have learned earlier that this corresponds to multiplying by a growth factor. What is the growth factor in this situation?
5. You can generate the numbers in column 4 using the constant multiplication feature of your calculator. Follow these steps:
   * 1. Type in 10,000 and press ENTER.
     2. Press \* 1.03 and press ENTER.
     3. Press ENTER 9 more times.

What do you notice?

1. Write an explicit equation for the function that gives the value of the CD after *x* years.
2. Substitute *x* = 10 in your function. Do you get the same result as in the table and in step (e)?
3. Suppose you take out a loan for $10,000 and agree to pay it back ten years later. The interest rate is 3%, but it is not compounded. (This is what is called “simple interest”).
4. Each year you pay the same amount for the interest. What is that amount?
5. What is the total amount you will pay by the end of the tenth year, including the principal plus the interest?
6. The situations in questions 1 and 2 illustrate the difference between compound interest and simple interest. One may be considered linear growth and the other exponential growth. Which is which? Explain your reasoning.
7. Write an explicit equation for each of these situations.
8. $500 is invested in a bond paying 8% interest compounded annually. What is the value of the bond after *x* years?
9. $1000 is invested in a bond paying 5% interest compounded annually. What is the value of the bond after *x* years
10. Which of the two bonds in questions (a) and (b) will be worth more after 30 years? How much more?
11. Sometimes interest is compounded more frequently than every year. For example, suppose the CD in question 1 pays the same rate of interest (3% per year) but the compounding takes place at the end of each month.
12. 3% per year is equivalent to \_\_\_\_\_\_% per month.
13. If the CD is held for 10 years, how many times will the interest be compounded?
14. Write an exponential function that shows how much this CD is worth after *x* months.
15. How much will the CD be worth after 120 months?
16. How does the value of this CD after 10 years compare with the value of the CD in question 1 after 10 years?
17. Explain why the two values in (e) are different.
18. If you visit a bank or a credit union you will see that certificates of deposit are often described by APY. This stands for annual percentage yield. Here’s how that works. Suppose the rate of interest is 6% and the interest is compounded monthly.
19. At the end of each month, what percent of the investment will be added on? This is the monthly rate of interest.
20. The growth factor for one month will be of the form 1 + *r*, where *r* is the monthly rate of interest. What is the growth factor for one month?
21. At the end of the year the principle will have been multiplied how many times by 1+*r*?
22. If *P* is the principal, at the end of the year the investment will be worth .
23. If the interest rate is 6% and the money is compounded every month, what will be the growth factor for the year?
24. What annual rate of interest corresponds with the growth factor in (e)? This is the APY.
25. Suppose instead of compounding every month, the bank compounds only twice a year. Suppose the interest rate is 6% per year. Find the APY for this investment.