**Unit 7: Investigation 2 (4 Days)**

**EXPONENTIAL GROWTH AND EXPONENTS**

*CCSS: N-RN 1, N-RN 2, F-IF 7e, F-LE 1, F-LE-3*

**Overview**

Students continue to explore exponential functions and distinguish between linear growth and exponential growth. By extending patterns based on properties of positive integer exponents, students learn the meaning of zero, negative and rational exponents, and begin to understand that the domain of an exponential function is all real numbers. The culminating activity is based on the classic “rice and chessboard problem” that emphasizes how an exponential function will eventually overtake a linear function.

**Assessment Activities**

**Evidence of Success: What will students be able to do?**

* Recognize that whereas linear growth patterns can be modeled by $y=mx+b$, exponential growth patterns can be modeled by $y=ab^{x}$.
* Explore patterns with positive integer exponents to justify the rules:

 $a^{m}a^{n}=a^{m+n}$, $\frac{a^{m}}{a^{n}}=a^{m-n}$, and $\left(a^{m}\right)^{n}=a^{mn}$.

* Extend the meaning of exponents to include zero and negative integer exponents.
* Extend the meaning of exponents to include rational exponents.
* Contrast linear and exponential growth.

 **Assessment Strategies: How will they show what they know?**

* **Exit Slip 7.2.1** assesses students’ understanding of the rules of exponents and the meaning of integer exponents.
* **Journal Entry 1** asks students to explain why raising a number to a negative power does not necessarily result in a negative number.
* **Exit Slip 7.2.2** assesses students’ understanding of rational exponents.
* **Journal Entry 2** asks students to explain what they have learned about exponential growth based on the “rice and chessboard” activity.

**Launch Notes**

Ask students to recall from Unit 1 the differences and similarities between arithmetic and geometric sequences. You may want to return briefly to **Activity 7.1.2 Is This a Good Deal?** to point out that the amounts paid each day form a geometric sequence. Geometric sequences are exponential functions (defined only on the integers) whereas arithmetic sequences are linear functions defined on the integers. This distinction leads us to **Activity 7.2.1 Exploring Growth Patterns.**

**Closure Notes**

Discuss the conclusions to **Activity 7.2.6 How Much Rice?** Be sure that students understand that even after they have converted the way rice is measured in Option 1 from cups to grains of rice, and even if they assume that the mathematician lives a very long life, the amount of rice obtained from Option 2 is far greater than the amount of rice obtained from Option 1. Put another way, the exponential function has overtaken the linear function.

**Teaching Strategies**

1. **Activity 7.2.1 Exploring Growth Patterns** uses block growth patterns to compare and contrast linear and exponential growth. We begin with linear patterns. Students are first asked to explain each pattern in words and then with an equation. Students review the meaning of the parameters *b* and *m* in the linear equation *y* = *mx* + *b.* Recognizing the initial starting amount and the amount *added* at each stage will be useful when extending to exponentials. This activity encourages students to develop the general equation for exponential growth$ y=ab^{x}$, and helps students begin to understand that *a* is the initial value and *b* is the growth factor (or constant multiplier).
2. **Activity 7.2.2 The Meaning of Integer Exponents** begins with a review of the meaning of exponential expressions with positive integer exponents. Students recognize that positive integer exponents represent repeated multiplication of the base. Students explore how exponents work and discover (or rediscover) shortcuts for exponential expressions, in particular examples such as:

$$2^{4}∙2^{3}= \left(2∙2∙2∙2\right)∙\left(2∙2∙2\right)= 2∙2∙2∙2∙2∙2∙2= 2^{7}$$

$$\left(2^{3}\right)^{2} = \left(2∙2∙2\right)^{2} = \left(2∙2∙2\right)∙\left(2∙2∙2\right)= 2∙2∙2∙2∙2∙2= 2^{6}$$

This idea is extended as students look for patterns in a table showing powers of 10 to construct a meaning for zero and negative integer exponents.

**Differentiated Instruction (For Learners Who Need More Help)**

Encourage students to write out expressions with positive integer expressions “the long way” as repeated multiplication as long as they find it necessary before they are expected to apply the rules of exponents. This will not only ensure that they get correct answers but will help them focus on the meaning of exponents.

**Exit Slip 7.2.1** may be given following this activity.

**Journal Entry 1**

A student in another class tells you that $2^{-5}$must be a negative number because of the minus sign in front of the five. Give an argument that would convince the student that they are incorrect.

In **Activity 7.2.3** **Exploring the Meaning of Rational Exponents** students are given an opportunity to make a conjecture about the meaning of$ 9^{\frac{1}{2}}$ and determine whether their conjecture provides a solution that fits the pattern of powers of 9. They also consider whether their conjecture satisfies the rules and meanings of exponents after reviewing some additional patterns.

You can discuss how the definition and meaning of rational exponents follows from extending the properties of integer exponents (as in **Activity 7.2.2**) to rational values, allowing for a notation for radicals in terms of rational exponents. For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.

**Differentiated Instruction (Enrichment)**

The emphasis in **Activity 7.2.3** is placed on unit fraction exponents. Encourage students who are able to discover on their own a meaning for $a^{\frac{m}{n}}$.

1. **Activity 7.2.4 Roots and Exponents** reinforces the connection between radical expressions such as $\sqrt{25}$, $\sqrt[3]{64}$, and $\sqrt[4]{16}$ and their equivalent exponential expressions $25^{\frac{1}{2}}$, $64^{\frac{1}{3}}$, and $16^{\frac{1}{4}}$. The activity begins with students filling in a table of powers. Students should already know all the squares in the table and have an understanding of place value that enables them to find all the powers of 10 without a calculator. Many students may also already know powers of 2 up through $2^{6}. $Encourage students to learn the remaining powers in the table.

Based on the previous activity students should see the connection between an integer exponent statement such as $3^{2}=9$ and the corresponding unit fraction power $9^{\frac{1}{2}}=3$. You may then show students how to find roots on the calculator. Point out that square roots are usually written without the index 2 and can be found on the calculator as 2nd $x^{2}$. In the MATH menu there is a special function for cube root (MATH 4). For higher routes the index must be entered before the $\sqrt[x]{}$ function (MATH 5). Several questions in this activity relate squares, square roots, cubes and cube roots to the geometry of squares and cubes.

**Differentiated Instruction (For Learners Who Need More Help)**

For students still struggling with the concepts of area and volume, questions 9, 10, 13, and 14 in **Activity 7.2.4** may be enhanced with the use of manipulatives such as square tiles and linked cubes.

**Exit Slip 7.2.2** may be given following this activity. Note that students should not need calculators for this assessment.

**Activity 7.2.5 Exploring an Exponential Function** demonstrates the connection between the graph of an exponential function and the meaning of negative and rational exponents. With a carefully chosen function ($y=16^{x}$) and a carefully chosen window, students use the trace feature of the calculator to find values of *y* when *x* is a multiple of 0.25. Note the importance of having a window with Xmin = –2.35 and Xmax = 2.35 to ensure that all multiples of 0.25 will appear. Students are asked to verify that the values found on the graph correspond to those specified by the definitions of exponents they discovered in the previous activities. They will also learn that appearances can be deceiving since in the first window the graph does not appear to extend into the second quadrant. To get a full picture of the behavior of this function, several windows should be used.

1. **Activity 7.2.6 How Many Grains?** provides an opportunity for students to contrast linear and exponential growth in a dramatic way. Question 2 in Activity 7.2.6a poses an open-ended question, “what additional information will you need?” Activity 7.2.6b has this question scaffolded so students have more guidance on how to approach the problem.

There are several versions of the story about the king and the inventor of chess on the Internet. Here is one. <http://mathforum.org/sanders/geometry/GP11Fable.html>. To figure out how many grains of rice are in one cup, you may have students perform an Internet search, or bring in rice and measuring cups to have them figure it out for themselves.

**Group Activity**

Have students work in groups of 2 or 3 on **Activity 7.2.6**. They may then present their conclusions to the class.

**Differentiated Instruction (For Learners who Need More Help)**

Assign **Activity 7.2.6b** rather than **Activity 7.2.6a.**

**Journal Entry 2**

In the story about the mathematician and the king, the king at first thinks that the mathematician is foolish not to ask for gold or diamonds instead of rice. What did the mathematician know that the king did not know about exponential growth patterns? Write at least one paragraph to explain your answer in detail.

**Resources and Materials**

* **Activity 7.2.1 Exploring Growth Patterns**
* **Activity 7.2.2 The Meaning of Integer Exponents**
* **Activity 7.2.3 Exploring the Meaning of Rational Exponents**
* **Activity 7.2.4 Roots and Exponents**
* **Activity 7.2.5 Exploring an Exponential Function**
* **Activity 7.2.6a How Many Grains?**
* **Activity 7.2.6b How Many Grains?**
* **Exit Slip 7.2.1 Exponential Expressions**
* **Exit Slip 7.2.2 Rational Exponents**
* Square tiles and linked cubes (possibly for **Activity 7.2.4**)
* Rice and measuring cups (possibly for **Activity 7.2.6**)
* Graphing Calculators
* <http://mathforum.org/sanders/geometry/GP11Fable.html>
* Bulletin board for key concepts