**Unit 7: Investigation 1 (2 Days)**

**A NEW FUNCTION FAMILY – EXPONENTIAL FUNCTIONS**

*CCSS: F-IF 7e, F-BF 2, F-LE 1a, F-LE 3*

**Overview**

This investigation uses the context of world population growth and world food production to introduce exponential growth patterns and illustrate how exponential growth compares to linear growth.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able To Do?**

Students will be able to distinguish between linear and nonlinear growth in tables or in graphs, recognize that nonlinear growth leads to average rates of change that are not constant, use a recursive feature of a graphing calculator to model exponential growth, and recognize that exponential growth occurs when there is a constant multiplicative pattern among function values.

**Assessment Strategies: How will they show what they know?**

* **Exit Slip 7.1.1** assesses student’s ability to use the constant multiplication feature on the calculator.
* **Exit Slip 7.1.2** asks students to compare and contrast linear and exponential growth.
* **Journal Prompt 1** asks students to consider causes of world hunger.
* **Journal Prompt 2** asks students to predict how world hunger will change if the world population continues to grow exponentially and world food production continues to grow linearly.

**Launch Notes**

You may begin this investigation by initiating a discussion on world hunger, a topic that often captures student interest. Consider assigning the opening journal prompt as homework before the first day of this unit so students can come in prepared to discuss their ideas. You may present the paragraph in **Activity 7.1.1a** or **Activity 7.1.1b** at the start of the first day to facilitate an in-class discussion.

You may also initiate this investigation by presenting data or information from the following websites.

* [www.fao.org/hunger/en/](http://www.fao.org/hunger/en/)

This site from the Food and Agriculture Organization (FAO) of the United Nations has an interactive hunger map. Click on the full screen link. When you click on a country the screen will have a pop-up with the name of the country and the percentage of population that is undernourished.

* [www.wfp.org/hunger/map](http://www.wfp.org/hunger/map)

This site from the United Nations World Food Program has an interactive world hunger map. Select among the language options, and then click on map. The map of world hunger will fill the screen. Under the Hunger drop-down menu you will find Hunger FAQ that can add to your class discussion. You might also check the Videos menu to find a clip your class might like.

* <http://www.stopthehunger.com/>

This site has with real time statistics on world population, the number of undernourished people in the world, the amount of global food aid provided daily, and the amount of food wasted in America.

* [www.youtube.com/watch?v=4BbkQiQyaYc](http://www.youtube.com/watch?v=4BbkQiQyaYc)

This is a seven-minute video clip by Population Connection. (You can purchase the clip from Population Connection.) It traces world population growth from 1 AD through 2030 AD by placing dots in the appropriate area of the globe for each new one million people in that area. It also provides historical references such as the Roman Empire, Golden Age of India, Mayan Empire, Rise of Islam, Mongols in China, and the Industrial Age. You might check with teachers in other disciplines to see if you can connect your discussion to ideas from their classes.

* [www.**youtube.com**/watch?v=HsAracLBCxI](http://www.youtube.com/watch?v=HsAracLBCxI)

This video presents the “myth of overpopulation” and describes factors that lead to global hunger.

**Closure Notes**

The investigation culminates with students successfully demonstrating their ability to compare and contrast the exponential growth of world population and the linear growth of world agriculture production. This may be done through written response to the questions posed in **Activity 7.1.5**.

**Teaching Strategies**

1. In **Activity 7.1.1** **Is Population Growth Linear?**, students are introduced to the context of world hunger and presented with world population data from 1804 to 2010. Their task is to determine whether or not the data are linear. At first glance, the average rates of change (Δ*y*/Δ*x*) are close in value, and depending on the chosen scale, the graph could look somewhat linear. Additionally, the correlation coefficient for the least-squares linear regression line is high enough that students could argue that the relationship is almost linear. However, when the graph is drawn, students should observe that the data points appear to lie along a curve rather than a straight line.

While students will make arguments on both sides, the ultimate conclusion should be that the data are NOT linear. In particular, the difference between successive function values is consistently increasing. This activity will likely lead to the discovery that sometimes deciding whether or not real data are linear can be complex and should involve more than one tool (graphs, tables and differences, regression equations and correlation coefficients.)

This activity comes in two forms, **Activity 7.1.1a (open ended)** and **Activity 7.1.1b (scaffolded).** Use one or both activities depending upon the needs of your students.

|  |
| --- |
| **Group Activity**  **Activity 7.1.1** can be implemented as a student debate. Begin by forming groups of two to four students. Prompt the groups to first brainstorm about how they can decide whether or not the data are linear. One student can be the recorder (to write down the group’s ideas) another can be the reporter (to explain the ideas to others.) Once they recall their prior knowledge about the properties of linear tables (constant rates of change), graphs, and equations, and how to use linear regression and the correlation coefficient, each group is assigned to defend one of two positions – the first is that the world population data are linear, the other is that the data are NOT linear.  Finally, bring the class together, perhaps with all the groups on ‘Team Linear’ on one side, and the groups on ‘Team Nonlinear’ on the other. The debate can go back and forth between the two teams. On each turn, the team gives one reason or comment in support of their position, with a different group (randomly selected) from each team taking the turn each time. You may want to assign a group of judges to critique the arguments made on both sides.  While students will make arguments on both sides, the ultimate conclusion should be that the data are NOT linear. This activity will lead to the discovery that sometimes deciding whether or not real data are linear can be complex and should involve more than one tool (graphs, tables and differences, regression equations and correlation coefficients.) |

|  |
| --- |
| **Differentiated Instruction (For Learners who need more help)**  **Activity 7.1.2b** is an alternate version of the activity that provides scaffolds for students to use to determine whether or not data are linear. |

|  |
| --- |
| **Journal Prompt 1**  World hunger is an issue we are all be concerned about. Crop yields are increasing but so is the world population. Scientists need to know if one quantity is growing much faster than the other. What are some of the factors that affect world hunger?What could happen if the population is growing much faster than the amount of food produced? |

**Activity 7.1.2** **Is it a Good Deal?** presents students with a situation that involves exponential growth. Through the exploration of a simple doubling model, in which successive function values change based on a multiplier of 2, students will discover that this new pattern, exponential growth, *eventually* grows very, very large, even when you begin with a very small initial amount.

Students should recall the activities from Unit 1 and recognize that this function as an example of a geometric sequence. You do not need to point out this connection to students at this point in this investigation. Students will make connections to Unit 1 on their own as this Unit 7 progresses. This activity may be assigned for homework.

You may assign **Exit Slip 7.1.1 Amount Earned** to check students’ ability to use a recursive feature of the calculator to find values in a multiplicative pattern.

1. In **Activity 7.1.3** **A Closer Look at World Population Data**, students explore how the world population data fits a pattern determined by repeated multiplication by a constant multiplier. This activity is very brief. It also provides an opportunity to discuss the fact that mathematical models can only *approximate* real-world data.

**Activity 7.1.4 World Agriculture Production** presents students with data on global agriculture production. The students’ task is to justify that a linear model is a good fit for the data and to interpret the meaning of the slope and *y*-intercept of the regression line. Since these are real data, the differences are not exactly constant. Students should observe, however, that the differences in successive outputs in the agriculture index data are not consistently increasing, as were the population data. For example, sometimes the index will increase by 9 over two years, but other times it will only increase by 6 over two years. Students should use the interpretation of the slope to describe how the data are increasing in at a constant rate.

**Activity 7.1.5** **Population and Food Production** provides students with some additional information that relates to the issue of world hunger. Students should compare the exponential growth of the world population with the linear growth of agriculture production. Students should also consider other factors that may determine whether or not there will be enough food in the future to feed the global population. As they consider the long term effects out to the year 2055, they should be reminded that predictions based on extrapolation are less reliable than those based on interpolation. This activity can be used as a writing homework assignment, as a prompt for small group or classroom discussion, or as a prompt for a journal entry.

Assign **Exit Slip 7.1.2 Linear Growth vs Exponential Growth** tocheck students’ ability to list similarities and differences between linear growth and exponential growth.

**Differentiated Instruction (For Enrichment)**

Ask students to find an explicit rule for the exponential function based on the multiplicative relationship demonstrated in **Activity 7.1.3** and use it to verify the prediction for world population in 2055 found in **Activity 7.1.5**.

|  |
| --- |
| **Journal Prompt 2**  The data we explored shows that world agriculture production is growing at a linear rate and the world population is growing at an exponential rate. Describe why you think we will have increased hunger or decreased hunger in the world if these trends continue. Support your ideas with information that can be researched. |

**Resources and Materials**

* **Activity 7.1.1a** Is Population Growth Linear?
* **Activity 7.1.1b** Is Population Growth Linear?
* **Activity 7.1.2** Is it a Good Deal?
* **Activity 7.1.3** A Closer Look at World Population Data
* **Activity 7.1.4** World Agriculture Production
* **Activity 7.1.5** World Population and Food Production
* Bulletin board for key concepts
* Graphing Calculators
* Student Journals
* Projector
* Computers
* Rulers