Unit 7

An Introduction to Exponential Functions

5 weeks or 25 days

(approximately 42 minute lessons)

Essential Questions

- Given real-world data, how can we determine whether it models linear or exponential growth (or decay)?
- What characterizes exponential growth and decay?
- What are some real world models of exponential growth and decay?
- What are the limitations of exponential growth models?

What Students Need to Be Able to Do

- Determine whether real-world data best models linear or exponential growth (or decay)
- Recognize exponential models in various forms and express them in other forms (situation, data table, graph, and equation)
- Identify and explain the meaning of the parameters of an exponential function
- Identify similarities and differences between linear models and exponential models
- Explain the limitations of exponential growth and decay models
- Understand and work with exponential algebraic expressions

What Students Need to Know

Unit Content

Investigation 1
(2 days)

A New Function Family—World Population Growth

Investigation 2
(3-4 days)

Exponential Growth and Working with Exponents

Investigation 3
(4 days)

Exploring Parameters of Exponential Functions

Mid-Unit Review and Quiz
(2 days)

Unit Content

Investigation 4 (2 days) — Modeling Exponential Growth & Decay with Real Data

Investigation 5 (4 days) — Exponential Patterns as Percent Change

Investigation 6 (2 days) — The Mathematics of Global Warming

Performance Task (3 days) — The Consequences of Global Warming

End-of-Unit Review and Test (2 days)

Appendix A (1 or 2 days) Meaning and Laws of Rational Exponents

Inv. 1: A New Function Family (2 days)

Students will:

- Review ideas about linearity by examining world population data (Is it linear? -No)
- Discover that real data is complex and it can be difficult to distinguish between linear and nonlinear
- Discover that the new (exponential) growth pattern is characterized by a 'constant multiplier' recursive rule
- Compare linear and exponential growth patterns to discover that the exponential growth will *eventually* 'overtake' linear growth (Is it a Good Deal?)
- Consider/Discuss consequences of linear agricultural growth and exponential population growth on world hunger

Inv. 2: Exponential Growth and Working with Exponents (3-4 days)

Students will:

- Use block growth patterns to compare and contrast linear and exponential growth
- Explain block growth patterns in words and then with an equation (function rule / explicit rule)
- *Develop* the general equation for exponential growth $y = a \cdot b^x$
- Review and explore the *meaning* of exponents and how to work with them (including basic exponent laws - Unit 7 Appendix A contains the extension of integer exponents to rational exponents)

Inv. 3: Exploring Parameters of Exponential Functions (4 days)

Students will:

- Explore the graphical effects of parameters a and b given exponential functions of the form $y = a \cdot b^x$.
- Describe the meanings and different possible "cases" for values of parameters a and b.
- Use knowledge of parameter meanings and calculator (plots and graphs) to explore and to find exponential equation models to fit real data.
- Write equations and sketch graphs that model a variety of given exponential *and linear* situations.

Mid-Unit Review and Quiz (2 days)

To assesses whether students can:

- Distinguish between linear and exponential functions/models in various forms (data table, equation, graph, situation)
- Express exponential (and linear) functions in various forms given one form (situation, data table, graph, and equation)
- Identify and explain the meaning of the parameters of an exponential function
- Identify similarities and differences between linear models and exponential models (ex: roles of parameters, additive vs multiplicative growth/decay, exponential eventually overtakes linear)
- Understand and work with exponential algebraic expressions

Inv. 4: Modeling Exponential Growth & Decay with Real Data (2 days)

Students will :

- *Collect* data through hands-on activities and then find an exponential model that fits the data
  (M&Ms and Bouncing Balls)
- Find an exponential model to fit a table of real data that models exponential growth (Facebook)

Inv. 5: Exponential Patterns as Percent Change (3 days)

Students will:

- Review meaning of percents and converting numbers from form to form (fractions, decimals, percents)
- Practice and review calculating with percents
- Calculate exponential growth/decay factors (parameter b) given constant percent change (rate of change)
- Identify situations as linear or exponential growth/decay and write function models for them

- Enrichment - explore compounding situations

Inv. 6: The Mathematics of Global Warming (2 days)

Students will:

- Create and analyze the exponential models that fit real data related to global warming
  - Describe the real world meanings of parameter values from the exponential models
  - Calculate percent changes from the exponential models
  - Make projections based on the exponential models
- Discuss possible consequences of and responses to global warming

Performance Task: The Consequences of Global Warming (3 days)

- Student research project about contributors and/or responses to global warming and the potential consequences (student choice)
  - Builds on Investigation 6 which explores data on rising levels of carbon dioxide in our atmosphere and on increasing use of solar power and how they relate to the corresponding rise in global temperature.

End-of-Unit Review and Test (2 days)

To assesses whether students can:

- Distinguish between linear and exponential functions/models in various forms (data table, equation, graph, situation)
- Express exponential (and linear) functions in various forms given one form (situation, data table, graph, and equation)
- Identify and explain the meaning of the parameters of an exponential function
- Identify similarities and differences between linear models and exponential models (ex: roles of parameters, additive vs multiplicative growth/decay, exponential eventually overtakes linear)
- Understand and work with exponential algebraic expressions
- Explain the limitations of exponential growth and decay models

Common Core Content Standards

(Priority Standard are in Bold)

- N-RN 1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*
- **N-RN 2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.**
- A-SSE 1b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For ex., interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*
- A-SSE 3c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $[1.15^{(1/12)}]^{(12t)} \approx 1.012^{(12t)}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Common Core Content Standards
(Priority Standard are in Bold, Continued)

- F-IF 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
- e. Graph exponential ... functions, showing intercepts and end behavior...
- **F-BF 2. Write ... geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.***
- F-LE 1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
- **a. Prove ... that exponential functions grow by equal factors over equal intervals....**
- c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- **F-LE 2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).**
- **F-LE 3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly,** quadratically, or (more generally) as a polynomial function.
- **F-LE 5. Interpret the parameters in a ... exponential function in terms of a context.**

Common Core Standards for Mathematical Practice

(Bold Standards to be emphasized in this Unit)

- **1. Make sense of problems and persevere in solving them.**
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- **4. Model with mathematics.**
- 5. Use appropriate tools strategically.
- **6. Attend to precision.**
- 7. Look for and make use of structure.
- **8. Look for and express regularity in repeated reasoning.**