Mathematics Instructional Cycle Guide

The Normal Distribution (HSS-ID.A.4)

Created by Andrew Hill, 2014 Connecticut Dream Team teacher

CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

Summarize, represent, and interpret data on a single count or measurement variable.

HSS-ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

MP.3 Construct viable arguments and critique the reasoning of others.

- *Students come up with approximate percentiles for values in a normal distribution and must defend their approximations if they differ from those of their peers.*

MP.4 Model with mathematics.

- *Students construct a normal model given a real-world normally distributed random variable and its mean and standard deviation. Students use this model to analyze, interpret, and predict information.*

MP.5 Use appropriate tools strategically.

- *Students use tables and calculators to find percentiles for values of a normally distributed random variable.*

WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings ***(page 3)***
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint ***(page 4)***
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed ***(page 8)***
- Supporting lesson materials ***(page 14)***
- Precursory research and review of standard HSS-ID.A.4 and assessment items that illustrate the standard ***(page 21)***

HOW TO USE THIS DOCUMENT

1) Before the lesson, administer the Normal Distribution Mathematical Checkpoint individually to students to elicit evidence of student understanding.
2) Analyze and interpret the student work using the Student Response Guide
3) Use the next steps or ***follow-up lesson plan*** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

MATERIALS REQUIRED

- “Horace and Gertrude's Test Score Quandary” projections or worksheets
- Normal distribution z score tables (at least one per 3-4 students)
- Extension task worksheets
- “Eunice and Winthrop's Bullfrog Bickering” exit slips
- “The Normal Distribution on the Ti-83/84” sheets for reference as needed

TIME NEEDED

Normal Distribution Checkpoint administration: **15 minutes**
Follow-Up Lesson Plan: **1 to 2 instructional blocks**

Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

<table>
<tr><th colspan="3">Step 1: Elicit evidence of student understanding</th></tr>
<tr><th colspan="3">Mathematical Checkpoint</th></tr>
<tr><th>Question(s)</th><th colspan="2">Purpose</th></tr>
<tr><td rowspan="2">On a particular standardized test, the scores fall approximately into a normal distribution with a mean of 18 and a standard deviation of 6. Student A earns a score of 24 and student B earns a score of 12.

Do you think it would be accurate to say that student A performed “twice as well” as student B? Explain, using the concept of percentile to justify your answer.</td><td>CT Core Standard:</td><td><u>HSS-ID.A.4</u> Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.</td></tr>
<tr><td>Target question addressed by this checkpoint:</td><td>How do students use the empirical rule for the normal distribution to compare specific values of a normally distributed random variable? Do the students understand the difference between comparing the variable values and comparing the percentiles represented by those values? Can the students explain the difference in order to justify their comparison?</td></tr>
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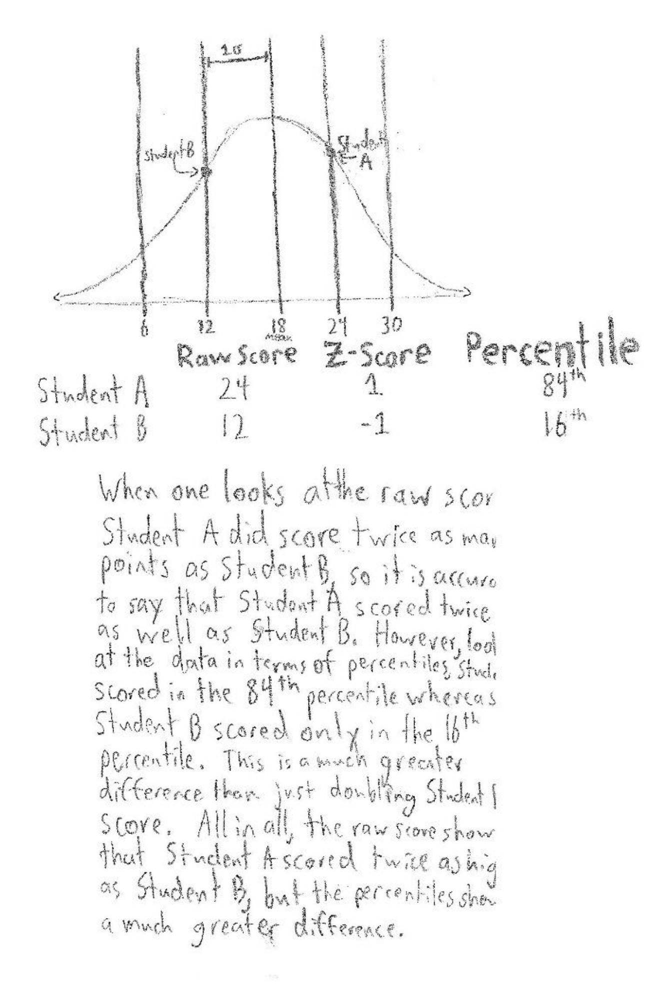
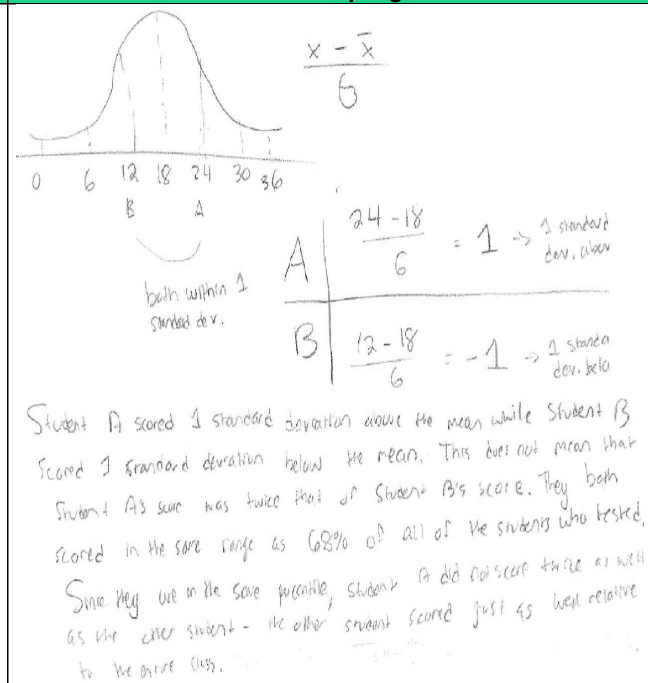
Step 2: Analyze and Interpret Student Work

Student Response Guide

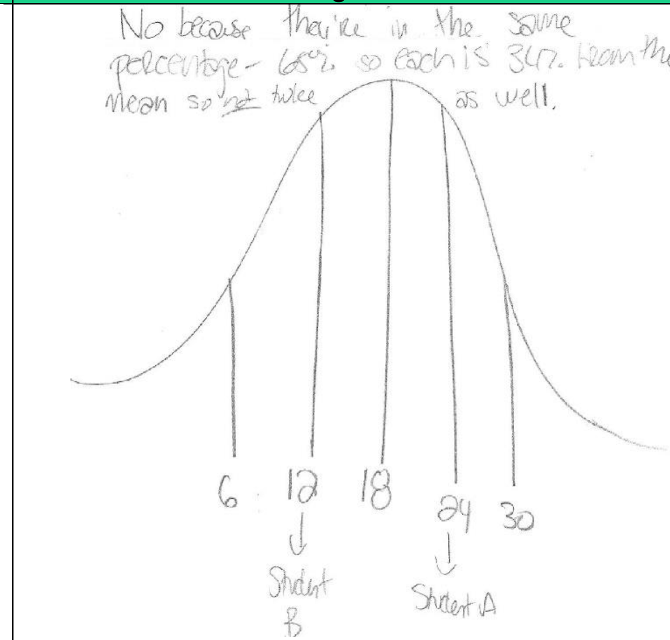
Got It

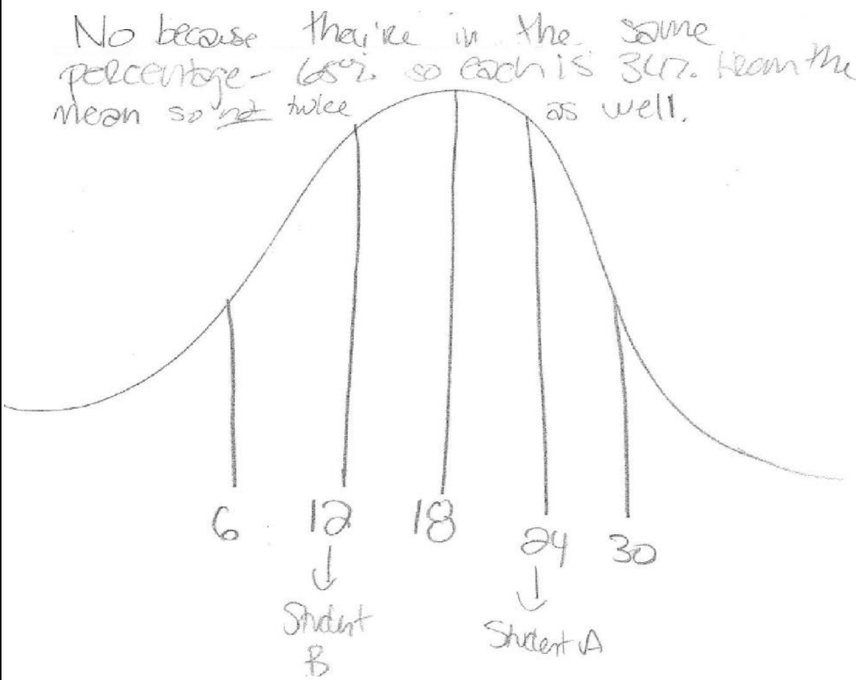


Developing



Getting Started



<table>
<tr><th colspan="2">Getting Started</th></tr>
<tr><th>Student Response Example</th><th>Indicators</th></tr>
<tr><td>
</td><td>
Student shows some understanding of how mean and standard deviation translate to a visual model of a normal distribution
Student shows some understanding of how the scores translate to percentages
Student answer shows little understanding of the purpose of z-scores or of the meaning of the empirical rule in general
Student does not demonstrate understanding of how to calculate and/or interpret a percentile
Student answer shows confusion between likelihood of a result and what the result represents (in this case a test score)
Student reaches an unreasonable conclusion with few details as to where the conclusion came from
</td></tr>
<tr><th>In the Moment Questions/Prompts</th><th>Closing the Loop (Interventions/Extensions)</th></tr>
<tr><td>P: Tell me how you went about this problem.

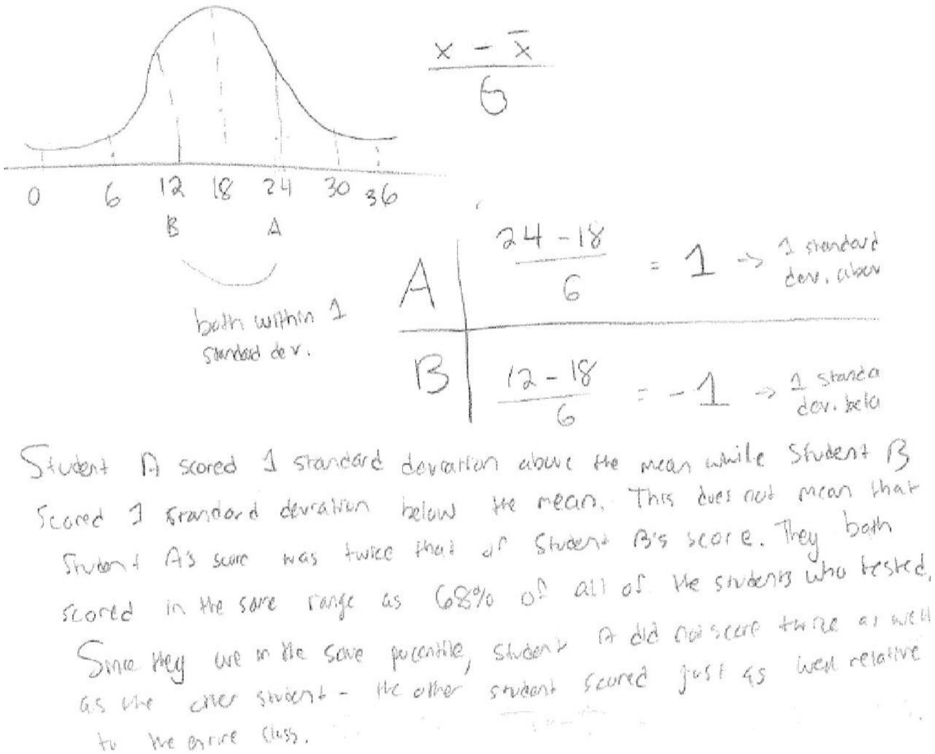
Q: How did you arrive at 34% for each score?

Q: How did you draw the conclusion you came to?

Q: What did the numbers in this problem mean? What did they represent?

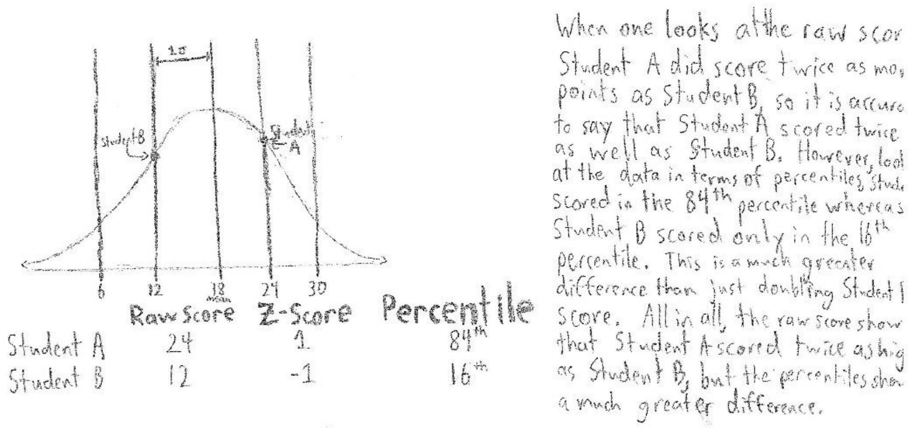
Q: If I just showed you the students' test scores without mentioning anything about normal distributions, how would you compare their performance?

Q: If one student did better than the other, how could we measure how much better?</td><td>Provide students with more exercises concerning calculating z-scores and comparing different scores to the rest of the population. Tie the ideas into the concept of percentiles. Ask students to justify their comparisons with specific details.</td></tr>
</table>

Developing	
Student Response Example	**Indicator**
	• Student shows understanding of how mean and standard deviation translate to a visual model of a normal distribution • Student may demonstrate understanding in how to calculate a z score but may not know what the z score means • Student shows some understanding of the empirical rule and how it relates to the z scores • Student shows confusion about how to use percentiles to compare values in a probability distribution
In the Moment Questions/Prompts	**Closing the Loop (Interventions/Extensions)**
P: Tell me why Student B did not perform twice as well as Student A. **Q:** What do the z scores you calculated mean? **Q:** How did either student perform compared to the rest of the class? **Q:** How did each student perform compared to the other?	Provide students with exercises concerning calculating and interpreting percentiles in a distribution. Follow up with exercises concerning calculating and interpreting percentiles for scores in a normal distribution.

Got it

Student Response Example



Indicators

- Student shows understanding of how mean and standard deviation translate to a visual model of the normal distribution
- Student correctly calculates z scores based on the provided information
- Student converts z scores to percentiles correctly using the empirical rule
- Student demonstrates understanding of how to use percentiles to compare values in a probability distribution

In the Moment Questions/Prompts

P: Explain to me how you arrived to your conclusion.

Q: What score do you think Student A could have gotten if Student B really did perform “twice as well”?

Q: How much better would Student B have done if the mean were 24 instead of 18?

Q: In general, how does changing either the mean or the standard deviation have effected the result of this problem?

Q: Can you find a mean and standard deviation that would have made Student B's percentile twice that of Student A's with the same score?

Closing the Loop (Interventions/Extensions)

Provide two scores and their respective percentiles from a normal distribution with an unknown mean and standard deviation and have the students solve for the mean and standard deviation. Include examples that use the empirical rule and ones that require using a calculator or table to convert percentiles to z scores.

See extension task in follow up lesson plan.

Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction	
Lesson Objective:	Interpret values of a normally distributed random variable in terms of percentile.
CC Content Standard(s):	**HSS-ID.A.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
Targeted Standard(s) for Mathematical Practice	**MP.3 Construct viable arguments and critique the reasoning of others.** • *Can students give a reasonable estimate for the percentile of a value of normally distributed random variable? Can they justify their estimates, even if their estimates differ from those of their peers?* **MP.4 Model with mathematics.** • *Are students able to relate the description of a real-world normally distributed random variable to its graphical representation? Do the students understand how the mean and standard deviation determines the exact center and spread of any normal model? Can students connect the mean and standard deviation with the area under a normal curve? Can students relate the area under a probability curve to the concept of percentile?* **MP.5 Use appropriate tools strategically.** • *Can students use calculators, tables, and/or spreadsheets to find percentiles for values of a normally distributed random variable?*

Mathematical Goals	Success Criteria
• Understand how the shape, center, and spread of a normal distribution connect to the percentile of any value within the distribution • Understand that a z score represents a number of standard deviations above or below the mean • Understand how the empirical rule helps find percentiles for scores that are 1, 2, or 3 standard deviations above or below the mean for a normally distributed random variable • Understand how resources such as calculators, spreadsheets, and tables can find percentiles for any score and why such resources are necessary	• Sketch a bell curve model to represent a normally distributed variable with the mean and standard deviation values accurately labeled • Convert a value from a normally distributed random variable to a z score • Interpret what the z score for a value means in terms of mean and standard deviation • Use the empirical rule to calculate percentiles for values 1, 2, or 3 standard deviations above or below the mean • Use appropriate tools such as calculators, tables, and spreadsheets to find percentiles for any value in a normal distribution • Compare multiple values in a normal distribution using percentiles

Lesson Launch (Probe and Build Background Knowledge)

Purpose: ***Activate and assess background knowledge concerning percentiles and how they are used to compare values.***

Write the following questions on the board and have the students read them and reflect on the answers.

- *Have you ever seen or heard the term "percentile" used? If so, what was the context?*
- *What does the term "percentile" mean?*
- *Is it always a good thing to have a high percentile ranking? If not, can you think of a specific example situation?*

Have the students discuss their thoughts with other students. After a few minutes have groups of students share their thoughts. Most students by Algebra II should have seen percentiles used somewhere, such as in the results of physicals or standardized test scores.

Instructional Task

Purpose: ***Introduce the test score problem and provide students with the time and resources to reason through it and produce a solution.***

Engage (Setting Up the Task)
1) Present the following situation to the class. After everyone has read it, show question *a*.

Horace and Gertrude's Test Score Quandary

Horace and Gertrude have both taken the SAT I and the ACT, and they are comparing their mathematics scores. Horace scored a 30 on the ACT mathematics section and a 570 on the SAT mathematics section. Gertrude scored a 24 on the ACT Mathematics section and a 650 on the SAT Mathematics section. They read that the SAT I mathematics scores are normally distributed with a mean of 500 and a standard deviation of 100, while the ACT Mathematics scores are also normally distributed but with a mean of 20 and a standard deviation of 5.

a) Horace and Gertrude want to send their best mathematics test scores to colleges. How can they know which scores are their best if the scales for the tests are different?

b) How would you rank the four mathematics test scores from highest to lowest?

c) Horace and Gertrude each had a goal of getting at least one test score to be in the top five percent. Can you tell for sure if either of them succeeded using the empirical rule? If so, who and with what score?

d) Use the empirical rule to approximate the percentiles for each mathematics score. Justify your approximation.

e) What would we need in order to find the EXACT percentiles for each score without approximating?

f) Horace and Gertrude plan to help each other study to retake the tests. If they each want to get at least one of their scores in the top 1%, what score should they aim for on either test?

2) Have a list of students who demonstrated mastery of percentiles from the checkpoint exercise and use them as team "captains" to work with a group of three to four students to answer question *a*. After a minute, have groups share what they had discussed. If they mention using percentiles, return a probing question such as: "*That would be a great idea. Do we know the percentiles of all of our scores?"* or "*Can we compare them at all, even if we don't know their exact percentiles?"* The class should come up with the idea of comparing z scores.

3) Have students work in groups to answer question *b*.

Explore (Solving the Task)

4) Show the students questions *c* and *d* and tell them that now they would have to find percentiles, even when they may not know their exact values. They will have to share and defend their answers with the rest of the class when they are finished. As the groups work, circulate around the room and pose questions to guide their thinking as necessary.

Possible probing questions:

- *What are the percentages from the empirical rule? What do they mean?*
- *How would we use empirical rule percentages to find percentiles for z-scores 1, 2, or 3 standard deviations above or below the mean?*
- *What if a score falls just above or just below 1, 2, or 3 standard deviations? What would happen to the percentile?*
- *If one value has a higher z score than another, how would their percentiles compare?*

5) As groups finish question *d*, stop the class and have representatives from each group state their approximations. When approximations differ, have students defend their answers. After a few minutes, show question *e*. Have the groups take a minute to discuss it and then share with the class. They should reach the conclusion that they would some sort of reference to know percentiles for any score in a normal distribution.

6) Have one normal distribution percentile chart copied for each group and distribute them. Have the groups use the chart to find the exact percentiles for each score.

7) Once groups have finished, show question *f*. As students work on it, circulate around the room and give guiding questions as necessary. If any groups finish their work noticeably early, present the extension task for enrichment.

Possible probing questions:

- *How is the chart/table set up? What do we need to know about our values before we use it?*
- *What is if we knew our percentile and wanted a z score. How would we use the chart/table?*

Elaborate (Discuss Task and Related Mathematical Concepts)

8) Bring the class back together and have student representatives from each group (preferably not the captain) come up to discuss how their groups answered questions *e* and *f*, including how they decided to use the table to answer the question. As students share work, pose question to further the discussion:

- *How close were your approximate percentiles to the ones you came up with using the table/chart?*
- *Are there times when we do not need the exact percentiles to know things about normally distributed scores? What can we use then?*
- *How important do you think a reference like this table/chart was (is) for statisticians?*
- *Where else do you think you might find this information about percentiles and normal distributions?*

9) If the class has TI-83 or TI-84 calculators widely available, use the reference "The Normal Distribution on the Ti-83/84" to show how the calculator can be used to arrive at the same answers.

10) Pose the following questions to close the conversation:

- *What if our score distributions were not normal? Could we have used these methods? What ideas do you think would still apply?*
- *What real-world variables do you think would not be normally distributed? Why? (example: human weight, most classroom test scores)*
- *Why do you think mathematicians care so much about the normal distribution versus other distributions?*

Checking for Understanding

Purpose: Pose the following question at the end of the class as an exit slip to elicit evidence of students' understanding of z scores and percentiles.

Eunice and Winthrop are arguing over whose pet bullfrog is the more impressive jumper. Eunice says, "The other day my frog jumped a new record: 6.5 feet!" Winthrop counters, "That's nothing. My frog's trainer says his jumps put him in the 96th percentile of all bullfrogs." Not being able to settle their argument, Eunice finds a source that says that bullfrog jumps are normally distributed with a mean of 4 feet and a standard deviation of 1.6 feet.

If Eunice's source is correct, whose bullfrog is the better jumper? Explain how you determined your answer.

Common Misunderstanding

Purpose: ***Address some common misunderstandings students have about interpreting z-scores, the empirical rule, and percentiles.***

Remind students about the normal distribution checkpoint problem, writing it up on the board if necessary. Then write the two sample answers:

Carl: The two students did equally well because both of their scores were one standard deviation from the mean.
Kristina: But Student A really did score twice as well, because that score was in the 68^{th} percentile, while student B's was in the 34^{th}.

- Instruct students to think about why Carl might have thought the scores were "equally" good, but why that does not make sense in the context of the problem.
- *Take the concept of mean and standard deviation out of the problem. How should we know that the students did not perform equally well?*
- Ask the students to discuss with a partner: *Is there any truth to Carl's statement? In what way(s) are the two scores "equal"?*
- Ask a student to explain again what the definition of a percentile is and how it can help us interpret scores in a distribution.
- Ask students to think about where Kristina got her numbers from.
- *What does Kristina seem to understand about the problem that Carl does not?*
- Have students discuss with each other how they would explain the difference between the numbers in the Empirical rule versus percentiles.
- *How could Kristina use the numbers she has to come up with the correct percentiles?*

Closure

Purpose: ***Allow students to reflect on their understanding of the normal distribution and percentiles.***

Hand out the following questions on a piece of paper to students at the end of the lesson.

1) What is something you learned today about percentiles and the normal distribution that you did not know before class? (Or, alternatively, what is something you understand better now than you did?)

2) What is something that came to mind during class about today's material that did not get addressed or answered for you?

3) What is something about distributions and percentiles that was not part of today's material that you would be interested to know about?

Extension Task
Purpose: ***Provide an extension task for students who have demonstrated a thorough understanding of how to find percentiles for values of normally distributed variables. This extension will require them to use the same tools and concepts but to work backwards to find the missing mean and standard deviation of a normal distribution using both the empirical rule and a table or calculator.*** You and a friend take a new standardized test. You are told that the results are normally distributed but are not told the mean or standard deviation of the scores. Once your scores are calculated, the proctor tells you that you received a score of 32, which places you in the 16th percentile. The proctor then tells your friend that his score of 56 put him in the top 2.5% of all people they had tested so far. a) Given the above information and the empirical rule, find the mean and standard deviation of the distribution of test results without a calculator. Explain how you found your answer. b) Using a table or your calculator, answer the same question if your score actually put you in the 30th percentile and your friend actually scored in the top 1%.

Horace and Gertrude's Test Score Quandary

Horace and Gertrude have both taken the SAT I and the ACT, and they are comparing their mathematics scores. Horace scored a 30 on the ACT mathematics section and a 570 on the SAT mathematics section. Gertrude scored a 24 on the ACT Mathematics section and a 650 on the SAT Mathematics section. They read that the SAT I mathematics scores are normally distributed with a mean of 500 and a standard deviation of 100, while the ACT Mathematics scores are also normally distributed but with a mean of 20 and a standard deviation of 5.

a) Horace and Gertrude want to send their best mathematics test scores to colleges. How can they know which scores are their best if the scales for the tests are different?

b) How would you rank the four mathematics test scores from highest to lowest?

c) Horace and Gertrude each had a goal of getting at least one test score to be in the top five percent. Can you tell for sure if either them succeeded using the empirical rule? If so, who and with what score?

d) Use the empirical rule to approximate the percentiles for each mathematics score. Justify your approximation.

e) What would we need in order to find the EXACT percentiles for each score without approximating?

f) Horace and Gertrude plan to help each other study to retake the tests. If they each want to get at least one of their scores in the top 1%, what score should they aim for on either test?

Horace and Gertrude's Test Score Quandary

Student Response Guide/Answer Key

a) Even though the two tests have different means and standard deviations, they are both normally distributed. Therefore comparable z-scores on either test correlate to comparable achievement compared with the entire population. Horace and Gertrude can find the z-scores on both tests and know that whichever one is higher represents their “better” score.

b) First, find the z-scores of each test.

Horace ACT: z = (30 – 20)/5 = 2

Horace SAT: z = (570 – 500)/100 = .7

Gertrude ACT: z = (24 – 20)/5 = .8

Gertrude SAT: z = (650 – 500)/100 = 1.5

Then rank the test scores in order from highest to lowest z-score: Horace ACT, Gertrude SAT, Gertrude ACT, Horace SAT.

c) Using only the empirical rule, we can only be sure that a score is in the top 5% if it has a z-score of at least 2. Such a score would be in at least the 97.5th percentile. Only Horace's ACT score meets this criteria.

d) Students should all have the answer of 97.5th percentile for Horace's ACT score, since it has a z-score of exactly 2. Students should also be able to determine that a z-score of exactly 1 lies in the 84th percentile from the empirical rule and should use that information to guide their responses. Reasonable answers may include:

Horace ACT: 97.5 (all students should have this)
Gertrude SAT: All answers should fall between 85 and 97, though 90-94 would be the most reasonable
Gertrude ACT: All answers should fall between 51 and 83, though 70 – 80 would be the most reasonable
Horace SAT: Answers should be similar to those given for Gertrude ACT, though slightly less.

e) We would need a reference that could provide us with the exact area under the normal curve below any given value. We could then look up the area under each z-score. The exact percentiles for each score are:

Horace ACT: 97.72
Gertrude SAT: 93.32
Gertrude ACT: 78.81
Horace SAT: 75.80

f) Using the table, students should find that a score in the 99th percentile has a z-score of about 2.3 (actually about 2.326). Therefore Gertrude and Horace have to obtain scores that are at least 2.3 standard deviations above the mean (preferable 2.4 to be sure to get at least 99%).

SAT: 2.3*100 + 500 = 730 (or 740 for a z-score of 2.4)
ACT: 2.3*5 + 20 = 31.5 (or 32 for a z-score of 2.4)

Normal Distribution Extension Task

Student Response Guide/Answer Key

a) mean: 40; standard deviation: 8
You are told that your score was in the 16th percentile, which according to the empirical rule corresponds to a z-score of -1. Your friend's score in the 97.5th percentile corresponds to a z score of 2. This means that your scores are three standard deviations apart. Dividing the difference in your actual scores (24) by three yields a standard deviation of 8. It follows that the mean is both 8 higher than your score of 32 and 16 less than your friend's score of 56, making it 40.

b) mean: 36.38; standard deviation: 8.42 (answers will vary slightly due to rounding)
The overall process is the same for this problem except that students must use their calculator or a table to find the z-scores that correspond with the given percentiles. A score in the 30th percentile corresponds to a z-score of approximately -.52 and a score in the 99th percentile corresponds to a z-score of approximately 2.33. Therefore the two scores are 2.85 standard deviations apart. Dividing 24/2.85 yields a standard deviation of approximately 8.42. The mean lies .52 standard deviations above a score of 32, so 32 + (8.42)(.52) = 36.38.

Exit Slip

Eunice and Winthrop's Bullfrog Bickering

Eunice and Winthrop are arguing over whose pet bullfrog is the more impressive jumper. Eunice says, “The other day my frog jumped a new record: 6.5 feet!” Winthrop counters, “That's nothing. My frog's trainer says his jumps put him in the 96th percentile of all bullfrogs.” Not being able to settle their argument, Eunice finds a source that says that bullfrog jumps are normally distributed with a mean of 4 feet and a standard deviation of 1.6 feet.

If Eunice's source is correct, whose bullfrog is the better jumper? Explain how you determined your answer.

Eunice and Winthrop's Bullfrog Bickering

Student Response Guide/Answer Key

Students may choose one of three routes to answer this question. They may 1) convert Eunice's frog's jumping distance to a percentile, 2) convert Winthrop's frog's percentile to a jumping distance, or 3) convert both numbers to z scores. Students will need to use their calculator or a table to correctly go back and forth between percentiles and z scores regardless of their solution method.

The correct information for both frogs is as follows, with z scores rounded to two decimal places:

	Jumping Distance	z score	Percentile
Eunice	6.5 feet	1.57	94^{th}
Winthrop	6.8 feet	1.75	96^{th}

Regardless of the student's method of comparison, Winthrop's frog is the better jumper.

The Numeral Distribution and Percentiles

Closing Thoughts

1) What is something you learned today about percentiles and the normal distribution that you did not know before class? (Or, alternatively, what is something you understand better now than you did?)

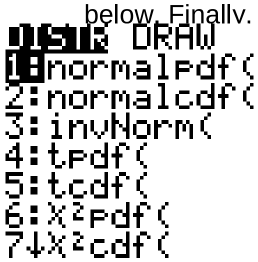
2) What is something that came to mind during class about today's material that did not get addressed or answered for you?

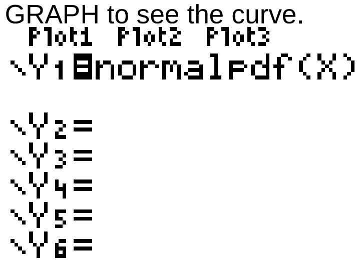
3) What is something about distributions and percentiles that was not part of today's material that you would be interested to know about?

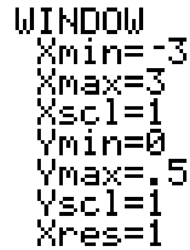
The Normal Distribution on the TI-83/84

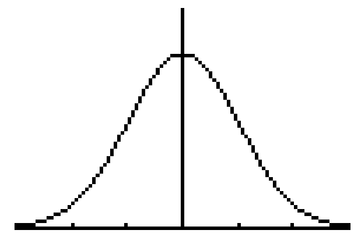
Calculator Note: E99 or -E99 can be used for limits of infinite or negative infinite, respectively.

1. **Displaying a normal curve.**
   To see a graph of a normal curve, clear out the Y= screen and from there go into the Distr (Distribution) menu. Press ENTER on option 1, normalpdf((normal probability density function). Enter normalpdf(X) into Y1. Then go to the Window and enter the suggested parameters shown below. Finally, press GRAPH to see the curve.

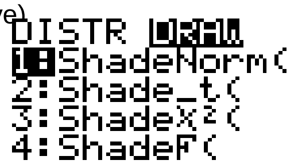


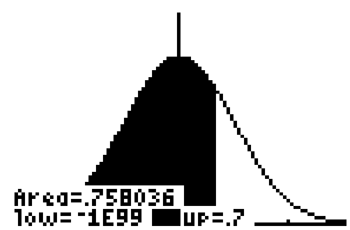




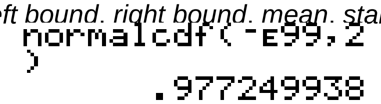


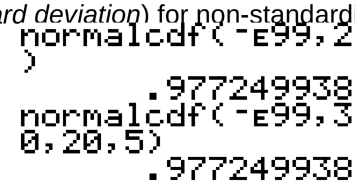
2. **Showing the area under a normal curve.**
   To see a visual representation of the area under a normal curve, go to the Distribution menu and scroll over to DRAW. Press enter on the first option, ShadeNorm(. On the main screen, put in the left bound of the area as a z-score followed by a comma and the right bound. Press enter to display the shaded graph and area (make sure the window settings are the same as those listed above).



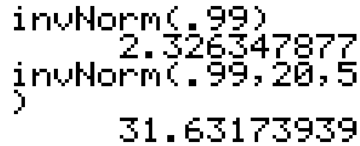


3. **Finding an area under a normal distribution on the home screen.**
   Under the Distribution menu, use the second option normalcdf((standing for normal cumulative distribution function). Then enter the normalcdf(*left bound, right bound*) for standardized (z) scores, or
   normalcdf(*left bound, right bound, mean, standard deviation*) for non-standardized scores.





4. **Finding a standardized or non-stardardized score from a percentile on the main screen.** Under the Disribution menu, use the third option inversenorm(. Then enter inversenorm(*percentile*) to return a standardized score or inversenorm(*percentile, mean, standard deviation*) for a non-standardized score



Research and review of standard

Content Standard(s):

Summarize, represent, and interpret data on a single count or measurement variable.

Standard: <u>HSS-ID.A.4</u> Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Standard(s) for Mathematical Practice:

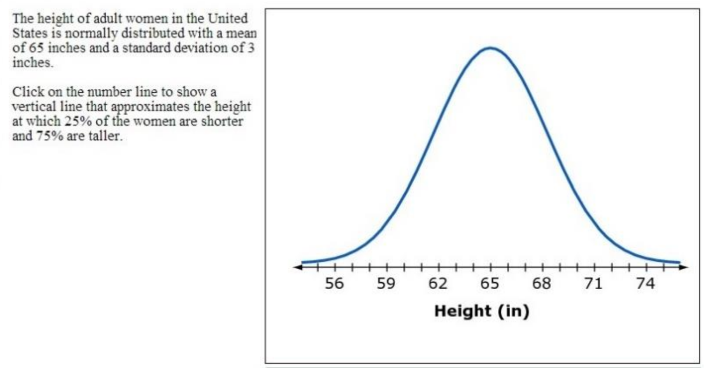
MP.4 Model with mathematics.
Identify important quantities in a practical situation and map their relationships using a visual representation.

MP.5 Use appropriate tools strategically.
Use available tools such as a calculator, table, or spreadsheet to solve a mathematical problem.

Smarter Balanced Claim

Claim 4 Modeling and Data Analysis
Students can analyze complex, real-world scenarios and construct and use mathematical models to interpret and solve problems.

Smarter Balanced Item



CPR Pre-Requisites
(Conceptual Understanding, Procedural Skills, and Representations)

Look at the Progressions documents, Learning Trajectories, LZ lesson library, unpacked standards documents from states, NCTM Essential Understandings Series, NCTM articles, and other professional resources. You'll find links to great resources on your PLC Platform.

Conceptual Understanding and Knowledge
- Understand that the mean determines the "balancing" point of a probability density curve
- Understand that the mean of a symmetric distribution, such as the normal distribution, lies at at the location of the axis of symmetry
- Understand how the standard deviation determines the spread of a bell curve relative to the number line
- Understand how the empirical (68-95-99.7) rule and the standard deviation can be used to approximate percentages of total data in a given range about the mean of any normally distributed data set
- Understand how to use basic operations with percentages to convert an empirical rule percentage to a tail percentage
- Understand how to estimate a percentile that lies "a little above" or "a little below" the a percentile computed directly from the standard deviation

Procedural Skills
- Adding, subtracting, and dividing percentages
- Estimating values

	• Using a number line • Calculate mean and standard deviation of a data set • Using technology (calculators, spreadsheets, tables) to calculate area under the normal curve **Representational** • Express normally distributed data with a bell curve with the correct center and spread • Represent a percent of the distribution of a random variable as an area between 0 and 1 under the probability density curve • Represent a particular percentile of a distribution as a domain value with that percent of the area between the probability density curve and the number line falling to the left of that value. **Social knowledge** • Know the definition of the term "mean" • Know the definition of the term "standard deviation" • Know how to express mean, standard deviation, and z scores symbolically

Standards Progression

**Look at LearnZillion lessons and expert tutorials, the Progressions documents, learning trajectories, and the "Wiring Document" to help you with this section*

Prerequisite Standards	Corequisite Standards	Future Standards
6.SP.A.2 – Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. 6.SP.A.3 – Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. 7.SP.B.4 – Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.* HSS-ID.A.2 – Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	HSS-IC.A.2 – Decide if a specific model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* HSS-IC.B.6 – Evaluate reports based on data.	HSS-MD.A.2 – Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. HSS-MD.A.3 – Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple choice test where each question has four choices, and find the expected grade under various grading schemes.*

Common Misconceptions/Roadblocks
What characteristics of this problem may confuse students? • Students may not realize that the problem is asking for a percentile rather than a percentage about the mean • Student may expect the answer to be exactly 1, 2, or 3 standard deviations above or below the mean rather than an estimation **What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?** • Student does not know the empirical (68-95-99.7) rule for normal distributions • Student does not know how to transfer from the empirical rule to calculating percentiles • Student does not divide the difference between 100 and the percentage of the data in a given range about the mean by 2 in order to find the percentage of the area in a single tail • Student does not know how to estimate a percentile when they do not explicitly know the actual percentile a variable value represents **What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?** • Student may believe the 25^{th} percentile of a distribution can always be found exactly one quarter through the domain.