**Unit 3: Investigation 2 (3 Days)**

**WHAT IS A FUNCTION?**

**CCSS: 8 F-2, 8 F-5, A-CED 2, A-CED 10, F-IF 9**

**Overview**

Students will examine relations and functions presented by tables, graphs and verbal descriptions, identify the input and output variables, classify relations as functions or non-functions, and examine the domains of selected real world functions. Students will also perform an experiment and collect data that can be modeled by a function.

**Assessment Activities**

**Evidence of Success: What students will be able to do?**

Identify functions and non-functions in real world contexts, determine the input variable and the output variable and represent functions by tables and graphs and words.

**Assessment Tools: How they will show what they know.**

* **Exit Slip 3.2** asks students to determine a relation that is a function and to determine a relation that is not a function.
* **Journal Entry** asks students to describe how they can determine whether or not a table of values or a graph represents a function.

**Launch Notes**

The first session may begin with the video about the use of tap water v. bottled water (1.5 minutes) at http:www.bofunk.com/video/2402/bottled\_water\_water\_vs\_tap\_water.html. This leads into the examination of US Bottled Water Market data during the investigations.

**Closure Notes**

The investigation culminates with students successfully demonstrating their ability to determine relations that are functions and non-functions. This may be done with **Exit Slip 3.2**. Students also will demonstrate their ability to represent functions in multiple representations via the completion of the data collection activities.

**Important to Note: vocabulary, connections, common mistakes, typical misconceptions**

Students may be confused when two distinct inputs produce the same output. To help students understand this idea, you may discuss example such as these (1) a ½ or ¾ ounce letter costs the same, (2) filling two cars with 10 gallons of gas at the same pump results in the same charge, and (3) everyone has the same number of ears.

While students should be able to express the meaning of key terms in their own words, it is important that they have acceptable definitions in their notes. At some point you should help them fill in the blanks on the first page of **Activity 3.2.2** to read:

A function is a ***correspondence*** between two sets, a first set and a second set, where each element from the first set is paired with ***exactly one*** element from the second set.

The values we use for the first set are called ***inputs*** and are the same as the ***independent*** variable. The values in the second set are often called ***outputs*** and are the same as the ***dependent***variable.

The set of all possible inputs is called the ***domain*** of the function.

The set of all possible outputs is called the ***range***of the function.

When we present a relation in a table the left column always contains the inputs, also called the ***independent variable***. The dependent variable is always in the right column.

**Teaching Strategies**

1. You may begin by polling the class to find out how many students buy bottled water. Ask the class for their opinion as to the merits of bottled vs. tap water. You may also show an amusing 1.5 minute video on how consumers our sometimes persuaded that bottled water is better than tap water. It is found at http:www.bofunk.com/video/2402/bottled\_water\_water\_vs\_tap\_water.ht

A transparency of the Table 1 from **Activity 3.2.1** may be projected to show how bottled water consumption has increased over the years. This activity further develops the idea of a relation between two sets of objects, which was introduced in Investigation 1. Students will understand that some relations are very special and are called functions. A function is a correspondence between two sets and each input of a function will produce just one corresponding output.

Students will make a graph of US Bottled Market data. Students should then examine Table 2 and make another graph. (Note: These data will be used again in Unit 4.) Students may notice that the points on the graph up through 2007 appear to lie on a line, so we can say that the function appears to be linear. You may discuss why sales may have decreased in 2008.

As students work in groups to complete **Activity 3.2.1** you may walk around to see how students are participating. If many students are unclear about the main idea, you introduce additional examples to the group or, if needed, the whole class.

**Differentiated Instruction (For Learners Needing More Help)**

Before making the graph, some students might need to review graphing in the first quadrant of the coordinate plane and be given guidance in choosing appropriate scales for the axes. If a subset of the class needs this review you may work with that group to produce the graph for Table 1while other students go ahead and work independently or in pairs to graph the data in Table 2.

1. The next set of data to be considered is found in **Activity 3.2.2a Hartford Precipitation.** Students begin by using month as the domain. Later the domain becomes the number of inches of precipitation. This allows students to explore non-functions and to gain a deeper understanding of the concepts of input, output, function, domain, and range. At some point you may help the students fill in the blanks in the guided notes at the top of the first page. This activity is also designed to help students see the connection between a table and a graph that represent the same relation.

When students have plotted the data from Table 3, you may demonstrate that the vertical line test can be used to show that this relation is not a function.

Note that on the Activity Sheet zigzag lines indicate breaks on the y-axis for the first graph and on the x-axis for the second graph. This allows for the display of the precipitation data (all of which are between 2.5 and 4.0 inches) to be spread out. You may want to discuss with students the meaning of the zigzag and which it is convenient to use it for this purpose.

**Differentiated Instruction (Enrichment)**

Use the worksheet for **Activity 3.2.2b** instead of **Activity 3.2.2a**. Have students work in groups to decide what scale is appropriate for each axis.

1. Real world examples of functions and non-functions are examined in **Activity 3.2.3 Functions Everywhere**. In this activity student identify the input output variables and fill in blanks to indicate that the dependent variable is a function of the independent variable.

Ask students to share examples of functions and non-functions with the class. Encourage students to think creatively about functions corresponding to quantitative and qualitative relationships. **Exit Slip 3.2** may be used after this activity.

**Differentiated Instruction (For Learners Needing More Help)**

Students who are still wrestling with the concept of function may be given additional illustrative examples and you may also try to draw examples from the students’ experiences.

1. Next we suggest that you have students perform an experiment in which they collect data that can be modeled with a function. We offer two alternatives **Activity 3.2.4 Celsius and Fahrenheit** and **Activity 3.2.5 The Raven and the Jug.**

**Group Activity**

**Activities 3.2.4** and **3.2.5** are suited for group work. Activity 3.2.4 has different roles for 10 students and may be done by the entire class. Alternatively, some students may be assigned several tasks so that the work can be done by smaller groups. For Activity 3.2.5 have students work in groups of three or four. Assign roles to group members. For example, one student can coordinate materials, another student can read measurements, a third student can record data, and a fourth student can report on the group’s conclusions. In a group of three the first and fourth roles may be combined.

**Activity 3.2.4 Celsius and Fahrenheit** examines the relationship between the Celsius and Fahrenheit temperature scales.You do NOT need a lab setting for this activity. It can be performed with just containers and thermometers.

The class will need two containers of water, one with hot water and one with cold water (Use a few ice cubes in the water if available.) Place one Fahrenheit and one Celsius thermometer in each of the containers. You might have two students for each thermometer make simultaneous readings and agree on the reading to reduce the likelihood of reading and recording errors. Have a student read a timer or stop watch for the class. They will work in pairs as described above and when the timer says read, pair one will read and record the Celsius temps and pair two the Fahrenheit. Collect eight to ten data points. After recording the data, have each student copy the data onto the Activity sheet 2.4 Student Handout – Relating Celsius and Fahrenheit

Note: For classes with access to a temperature probe, see instructions at

<http://education.ti.com/calculators/downloads/US/Activities/Detail?ID=8521&MICROSITE=ACTIVITYEXCHANGE>.

Each student should complete the temperature data table showing the relationship between the Fahrenheit measurement and Celsius measurements. Using the table, have students construct a Fahrenheit versus Celsius graph. Instruct students to carefully label the axis and to determine an appropriate scale to the plot points. Students should recognize the (approximately) linear relationship between the variables from their graphs. Time permitting, have students do a Celsius vs. Fahrenheit table and graph. Or, assign this for homework.

As an alternative to **Activity 3.2.4** you may consider using **Activity 3.2.5 the Raven and the Jug.** This also involves the collection of data and making a graph from a table. For this experiment you will need jars that are approximately cylindrical in shape so that the cross-sectional area is constant, marbles, a source of water, and rulers.

**Journal Entry**

How can you tell from a table of values whether or not a relation is a function? How can you tell from the graph whether or not it is a function?

**Differentiated Instruction (Enrichment)**

Students who are comfortable with the concept of function can be asked to identify examples of multivariable function. For example, the area of a rectangle is a function of its length, *l*, and its width, *w*.

**Resources and Materials**

* **Activity 3.2.1 –** Bottled Water
* **Activities 3.2.2a –** Hartford Precipitation
* **Activities 3.2.2b –** Hartford Precipitation
* **Activity 3.2.3** – Functions Everywhere
* **Activity 3.2.4** – Celsius and Fahrenheit
* **Activity 3.2.5** – The Raven and the Jug
* **Exit Slip 3.2** – Relations
* Bulletin board for key concepts
* Student Journals
* Thermometers or temperature probes
* Stop watch
* Containers
* Hot and cold water (Ice cubes are helpful.)
* Jars, marbles, water, and rulers
* [www.bottledwater.org](http://www.bottledwater.org) on that site can download Beverage Marketing 2008 Market Report Findings
* [www.weather.com](http://www.weather.com) for the precipitation data (A search of temperature and precipitation in Hartford CT)
* Amusing video on consumers preference for bottled water http:www.bofunk.com/video/2402/bottled\_water\_water\_vs\_tap\_water.ht