Connecticut Common Core Algebra 1 Curriculum

Professional Development Materials

Unit 8 Quadratic Functions

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Unit 8 Performance Task: Stopping Distance*

Unit 8 Mid-Unit Test*

Unit 8 End-of-Unit Test*

*** These items appear only on the password-protected web site.**

Rolling Ball – TI-84 + and CBR-2

In this activity you will roll a ball down a ramp and examine a graph that results from the data collection.

Equipment & Materials

- CBR-2 unit
- TI-84 + calculator and Standard-B to Mini-A USB cable (square end and small trapezoidal end)
- Two 1 by 2 boards at least 8 feet long
- Smooth ball (approximately 8” in diameter - play balls from Stop & Shop work well)

CBR Setup

1. With the calculator turned off, connect the CBR-2 to the calculator using the link cable. The square plug goes into the side of the CBR-2, and the trapezoidal plug goes into the top of the calculator.

2. The Tic-Tic-Tic sound and light indicates that there is a connection but data are not being collected.

3. The EasyData app is already installed in TI-84 calculators. The calculator may turn on and automatically launch the program when the CBR-2 motion detector is connected. If the calculator doesn’t turn on, turn it on, find the key marked APPS and select #6 – Easy Data. The CBR-2 will be flashing.

4. From the MAIN MENU, select SETUP by pushing the WINDOW button.
   Choose Time Graph by pressing 2. Then push the ZOOM button to edit the Time Graph Settings.

5. Set the sample interval to .02. Push ZOOM to get to the Number of Samples window. Set the number of samples to collect to 100.

6. Push ZOOM to see the final settings. Note that the experiment length will be 2 seconds (the number of samples multiplied by the sample interval). Note the flashing light and the ticking only indicate the CBR-2 is ready to collect data. It is not collecting data.

7. Select OK by pressing the GRAPH key.
   Respond START by pressing Zoom. The CBR-2 will stop ticking.

A Data Deletion window will appear reminding you that the selected function will overwrite any prior data collection. Be prepared to collect the data before pressing GRAPH. **The device will start collecting the data as soon as you press the GRAPH button to respond OK.**

So you can see how it works you will collect some data just for a trial. Put the CBR-2 data collector facing up and the calculator on a table next to it. Hold one of your hands over the data collector screen, press GRAPH and move your hand up and down slowly. After a while the CBR-2 will stop collecting data, transfer the data to the calculator and show you a graph.

To use the app again after you see a graph press TRACE to select the Main menu and then ZOOM to select Start.

Create a Ramp and Set up the Experiment

Find a low step with space in front of it. Create a ramp by placing the boards parallel to each other about 4” inches apart with one end of each board on the step. Place the ball at the top of the ramp and release it to test how smoothly it rolls.

Place the CBR-2 at the bottom of the ramp hinged at approximately 90° to the ramp.
In this activity, you will roll a ball down the ramp and record its motion with the CBR-2.

1. Have one person at the top to let go of the ball and another at the bottom with the calculator and CBR-2.

2. Have the person with the calculator count down to release the ball at the same time he or she presses GRAPH to start collecting the data.

3. When the CBR-2 stops collecting the data, it will transfer it to the calculator and display a graph.

4. When you have a satisfactory graph, press TRACE to select the Main menu and select GRAPH to quit the app. Disconnect the cable from the calculator.

Modeling the Data with a Quadratic Function

You are going to use the collected data that are stored in L1 and L6. There are data in L7 and L8 that you will ignore for this experiment.

Press 2^{nd} Y= to be certain that your Plot1 is turned on and that it is showing the data in L1 and L6.

Press STAT, select CALC, and 5: QuadReg. Scroll down to Xlist and Ylist making sure that entries are L1 and L6 respectively. Scroll down to Store RegEQ: enter VARS, Y-VARS, 1: Function and 1:Y1. Scroll down to CALCULATE and hit ENTER.

When the QuadReg appears, the parameters for the quadratic function are shown. Now go to Y=, Y1 will now contain the quadratic function from the quadratic regression formula in your calculator. Scroll left until you reach the column before Y1. Hit enter until you see –O. Press ZOOM and 9 for StatPlot.

Galileo in Dubai

Galileo was a great Italian scientist in the 17^{th} Century. Before Galileo conducted some experiments, most people thought that heavy bodies fall faster than light ones do. In fact, if you have not before thought about this you might be thinking so too. Galileo studied the motion of falling bodies by climbing up the Tower of Pisa and dropping objects of different weights. He measured the distance an object falls in a given amount of time. The Tower of Pisa, is now called the "Leaning Tower" since it is no longer completely vertical. It is 183 feet high on one side and 186 feet high on the other.

Let us pretend to repeat Galileo's experiment and drop a heavy object from the top of the 2723-foot Khalefa Tower in Dubai, the world's tallest building in 2012. The table of values is given below. One variable is the time x the object has been falling, measured in seconds. The other variable is the total distance y the object has fallen in x seconds, and it is measured in feet.

Time object has fallen, *x*	**Distance object has travelled, *y***
0	0
1	16
2	64
3	144
4	256
5	400
6	?

1. Graph the data above using time in seconds as the independent variable. Label your axes and title your graph. Plan your scale so that your graph uses at least half of the graph paper. Do not connect the points with straight line segments. Your teacher will give you graph paper. Note: Your graph for this data should look very much like your TI-made graphs in class and from the experiment in **Activity 8.1.2**.

2. Earlier in this course you studied *direct variation*. Galileo, before he performed his experiments thought that the distance an object would fall would be directly proportional to the time it took to fall that distance. That means he thought he would come up with an equation of the form $y = kx$, where k is a constant. Does your graph support this idea of Galileo's? _______ Explain why or why not.

3. The same table appears below. Another column has been added so that you can find the change in y. After you compute Δy it should help explain why your graph looks the way it does. Should your points appear to lie on a line? _______ Explain.

x	***y***	**First Difference** **Δy**
0	0	-----
1	16	
2	64	
3	144	
4	256	
5	400	
6	?	

4. Could your table be describing an exponential function? _______ Why or why not?

5. We can examine the table more carefully to see if the data is quadratic by looking at the second differences. Find $\Delta(\Delta y)$.

x	***y***	**First Difference** **Δy**	**Second difference** **$\Delta\ (\Delta y)$**
0	0	-----	
1	16		
2	64		
3	144		
4	256		
5	400		
6	?		

Does the table describe a quadratic function? _______

6. Look at the table and the graph to model your function with an equation of the form $f(x) = ax^2 + bx + c$. You should find *b* and *c* based on the table and graph:

   b = __________ *c* = ____________. Explain how you found them.

   Now experiment with different values of *a* to find one that fits the data.

   a = ___________. Write your equation here: ___________________________________.

7. Do you notice a connection between the value of *a* and the second differences for *y*? Explain.

8. Using your equation, fill in the output in the table when time is 6 seconds.

9. Fill in the first and second differences the table for *t* = 6.

10. Do the first and second differences for *t* = 6 follow the pattern in the rows above? Explain.

11. Could we let the time be 1.5 seconds? _______ 2.6 seconds? _______ Why or why not?

12. Would we let the time be –3 seconds? _______ –2.6 seconds? _______Why or why not?

13. Use the equation to show how far the object will fall:

a. After 10 seconds?

b. After 15 seconds?

c. When do you think the object will hit the ground? Explain your reasoning.

14. For this real world problem:

a. What is a reasonable domain?

b. What is a reasonable range?

15. If the Khalefa Tower in Dubai had been built in Galileo's time, do you think he would have used it or the Tower of Pisa for his experiment? Why?

Exploring Parameters with Geometer's Sketchpad

Recall the **standard form** of a quadratic function $f(x) = ax^2 + bx + c$. In the Geometer's Sketchpad file for Standard Form, parameters a, b, and c each have sliders at the bottom of the first sketch. Drag each parameter to the right to increase the value or to the left to decrease the value.

Looking at Tab 1, describe how changing the following parameters affect the graph.

1) Increasing *a*?

2) Decreasing *a*?

3) $a > 0$?

4) $a < 0$?

5) $a = 0$? Why do you think this happens?

6) Increasing *c*?

7) Decreasing *c*?

8) Which part of a quadratic graph is always represented by the *c* parameter? Can you explain why this is so?

Now, select the other tabs at the bottom of the file and see if you can match the purple graph to the red graph by changing the parameters.

Recall the **vertex form** of a quadratic function $f(x) = a(x - h)^2 + k$. In the Geometer's Sketchpad file for Vertex Form, parameters a, h, and k each have sliders at the bottom of the first sketch. Drag each parameter to the right to increase the value or to the left to decrease the value.

Looking at Tab 1, describe how changing the following parameters affect the graph.

1) Increasing *a*?

2) Decreasing *a*?

3) $a > 0$?

4) $a < 0$?

5) Increasing *h*?

6) Decreasing *h*?

7) Increasing *k*?

8) Decreasing *k*?

Now select the other tabs at the bottom of the file and see if you can match the purple graph to the red graph by changing the parameters.

Finding *x*-intercepts of Parabolas

For each of the quadratic functions in 1 – 6 do the following work on a separate sheet of paper.

a. Sketch a graph (either by making a table or using a calculator).

b. Estimate the values of x at the x-intercepts (if there are any).

c. Set $y = 0$ and solve the equation for x. Leave a radical in the answer if you don't have an exact square root.

d. Find decimal approximations for the x-intercepts.

e. Compare the decimal approximations from (d) with the estimates you made in (b).

1. $y = (x-4)^2 - 7$

2. $y = (x+3)^2 + 22$

3. $y = -(x+2)^2 + 33$

4. $y = 2(x-8)^2 - 60$

5. $y = -\dfrac{1}{2}(x+5)^2 - 10$

6. $y = 4(x-6)^2$

7. All of the above functions were given in vertex form. The vertex form of a quadratic function is $y = a(x-h)^2 + k$. There is a relationship between the parameters a and k and the number of x-intercepts. Look at your solutions for equations 1 – 6 and then fill in the table below.

When	and	Then the number of solutions is (0, 1 or 2?)
a is positive	k is positive	
a is positive	k is negative	
a is negative	k is positive	
a is negative	k is negative	
a is positive or negative	k is zero	

8. Now write four quadratic functions of your own in vertex form. Without graphing the functions, make predictions for each function in the table below:

   a. $y =$

   b. $y =$

   c. $y =$

   d. $y =$

Function	For this function	and	How many *x*-intercepts will the graph have?	The vertex will be in which quadrant (or on which axis)?
a.	a is __________	k is __________		
b.	a is __________	k is __________		
c.	a is __________	k is __________		
d.	a is __________	k is __________		

9. Solve the problem from the opener for this investigation:

   A hitter at Fenway Park hits a ball with equation $y = -0.001(x-200)^2 + 44$ where y is the height of the ball and x is the horizontal distance of the ball from home plate. Both values are given in feet.

   a. How far will the ball travel before it hits the ground?

   b. If the ball is hit toward deep center field, where the fence is 420 feet will it travel far enough to be a home run?

10. There are two solutions to the quadratic equation in problem 9. Explain why only one of the solutions makes sense in the context of this problem.

Solving Quadratic Equations in Standard Form

We will explore a method for finding the x-intercepts for any quadratic function given in general form, $y = ax^2 + bx + c$. In other words we will learn how to solve any quadratic equation of the form $0 = ax^2 + bx + c$.

Here are the steps:

1. Find the x coordinate of the vertex using the formula $h = -\frac{b}{2a}$.
2. Substitute the value h into the function to find the value of the y-coordinate of the vertex, which we call k.
3. Rewrite the function in vertex form as $y = a(x-h)^2 + k$.
4. Set $y = 0$ and solve the equation of x using the method of undoing learned in this investigation.

There are other methods for solving quadratic equations, which you will learn later in this unit.

For now, on a separate piece of paper, use the above method to solve these equations.

1. $0 = x^2 - 8x + 15$

2. $0 = x^2 + 10x + 9$

3 $0 = 2x^2 - 4x - 6$

4. $0 = -2x^2 + 12x - 10$

5. $0 = x^2 - 10x + 3$

6. $0 = x^2 + 4x + 5$

7. $0 = x^2 - 5x - 2\frac{3}{4}$

Multiplying Polynomials

Vocabulary:

Polynomial: A mathematical expression containing one or more terms. Each term may be a constant, a number raised to a positive integer power, or the product of a constant times a number raised to a positive integer power.

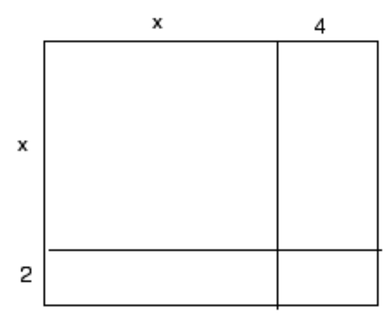
Monomial: A mathematical expression with ________ term.

Binomial: A mathematical expression with ________ terms.

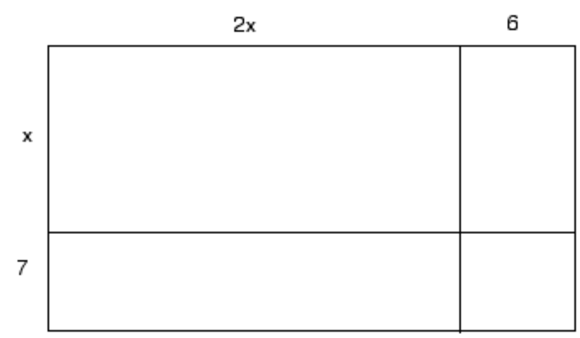
Trinomial: A mathematical expression with ________ terms.

Use an area model to show the product of two binomials. Then combine like terms to form a trinomial. You may, if you choose, use algebra tiles to assist you.

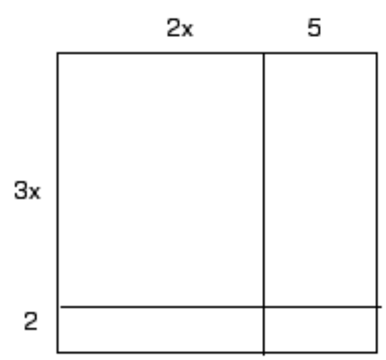
1. $(x + 2)(x + 4)$



2. $(x + 7)(2x + 6)$

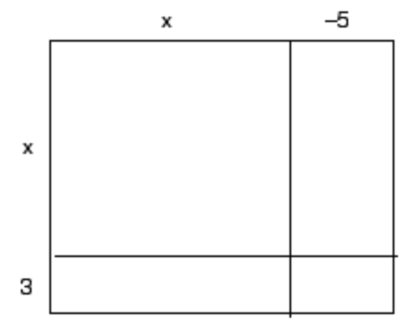


3. $(3x + 2)(2x + 5)$

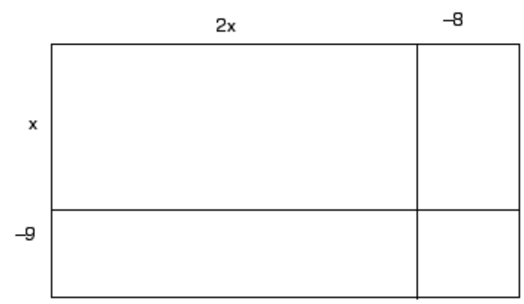


Continue to use area models, even when some terms are negative. (At this point you may not want to continue using algebra tiles.)

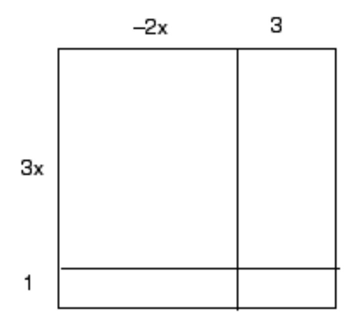
4. $(x + 3)(x - 5)$



5. $(x - 9)(2x - 8)$



6. $(3x + 1)(-2x + 3)$



Here is the product of a binomial and a trinomial

7. $(x - 9)(x - 2y + 4)$

Now try the method of applying the distributive property twice:

8. $(x-5)(x+7) = (x-5)x + (x-5)7 = x^2$ – ____ x + ____ x – ____
   = x^2 + ____ x – ____

9. $(x+4)(3x-10)$ = ( ) $3x$ – ( )10 = _____ + _____ – ______ – _____
   =

You may check your work for problems 8 & 9 by drawing an area model if you like.

Expand these products by any method you like. Show your work.

10. $6x(x-5)$

11. $x(x^2+4x-15)$

12. $(5x-1)(2x-3)$

13. $(x-4)(x+4)$

14. $(x+3)^2$

15. Which of problems 10–14 shows a monomial multiplied by a trinomial?

16. Which of the problems 10–14 has a binomial product?

17. In which of the problems 10–14 are the two binomial factors the same?

Solving Quadratic Equations by Factoring

1. Let's solve the equation $x^2 - 7x + 12 = 0$ by factoring. To do this, fill in the blanks for each step below.

   Step 1: Find all factor pairs for 12: 12×1, $6 \times$ __ ,

   Step 2: Factor the left side of the equation: $(x - \_\_\_)(x \_\_\_ 3) = 0$

   Step 3: Using the Zero Product Property, either $x - 4 = 0$ or _______ = 0

   Step 4: If $x - 4 = 0$ then If _______ = 0 then

   $x =$ _____ $x =$ _____

2. Now that you have experienced this process, solve each of the equations below by factoring the left side and using the Zero Product Property.

   a. $x^2 + 8x + 7 = 0$

   b. $x^2 - 8x + 7 = 0$

   c. $x^2 + 6x - 7 = 0$

   d. $x^2 - 6x - 7 = 0$

3. Describe any patterns you observe in solving equations 2(a) through 2(d).

4. Check 2(a) through 2(d) by graphing functions on a calculator.

5. Could you solve $x^2 + 6x + 7 = 0$ by factoring? Why or why not?

6. Solve each of these equations by factoring the left side and using the Zero Product Property.

   a. $x^2 + 16x + 15 = 0$

   b. $x^2 - 8x + 15 = 0$

   c. $x^2 + 2x - 15 = 0$

   d. $x^2 - 14x - 15 = 0$

7. Find another quadratic equation of the form $x^2 + bx + 15 = 0$ or $x^2 + bx - 15 = 0$ that you can solve by factoring and solve it.

8. Find a quadratic equation of the form $x^2 + bx + 15 = 0$ or $x^2 + bx - 15 = 0$ that you cannot solve by factoring and explain why you can't solve it.

9. Solve each of these equations by factoring.

   a. $2x^2 + 7x + 5 = 0$

b. $2x^2 + 11x + 5 = 0$

c. $2x^2 - 3x - 5 = 0$

d. $2x^2 + 9x - 5 = 0$

10. Find another quadratic equation with the same leading coefficient (that is, *a* = 2) that can be solved by factoring, and solve it.

11. Solve each of these equations by factoring.

a. $x^2 - 11x + 24 = 0$

b. $5x^2 + 4x - 1 = 0$

c. $3x^2 - 30x - 33 = 0$ (Hint: first find a common factor for all three terms)

d. $x^2 - 8x + 16 = 0$

e. How is the solution for (d) different from the other solutions on this page?

Using the Quadratic Formula

The Quadratic Formula gives solutions to the equation $ax^2 + bx + c =$ ______. It may also be used to find the ___-intercepts for the function $y = f(x) = ax^2 + bx + c$.

The quadratic formula states that $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$. This formula gives two possible values for *x*. The first is $-\frac{b}{2a} + \frac{\sqrt{b^2-4ac}}{2a}$. The second value is ____________________.

Notice that the first term in the formula, $-\frac{b}{2a}$, is the ___-coordinate of the vertex of the parabola.

$x = -\frac{b}{2a}$ is the equation of the parabola's line of _________________.

The expression under the radical symbol, $b^2 - 4ac$, is called the ________________.

1. Use the quadratic formula to solve the equation $x^2 - 4x - 10 = 0$.

   Step 1. Identify *a, b,* and *c*: *a* = _______ *b* = _______ *c* = ________

   Step 2. Substitute: $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$

   x = __________________

   Step 3. Write out the two values for *x*. Leave radicals in your answers.

   x =___________ or *x* = ____________

   Step 4. Use a calculator to find approximate values of *x* (to the nearest 0.001).

   x ≈ ___________ or *x* ≈ ____________

2. a. Solve the equation $x^2 + 10x + 15 = 0$ by completing the square.

   b. Then solve the same equation using the quadratic formula.

   c. Show that your solutions are equivalent.

3. a. Solve the equation $x^2 - 7x - 18 = 0$ by factoring.

b. Then solve the same equation using the quadratic formula.

c. Show that your solutions are equivalent.

d. Here's how one student solved this equation using the formula

$a = 1$, $b = -7$, c = –18.

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = -\frac{-7}{2 \cdot 1} \pm \frac{\sqrt{-7^2 - 4 \cdot 1 \cdot (-18)}}{2 \cdot 1}$$

$$x = -\frac{-7}{2} \pm \frac{\sqrt{-49 + 72}}{2} = \frac{7}{2} \pm \frac{\sqrt{23}}{2}$$

Find this student's mistake and correct it.

4. The function $y = -5x^2 + 10x + 0.5$ models the height of a soccer ball in meters x seconds after it has been kicked.

   a. Use the quadratic formula to find the maximum height and the time it takes the ball to reach the ground.

   b. Sketch a graph of the function based on the results of (a).

   c. Check your answer to (b) with a graphing calculator.

5. For each quadratic function, find the value of the discriminant $b^2 - 4ac$. Then use a calculator to make a graph and determine the number of x-intercepts.

Function	Value of $b^2 - 4ac$	Is the discriminant positive, negative, or zero?	Number of x-intercepts.
$y = x^2 + 2x + 5$			
$y = x^2 - 3x - 7$			
$y = 2x^2 + 10x + 5$			
$y = -x^2 - 4x - 8$			
$y = 3x^2 + 24x + 48$			

6. Based on the above table, is there a relationship between the discriminant and the number of x-intercepts? Make a conjecture and explain why it might be true.