**Direct Variation**

1. When building a roof, carpenters place posts every 2 feet along the horizontal support beam starting at the eave. The diagram below illustrates this.



1. Complete the table below.

|  |  |  |  |
| --- | --- | --- | --- |
| **Post** | **Horizontal distance from post to eave** (*x*) | **Height of Post**(*y*) | **Ratio***y* ÷ *x* |
| 1 | 2 | 1.5 | 0.75 |
| 2 | 4 | 3 |  |
| 3 |  | 4.5 |  |
| 4 |  | 6 |  |

1. Circle the appropriate words to make the statement true. Consider only the posts from the left eave to the center peak.

As the height of post (increases/decreases), the horizontal distance from post to eave (increases/decreases).

1. What do you notice about the last column? Does it represent anything we learned about equations of lines? If so, what?
2. One of the points is (0,0). What would this point mean in the context of the problem?
3. Use the slope formula to find the slope between the point (0,0) and (4,3).
4. Use the slope formula to find the slope between the two points (0, 0) and (2, 1.5).
5. Describe how you can find the slope in a direct variation problem.

1. Write a linear equation in *y* = *mx* + *b* form for the height of a post as a function of distance from eave.
2. Graph the linear equation. Label the axes and choose a proper scale.



1. A bicyclist traveled at a constant speed during a timed practice period. His distance varied directly with the time elapsed. After 10 minutes elapsed, his distance travelled was 3 miles.
2. Complete the table below.

|  |  |  |
| --- | --- | --- |
| **Elapsed Time**(*x*) in minutes | **Distance**(*y*) in miles | **Ratio***y* ÷ *x* |
| 10  | 3 | .3 |
| 20 | 6 |  |
| 30 |  |  |
| 40 |  |  |

1. Circle the appropriate words to make this statement true.

As the elapsed time (increases/decreases), the distance in miles (increases/decreases).

1. Does any value in the table represent the slope? If so, what?
2. One of the points is (0,0). What does this point mean in the context of the problem?
3. Use the slope formula to find the slope between the two points (0,0) and (10,3).
4. Use the slope formula to find the slope between the two points (0,0) and (20,6).
5. Write a linear equation in *y* = *mx* + *b* form.
6. Graph the linear equation. Label the axes and choose a proper scale.



1. When two variable quantities have a constant ratio, their relationship is called a *direct variation*. It is said that one variable “varies directly” with the other variable.

Fill in the blanks.

1. In the roof problem (problem 1) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ varies directly

with \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

1. In the bike problem (problem 2) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ varies directly

with \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

1. The constant ratio (also known as the slope) is called the *constant of variation*.
2. Looking back at the roof problem (problem 1), what was the constant of variation?
3. Looking back at bike problem (problem 2), what was the constant of variation?
4. Describe how to find the constant of variation (slope).
5. We can also say that *y is proportional to x,* which means the same as *y varies directly with x*. The *x* multiplied by the constant of variation will equal *y*.
6. In the roof problem 1, you multiply the length from the eave by .75 to get \_\_\_\_\_\_\_\_.
7. In the bike problem 2, you multiply the elapsed time by \_\_\_\_\_\_\_ to get \_\_\_\_\_\_\_\_\_\_.
8. In the roof problem, we could set up a proportions such as: $\frac{1.5 }{2}=\frac{3}{4}$. Write a proportion for the bike problem.
9. The formula often used for direct variation is $y=kx$, where *k* is the constant of variation. A direct variation equation is a special case of the slope-intercept form. We can rewrite it as $y=mx$.
10. What was the linear equation you found in problem 1?
11. What is the *y*-intercept of this equation?
12. What was the linear equation you found in problem 2?
13. What is the *y*-intercept of this equation?
14. In the roof problem, what special relationship exists among the 4 nested triangles formed by the roof line, the 4 different posts and the base?
15. What is an example of a direct variation problem compared to one that is not? Here are two scenarios, the first is direct variation, and the second is not.

Scenario 1: The cost of topsoil varies directly as the number of cubic yards purchased if you can pick it up in your own truck. 5 cubic yards costs $60.

Since cost varies directly as number of cubic yards, then cost is a function of cubic yards.

1. What is the independent variable *x*?
2. What is the dependent variable *y*?
3. What is he constant of variation *y*/*x*?
4. Write an equation for cost as a function of cubic yards purchased.

Scenario 2: If you have the company deliver it in their truck they charge a $50 delivery fee in addition to the $12 per cubic yard.

1. Write an equation for cost as a function of cubic yards purchased for customers that have the soil delivered.
2. Make a table of values for each situation, showing how much it costs in total for 1-6 cubic yards:

|  |  |
| --- | --- |
| **Customers that haul their own topsoil** | **Customers that have the topsoil delivered** |
|

|  |  |
| --- | --- |
| **# cubic yards** | **Cost (in $)** |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |

 |

|  |  |
| --- | --- |
| **# cubic yards** | **Cost (in $)** |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |

 |

1. When you double the amount of topsoil you buy from 2 yards to 4 cubic yards, is the cost also doubled?

|  |  |
| --- | --- |
| **Customers that haul their own** | **Customers that have the topsoil delivered** |
| Cost is /is not doubled when amount of soil is doubled.  | Cost is /is not doubled when amount of soil is doubled. |

1. When you triple the amount of topsoil, do you expect to pay triple the amount? (Answer yes or no.)

|  |  |
| --- | --- |
| **Customers that haul their own** | **Customers that have the topsoil delivered** |
| Yes or No? | Yes or No? |

**Conclusion**: In direct variation problems, if you multiply the independent variable by a number, the dependent variable is multiplied by the same number. For example, if you double the input, the output will double. In direct variation problems, the output is proportional to the input.

If the *y*-intercept is not zero, the linear relation is not a direct variation problem. Doubling the input will not result in doubling the output. The output is not a constant multiple of the input.