Unit 8

Quadratic Functions and Equations

5 weeks

Unit 8 Content

‣ Investigation 1: Introducing Quadratic Functions: Parabolas Everywhere [Standard Form] (4 days)

‣ Investigation 2: Quadratic Functions in Vertex Form (3 days)

‣ Investigation 3: Solving Quadratic Equations Using the Square Root Property (4 days)

‣ Mid-Unit Test (1 day review + 1 day test)

‣ Investigation 4: Quadratic Functions in Factored Form (4 days)

Unit 8 Content (continued)

▸ Investigation 5: Factoring Quadratic Trinomials (4 days)

▸ Investigation 6: Solving Quadratic Equations by Completing the Square and the Quadratic Formula (2 days)

▸ Performance Task: Stopping Distance (1 day)

▸ End-of-Unit Test (1 day review + 1 day test)

What students need to know

- Quadratic Function
- Quadratic Equation
- Quadratic Formula
- Parabola
- Factored form
- Vertex form
- Standard form
- Square Root Property
- Zero Product Property
- Completing the Square

- Vertex
- Line of Symmetry
- First and Second Differences
- Monomial
- Binomial
- Trinomial

What students need to be able to do

- Graph (quadratic functions)
- Find (line of symmetry, vertex)
- Solve (quadratic equation)
- Model (with quadratic functions)
- Solve (problems arising from quadratic models)
- Expand (product of two binomials)
- Factor (quadratic trinomial)
- Use (quadratic formula)

Essential Questions

▸ What can the zeros, intercepts, vertex, maximum, minimum and other features of a quadratic function tell you about real world relationships?

▸ How is the polynomial system analogous to the system of integers?

▸ How can technology support investigation and experimentation of the way that parameters affect functions?

Enduring Understandings

▸ Quadratic functions can be used to model real world relationships and the key points in quadratic functions have meaning in the real world context.

▸ Polynomials are closed under addition, subtraction, and multiplication.

▸ Dynamic software, graphing calculators, and other technology can be used to explore and deepen our understanding of mathematics.

Investigation 1: Introducing Quadratic Functions: Parabolas Everywhere (4 days)

▸ Distinguish, given a table of values, between the nonlinear patterns of exponential and quadratic growth

▸ Make a scatter plot by hand or technology with appropriate scaling and labels and recognize a graph that could be modeled by a quadratic function

▸ Recognize that for nonlinear growth, the average rates of change will not be constant

▸ Recognize that for quadratic growth, the average rates of change exhibit linear growth or in other words, the second differences are constant (when Δx is constant)

Investigation 2: Quadratic Functions in Vertex Form (4 days)

▸ Find the vertex of a parabola from its equation given in either vertex or standard form.

▸ Model real world information or objects by writing the equation of a parabola given the vertex and one other point.

▸ Transform a quadratic function in standard form to an equation in vertex form by finding $h = \dfrac{-b}{2a}$ and $k = f\left(\dfrac{-b}{2a}\right)$.

▸ Graph a parabola from an equation in vertex form.

Investigation 3: Solving Quadratic Equations Using the Square Root Property (4 days)

- Recognize the relationship between squares and square roots

- Recognize and distinguish quadratic functions in standard form and in vertex form

- Undo quadratic expresses to find solutions to equations.

- Solve equations of the form $a(x - h)^2 + k$ = constant.

- Find the x-intercepts of parabolas with functions given in vertex form.

Investigation 4: Quadratic Functions in Factored Form (4 days)

▸ Graph and find the vertex of quadratic functions in factored form

▸ Use the zero product property to find the intercepts of a quadratic function in factored form

▸ Multiply combinations of monomials, binomials, and trinomials

▸ Convert quadratic functions in factored form to standard form

Investigation 5: Factoring Quadratic Trinomials (4 days)

▸ Factor quadratic trinomials in various forms

▸ Check factorizations using multiplication

▸ Convert quadratic functions in standard form to factored form

▸ Solve a quadratic equation by factoring or determine that a quadratic equation cannot be solved in this way

Investigation 6: Solving Quadratic Equations by Completing the Square and Quadratic Formula (2 days)

▸ Solve a quadratic equation that cannot be factored by completing the square and by using the quadratic formula

Performance Task: Stopping Distance

▸ Students work in groups to analyze a real world scenario that can be modeled by a quadratic function. They investigate the effects of distance, speed, and reaction time to determine the safety of a turn in the road that leads to a railroad crossing. The groups will write a letter to the local transit authority to recommend road signs that can increase the safety of drivers approaching the turn.

End-of-Unit Review and Test (2 days)

A bridge has a parabolic arch stretching across the middle section like the image below.
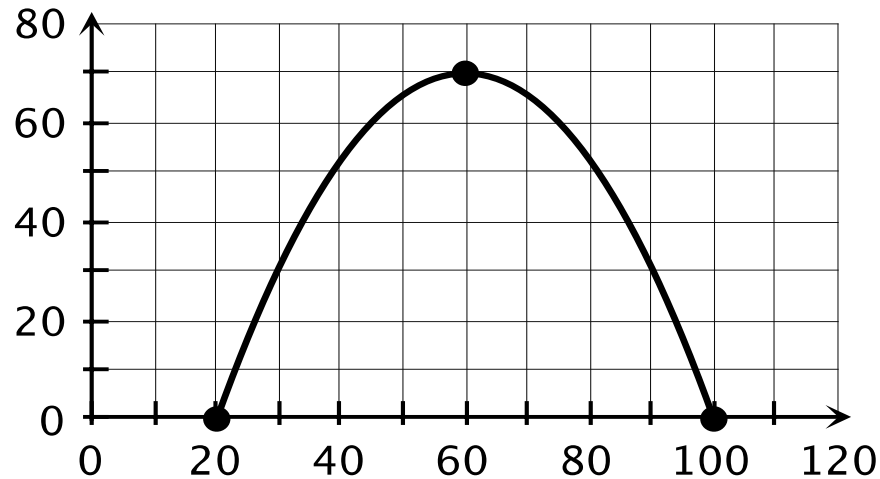
(a) Identify the *x*-intercepts.
(b) The highest point of the arch occurs at the coordinate (60, 70). Write an equation to model the arch.

Activity Exploration

Participants will break into four groups. Each group will participate in the following four workshops:

(30 minute rotation)

- **Workshop 1**: Identifying quadratic functions from table (Activities 8.1.4 and 8.1.7)

- **Workshop 2**: Using CBRs to collect data modeled by quadratic functions (Activities 8.1.3 and 8.2.5)

- **Workshop 3**: Using software to analyze effects of changes in parameters (Activity 8.2.3)

- **Workshop 4**: Using area models to expand and factor polynomials (Activities 8.4.5 and 8.5.2)

Priority Standards (CCSS)

▸ 8EE 2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

▸ **A-SSE 3. a Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.**

▸ A-REI 4. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation.

▸ **A-APR 1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.**

▸ A-CED 1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from ...quadratic functions ...*

Priority Standards (CCSS)

▶ A-CED 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

▶ **F-IF 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries... ***

▶ **F-IF 7a. Graph ... quadratic functions and show intercepts, maxima, and minima.**

▶ **F-IF 8a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.**

▶ **F-BF 3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology...**

3-2-1 Reflection in Informal Groups

▶ List 3 things that correlate between the activities and the assessments.

▶ List 2 things to change within the activities or assessments.

▶ List 1 thing that is most helpful to implement within the unit.