**Unit 4: Investigation 3 (4 Days)**

**CALCULATING AND INTERPRETING SLOPE**

***CCSS:*** CCSS: F-IF6, F-LE1a, F-LE1b

**Overview**

Students discover how to identify the slope of a linear function from a table, two ordered pairs, graph and the verbal description of a linear function. Students also learn how to interpret the slope in the context of real world situations.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Determine run, rise, and slope given two points in the coordinate plane.
* Identify the slope given the verbal description, graphic or tabular model of a linear function.
* Graph a line given a point and the average rate of change or slope.
* Graph a linear function by creating a table of values when given an equation for the linear function.
* Recognize rates in the form of units of the dependent variable per units of independent variable.
* Interpret the rate of change of the linear function in a real world context.
* Identify and graph horizontal and vertical lines.
* Determine whether lines are parallel or perpendicular.

**Assessment Strategies: How Will They Show What They Know?**

**Exit Slip 4.3.1** assesses students’ understanding of the relationship between slope and rate of change.

**Exit Slip 4.3.2** asks students to calculate the slope of a line and use the slope to determine its direction and steepness.

**Journal Entry** asks students to apply the concept of slope to a previously encountered function in context.

**Launch Notes**

Break out the motion detector again. Have students do 4 different walks with the motion detector recording their distance as a function of time. One walk should be increasing and steep, the other decreasing and steep. Two other walks should be increasing and almost flat, the other decreasing and almost flat. Using the table feature or the graph’s trace feature, obtain two coordinate pairs on each of the four graphs. Calculate the slope between two points on each of the four walks. Class discussion should draw the connection among fast walks, steeper graphs and slopes of larger magnitude (or absolute value). Slow walks correspond to flatter graphs and slopes smaller in absolute value. Increasing the distance from the motion detector results in a positive slope, whereas decreasing the distance results in negative slope.

**Closure Notes**

Slope is one of the fundamental concepts in mathematics. By the end of this investigation, students should have a very good understanding of slope and a facility for calculating and interpreting it.

**Teaching Strategies**

1. In **Activity 4.3.1 What is Slope,** students transition from a graphical understanding of slope to a numerical formulation of slope. Building upon student understanding of average rate of change and the method developed in Investigation 2 for finding the average rate of change between two data points, students will describe and use the run and rise between two points in the four-quadrant coordinate plane to find the rate of change, or slope. Students will quantify the rise, or the change in *y*, as *y2-y1* and the run, or the change in *x*, as *x2-x1*, and then calculate the slope of the line between two positions by taking the ratio of these two quantities. The teacher will explain that the letter “*m*” is often used to designate the slope of a line.

You should offer multiple contextual problems and allow students to select specific problems and work individually or in groups. Have students share their results and probe for their understanding of the meaning of the slope in the context of the situations. Students may now calculate slopes from data in the tables and graphs, verify that the coefficient of *x* is the slope and interpret the slopes as rates of change with appropriate units, e.g., 25 gallons per 10 minutes. Students will learn to express slope in unit rates as well, 2.5 gallons per minute. You may assign the **Activity 4.3.2 Calculating and Interpreting Slope.**

1. Students continue to explore and practice with the slope formula in **Activity 4.3.3 Positive and Negative Slope**. They will make the connection between the direction of the graph (increasing, decreasing or horizontal) and the sign of the slope (positive, negative or 0). Students may explore slope as the ratio of rise to run in the context of the pitch of a roof, or the slope of a mountain. Through calculating the slope of a roof using the legs of similar triangles, students will recognize that the slope of a line (or a roof) is the same regardless which two points on the line (roof) are chosen. You may need to discuss the properties of similar triangles in order for students to solve for unknown quantities within the context of the problems. You may assess students’ understanding of the connection between slope and rate of change with **Exit Slip 4.3.1**.
2. The focus of **Activity 4.3.4 Magnitude of Slope** is on identifying the steepness of a linear function by the magnitude of its slope. Use a class discussion to develop a rule of thumb for what is considered a steep slope and what is not so steep. Draw lines with slope +/- 4 or +/-5 on the board. Then draw lines with slope +/- $\frac{1}{3}$ or +/- 0.1. Ask students to vote on which lines are steep and which are not. Sketch more lines with slopes gradually closer to +/- 1, continuing to ask students to identify what is steep and what is not so steep. Direct the discussion so that students conclude that the larger the magnitude (the absolute value) of the number associated with the slope of a line, the steeper the line. You may use **Exit Slip 4.3.2** to determine whether students have grasped this relationship.
3. Next, students should explore tables, graphs and equations of horizontal lines with equations of the form *y = b* and vertical equations of the form *x = a.* The calculator can be used here. Equations for horizontal lines may be entered into the Y= menu; however, for vertical lines the DRAW menu must be used, perhaps only by the teacher as a demonstration. You may begin by presenting the table form of vertical and horizontal lines, and have students review what they have found in Investigation 1 about the slope of each type of line.

Horizontal lines have a slope of zero (*y = 0x + b*) (all run, no rise). These give rise to constant functions of the form *f(x) = b.* You may help students understand the concept of a constant function with these examples. (1) In a snack machine the input is the combination of buttons you push and the output is the item selected. A snack machine filled with identical potato chips may be considered a “constant function.” (2) We have seen that many numerical functions are associated with a rule such as “multiply by 3.” Suppose the rule is “multiply by 0.” Then we get the same output for every input.

Review the idea that vertical lines have undefined slopes (all rise, no run-division by zero is undefined) and horizontal lines have a slope of zero. Note that *x = a* is not a function. Though not functions, vertical lines play an important role in algebra. Use **Activity 4.3.5 Horizontal and Vertical Lines** to develop the concepts. **Activity 4.3.6 Additional Practice with Horizontal and Vertical Lines** may be assigned for homework.

**Differentiated Instruction (For Learners Needing More Help)**

Continue putting the major concepts on the index cards and the bulletin board. These include rise, run, positive slope, negative slope, zero slope, undefined slope, steeper, flatter, horizontal, and vertical. Have students choose or point to the concept they need to use when doing an exercise.

**Differentiated Instruction (Enrichment)**

Students might research the building codes and specifications for the slope of handicap ramps or the slope of a stairway. They can interview a builder and find whether builders refer to slopes of roofs or stairs as rise/run or run/rise. As another extension students might want to explore the construction of Egyptian or Mayan pyramids and compare the incredible geometry and specifications of each structure.

For another extension, students might research the history of “*m*” used to designate slope of a line. Do other countries use “m” for slope? Have them do a scavenger hunt for slope in math writings in other countries to learn about how universal the language of math is.

**Group Activity**

For variety, give one exit slip to each group, and have each group member solve one step of the problem, then hand the problem to the next group member. Continue to pass the problem around to each person until the problem is done. One of the rules is no talking, and no interfering with a person as they do their step.

**Journal Entry**

Choose one of the students’ favorite examples from Unit 2 such as Cab Fares, Weight Loss, Debit Cards, etc., which they analyzed before they knew about the vocabulary and formulas for linear functions. For example, you can ask them to identify the average rate of change, slope, and *y*-intercept, and state what the slope and *y*-intercept mean in context, and whether the function is increasing or decreasing.

**Resources and Materials**

* **Activity 4.3.1** What is Slope
* **Activity 4.3.2** Calculating and Interpreting Slope
* **Activity 4.3.3** Positive and Negative Slope
* **Activity 4.3.4** Magnitude of Slope
* **Activity 4.3.5** Horizontal and Vertical Lines
* **Activity 4.3.6** Additional Practice with Horizontal and Vertical Lines
* **Exit Slips 4.3.1** and **4.3.2**
* Straight Edges for drawing linear graphs
* Bulletin Board for key concepts
* Student Journals
* Graphing Calculators