# Mathematics Instructional Cycle Guide

Concept HSN.Q.A.1

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Connecticut Dream Team teacher

## CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

Reason quantitatively and use units to solve problems.

CCSS.MATH.CONTENT.HSN.Q.A.1

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

MP1 Make sense of problems and persevere in solving them.

MP2 Reason abstractly and quantitatively.

MP4 Model with mathematics.

MP6 Look for and make use of structure

## WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings **p. 2**
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint **p. 3-6**
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed **Step 3 and 4: pg. 0-3**
- Supporting lesson materials **Step 3 and 4: Pg. 4-6**
- Precursory research and review of standard **HSN.Q.A.1** and assessment items that illustrate the standard **Pg. 7-9**

## HOW TO USE THIS DOCUMENT

1) Before the lesson, administer the **Ice Cube** ***Mathematical Checkpoint*** individually to students to elicit evidence of student understanding.
2) Analyze and interpret the student work using the ***Student Response Guide***
3) Use the next steps or ***follow-up lesson plan*** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

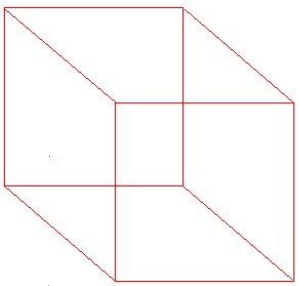
## MATERIALS REQUIRED

- **Dry Erase response boards**
- **Chart Paper to show student responses**

## TIME NEEDED

**Ice Cube Checkpoint** administration: **15 minutes**
Follow-Up Lesson Plan: **1-2 class periods**

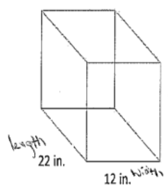
***Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.***

<table>
<tr><th colspan="3">Step 1: Elicit evidence of student understanding</th></tr>
<tr><th colspan="3">Mathematical Checkpoint</th></tr>
<tr><th>Question(s)</th><th colspan="2">Purpose</th></tr>
<tr><td rowspan="2">You and your family are planning a trip to the beach. You are going to bring a cooler to keep drinks and food cold. You have a rectangular cooler with a length of 22 inches, a width of 12 inches and a depth of 11 inches. You want to fill half of the cooler with ice. You have two 20-pound bags of ice.<br><br>One pound of ice equals 29 cubic inches.<br><br><br><br>(not drawn to scale)<br>Do you have enough ice to fill up ½ the cooler?<br>If yes, how much extra ice do you have, in pounds (to the nearest .1 pound)?<br>If no, how much more ice do you need, in pounds (to the nearest .1 pound?<br>Explain your choice</td><td>CT Core Standard:</td><td>HSN.Q.A.1<br>Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.</td></tr>
<tr><td>Target question addressed by this checkpoint:</td><td>Do the students understand how to convert units of volume?<br>Do students understand how to calculate volume of a rectangular prism?<br>Do students understand how to find a quotient and interpret the quotient?<br>Can the students make a decision and defend their choice?</td></tr>
</table>

## Step 2: Analyze and Interpret Student Work

## Student Response Guide

### Got It



2904
− 1160
1744

2904 $in^3$

20 20

580 580

1160

Do you have enough ice to fill up ½ the cooler?

No we do not.

If yes, how much extra ice do you have, in pounds (to the nearest ¼ pound)?

1452

If no, how much more ice do you need, in pounds(to the nearest ¼ pound?

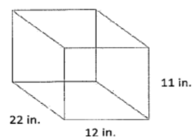
10.25 pounds of ice

Explain your choice

1160

We need 10.25 more pounds of ice because the cubic inches the cooler has is 2904 $in^3$. One 20 pound bag takes up 580 $in^3$. The two together takes up 1160 $in^3$. 10.25 more pounds of ice will take up 1457.25 $in^3$, but is 5.25 pounds of ice that should be left over. Lastly, half of the cooler is 1,452 $in^3$.

### Developing



One pound of ice equals 29 cubic inches.

580 $in^3$ of ice

Volume = 2,904 $in^3$

1/2 = 1,452 $in^3$

1452 − 580 = 872 $in^3$

Do you have enough ice to fill up ½ the cooler?

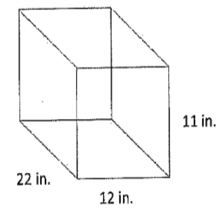
If yes, how much extra ice do you have, in pounds (to the nearest ¼ pound)?

If no, how much more ice do you need, in pounds(to the nearest ¼ pound?

Explain your choice

You would need 30 more pounds of ice to fill up half of the cooler because the total volume of the cooler is 2904 $in^3$. Half the volume of the cooler is 1452 $in^3$. To find out how much ice you have in cubic inches, you do 20 (pounds of ice) times 29 (cubic inches). The answer is 580, which does not fill up half of the cooler. To find out how much more ice you need in pounds, you first have to find it in cubic inches. To get this, you do 1452 minus 580, to get 872 $in^3$. Then, you have to divide that by 29 to find out how much ice you need in pounds. This rounds to 30 pounds.

### Getting Started



One pound of ice equals 29 cubic inches.

11
×12
×22

29 ) 2904 $in^3$
−29
00

100.14 pounds can fit

Do you have enough ice to fill up ½ the cooler?

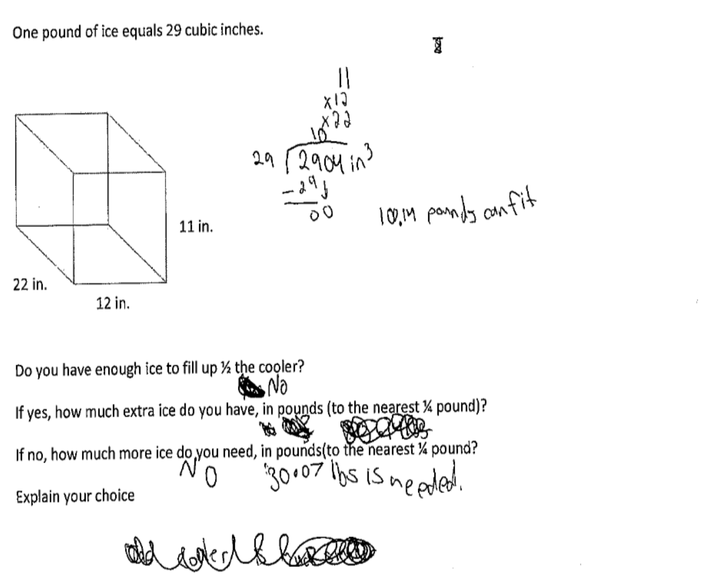
No

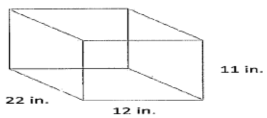
If yes, how much extra ice do you have, in pounds (to the nearest ¼ pound)?

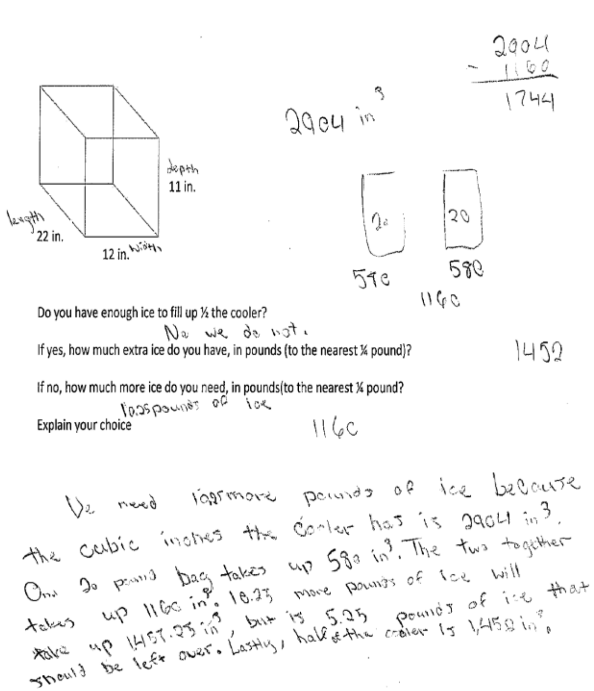
If no, how much more ice do you need, in pounds(to the nearest ¼ pound?

No 30.07 lbs is needed.

Explain your choice

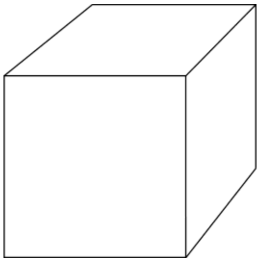
| Getting Started | |
|---|---|
| **Student Response Example** | **Indicators** |
|   | • Student response may not be accurate, but uses units consistently to solve the problem<br>• Student correctly calculates the volume of the cooler but it is not clear if he calculate half of the volume.<br>• The student may have correctly calculated that he needs about 50 lbs of ice but may not have realized how much ice he already has.<br>• There may be no explanation of solutions |
| **In the Moment Questions/Prompts** | **Closing the Loop (Interventions/Extensions)** |
| Q: What is the meaning of two 20 lbs and how did you use this information?<br><br>Q What does it mean to round to nearest .1 pounds?<br><br>P: Show me how you could approximate how much ice will fit without long division.<br><br>Q: You said you needed 30.07 lbs, what is the total amount needed?<br>Does 30 lbs plus what you have equal the total? Explain<br><br>Q: How much of the cooler do you have to fill?<br><br>P:Explain how you calculated 30.07 lbs | **Provide a review of definition of a pound. Provide practice on board for rounding to tenth.** |

<table>
<tr><th colspan="2">Developing</th></tr>
<tr><th>Student Response Example</th><th>Indicators</th></tr>
<tr><td>
One pound of ice equals 29 cubic inches.<br>
580 in³ of ice<br>

<br>
Volume = 2,904 in³<br>
1/2 = 1,452 in³<br>
1452 − 580 = 872 in³<br>
Do you have enough ice to fill up ½ the cooler?<br>
If yes, how much extra ice do you have, in pounds (to the nearest ¼ pound)?<br>
If no, how much more ice do you need, in pounds(to the nearest ¼ pound?<br>
Explain your choice<br>
You would need 30 more pounds of ice to fill up half of the cooler because the total volume of the cooler is 2904 in³. Half the volume of the cooler is 1,452 in³. To find out how much ice you have in cubic inches, you do 20 (pounds of ice) times 29 (cubic inches). The answer is 580, which does not fill up half of the cooler. To find out how much more ice you need in pounds, you first have to find it in cubic inches. To get this you do 1452 minus 580, to get 872 in³. Then, you have to divide that by 29 to find out how much ice you need in pounds. This rounds to 30 pounds.
</td><td>
<ul>
<li>The student correctly calculates the volume of the cooler and half of cooler</li>
<li>The student calculates the volume of one bag of ice but incorrectly determines that this is how much ice that he has.</li>
<li>The student uses a proper procedure to find the total but an earlier error leads to the incorrect solution.</li>
<li>The student is able to explain the answer although a miscalculation leads to an error in problem</li>
</ul>
</td></tr>
<tr><th>In the Moment Questions/Prompts</th><th>Closing the Loop (Interventions/Extensions)</th></tr>
<tr><td>
Q What does it mean to round off to .1 pounds?<br>
Q What does 2 bags of ice tell you about how much ice you have?<br>
Q: Explain how can you check your answer?<br>
Q: Explain how to write an equation to solve this problem?<br>
Q: What information is given in the beginning for problem?
</td><td>
Provide review of rounding off. Have the students re-read the problem and underline information. Have students explain solutions in their own words.
</td></tr>
</table>

| Got it | |
|---|---|
| **Student Response Example** | **Indicators** |
|  | • The student correctly calculates the volume of the cooler and half of cooler<br>• The student calculates the volume of two bags of ice<br>• The student uses a proper procedure to find the total using an equation or explains the procedure<br>• The student obtains the correct solution.<br>• The student may make a minor mathematical error including rounding off.<br>• The student had proper use of basic computation.<br>• The student correct converts pounds to volume. |
| **In the Moment Questions/Prompts** | **Closing the Loop (Interventions/Extensions)** |
| Q: How can you check your answer?<br>Q: Can you write an equation to solve this problem?<br>Q: What information is given in the beginning for problem?<br>Q: How did you round off?<br>Show me how you used division in this problem?<br>Show me how you can use an equation. | Have students analyze other approaches for solving the problem and explain the thinking behind those approaches. Have students walk around room and help others during group work. |

<table>
<tr><th colspan="3">Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction</th></tr>
<tr><td>Lesson Objective:</td><td colspan="2">Convert and use various units of volume to solve and defend a real world problem with rectangular prisms.</td></tr>
<tr><td>Content Standard(s):</td><td colspan="2">Reason quantitatively and use units to solve problems.<br><br>CCSS.MATH.CONTENT.HSN.Q.A.1<br><br>Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.</td></tr>
<tr><td>Targeted Practice Standard :</td><td colspan="2">MP1 Make sense of problems and persevere in solving them.<br>How do students explain the meaning of the problem in the Piñata problem and look for starting points to its solution? Are students able to explain the connection between different units of volume?<br><br>MP2 Reason abstractly and quantitatively.<br>Do students attend to the meaning of the quantities in the Piñata task to make meaning of the answer? Are students able to abstract the problem into an equation?<br><br>MP4 Model with mathematics.<br><br>Can students draw a model of the candy and inside of the Piñata? Can students make an equation to use for different types of candy?<br><br>MP7 Look for and make use of structure</td></tr>
<tr><td colspan="2">Mathematical Goals</td><td>Success Criteria</td></tr>
<tr><td colspan="2"><ul><li>Understand that the volume of a material can be formed into different shapes and that shapes can define volume.</li><li>Understand that different units describe the same amount of material when using unit conversation</li><li>Understand that equation can be written to represent real-life situations, and numbers and variables in an equation have contextual reference.</li><li>Interpret that quotients can have same or different units.</li><li>Understand the need to change units</li></ul></td><td>Convert from one unit of volume to another.<br>Write an equation to find how much of a smaller item fits into a larger container.<br>Interpret the meaning of their answer and if it makes sense</td></tr>
<tr><td colspan="3">Launch (Probe and Build Background Knowledge)</td></tr>
<tr><td colspan="3">Purpose: Assess and activate background knowledge about unit conversion and volume<br>Provide students with response boards. Project the following questions on the board and instruct the students to do the following:<br>• Discuss the problem with group member and work out problem. Draw picture if needed.<br>• Come to a consensus on the solution<br>• Be prepared to justify the solution</td></tr>
</table>

Example:
Find the volume of a prism with L=10in, W=20in, H=5in



If I had to fill the prism 1/4 full of water how many cubic inches would I need?
If I needed a prism with a volume of 1000 cubic inches and L=5in and W=7in, what is the height?
What equation can be made to solve this problem?
If I have 2000 cubic yards of water, how many cubic feet do I have? 1 cubic yard= 27 cubic feet

**Instructional Task**

**Purpose:** *Introduce the Piñata and provide students time to reason and problem-solve.*

**Engage (Setting Up the Task)**

1) Have the Piñata problem projected on the board for student to see and read silently.

2.) Facilitate discussion about the story frame using the following prompts/questions:
*Tell a partner what is going on in this story in your own words.*
*Students discuss what they will need to solve the problem? What formulas?*
*What mathematical questions could we ask about problem?*
*How might you go about finding solutions to your question(s)?*

3.)Instruct students to take 2 minutes to write how they may approach the problem using words, numbers, or pictures on their response boards.

4.) Facilitate a pair-share to have students share their thinking with a partner. Each partner will have 1 minute to share their approach. Switch roles after 1 minute.

5.) Explain students will now work on the Piñata task with their group. Students may use any tools that are available in the room. Provide copies of the task to each student, and specify time students will have to work on task.
If a document camera if unavailable, consider having students write their solutions on chart paper to facilitate the sharing of student work in the discussion.

**Explore (Solving the Task)**

*Provide students time to work on the Piñata problem in groups. Circulate to observe, question, and note students who are strategic candidates to share out responses. Possible questions/prompts to as students engage in task:*

**Focusing Questions**

- *What information do you know that can help you make sense of this problem?*
- *What type of model could you draw to show that information?*
- *What numbers and symbols do you think you may need in your equation?*
- *What does each part of the equation tell you?*

**Probing Questions**

- *Can you show and explain more about how you used the model you drew to find the solution?*
- *It's still not clear how you figured out that ___ was the number of bags of ice you need; can you explain it another way?*
- *How does your solution show unit conversion?*
- *How does your model show your solution?*
- *How can you check your solution?*
- *As you drew your model, what decisions did you make so that you could represent the problem?*

**Advancing Questions**

- *How might you prove your solution another way?*
- *Why did you decide you would need 10 lbs of ice?*

**Elaborate (Discuss Task and Related Mathematical Concepts)**

*How will you facilitate the sharing of student work and discussion to support students in making mathematical connections?*

7.) Call the class back together to facilitate a task discussion. Project or post the following questions for students to consider as others share their work.

- How is this approach the same as the approach we used in beginning problems?
- How is this approach different than the approach we used in beginning problems?
- Does this approach make sense to me? What questions do we have about this approach?
- Do we agree or disagree with this solution? Why?

**Checking for Understanding**

**Purpose:**

Q: What does it mean to round to nearest .1 pounds?

P: Show me how you could approximate how much candy will fit.

Q: what is the total amount needed?

Q: How did you figure out how much candy you need?

Q: How of much of each candy do you need to fill?

P: Explain how you calculated the final answer?

Q: Explain how can you check your answer?

Q: Explain how to write an equation to solve this problem?

Q: What information is given in the beginning for problem?

Q: Show me how you used division in this problem?

Q: Why will the entire container not be fill with candy?

**Common Misunderstanding**

**Purpose:** *Address a common misunderstanding about how to convert measurements.*

*Convert the following into square feet. Round to nearest tenth*

1. *1000 square inches*
2. *10000 square inches*
3. *4000.53 square inches*

*Convert the following into square inches. Round to nearest tenth*

1. *23 square feet*
2. *10 square feet*
3. *4 square feet*
4. *1.5 square feet*

*Have students pair up with another group and discuss if their answers make sense to each other.*
*Have students take two minutes each explaining how they arrived at their solution.*
*Have other group ask questions about their solution and what they do not understand.*
*Give students 10 minutes for revised the solution based on feedback.*

## Checking for Understanding

**Purpose:**

Q: What does it mean to round to nearest .1 pounds?
P: Show me how you could approximate how much candy will fit.
Q: what is the total amount needed?
Q: How did you figure out how much candy you need?
Q: How of much of each candy do you need to fill?
P: Explain how you calculated the final answer?
Q: Explain how can you check your answer?
Q: Explain how to write an equation to solve this problem?
Q: What information is given in the beginning for problem?
Q: Show me how you used division in this problem?
Q: Why will the entire container not be fill with candy?
Does you answer make sense?

## Closure

***Provide students an opportunity to self –assess their own learning related to the success criteria by projecting the questions below or providing students with a copy of self-assessment to complete.***
Circle the number you feel best matches your level of success with each item.

*I can convert between two different measures of volume*

| **Not at all** | | | | **Absolutely** |
|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** |

*I can write an equation to model a real life situation*

| **Not at all** | | | | **Absolutely** |
|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** |

*I can justify my answer*

| **Not at all** | | | | **Absolutely** |
|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** |

After this lesson, I feel like I need more time learning….

________________________________________________________________________________
________________________________________________________________________________
________________________________________________________________________________
__________________________________________________

## Extension Task

**Purpose:** ***Provide an extension task for those students who are ready to deepen their understanding of conversions and volume. This extension task context uses different shape container and candy.***

*Your friends don't like your candy choice and want to use large Smarties that come in a cylindrical shape. The packages have a diameter of one inches and a height of 6 inches. The tech ed. Department says that can make any shape to hold the candy and the volume can hold 30 sq feet of candy. You need to give them Height, Width, Length and/or Diameter of the container.*

*Design a container with the given information.*
*Determine how much candy is needed to fill the container.*

Piñata Problem

Your high school class wants to build and fill a large Piñata for charity. The part of the Piñata that will hold the candy will be a rectangular prism and measure 5 feet long, 3 feet wide and 2 feet tall. The class decides they will use types of two types of candy shapes: rectangular prism and spherical.

Part A

You find and measure the Jolly ranchers to be 1 in by 1in by .5in. These candies will fill up half of the piñata. How many candies will you need? Hint: 1728 square inches=1 square foot. Show work.

Part B

There are 90 candies in each bag, about how many bags will you need? Show work.

Part C

Your friends are in charge of finding out how many spherical candies you need. You measure the diameter of the candy as 1 inch. You think you can help them by writing an equation to solve the problem based on your work. Make an equation that they can use once to find the number of candies needed. Label each variable that you use.

Part D

How many spherical candies do you need? If there are 70 candies in each bag, how many bags do you need? Show work

Part E

Once you figure out how many candies you need, another friend says that you won't need that many candies. Explain why he is correct. Why might there be error or that all the candies may not fit.

**Student Self-Assessment**

Think about your learning…..

Circle the number you feel best matches your level of success with each item.

*I can convert between two different measures of volume*

| **Not at all** | | | | **Absolutely** |
|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** |

*I can write an equation to model a real life situation*

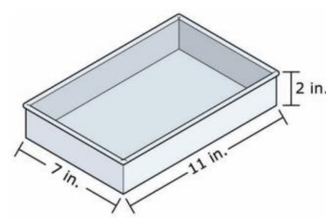
| **Not at all** | | | | **Absolutely** |
|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** |

*I can justify my answer*

| **Not at all** | | | | **Absolutely** |
|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** |

After this lesson, I feel like I need more time learning….

________________________________________________________________________________
________________________________________________________________________________
________________________________________________________________________________
________________________________________________________________________________
_____________

| Research and review of standard | |
|---|---|
| **Content Standard(s):** | **Standard(s) for Mathematical Practice:** |
| Reason quantitatively and use units to solve problems.<br>CCSS.MATH.CONTENT.HSN.Q.A.1<br>Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | MP1 Make sense of problems and persevere in solving them.<br>MP2 Reason abstractly and quantitatively.<br>MP4 Model with mathematics.<br>MP6 Look for and make use of structure |
| **Smarter Balanced Claim** | ***Smarter Balanced Item*** |
| *Claim 2: Problem Solving*<br>*Students can solve a range of complex, well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.* | Hannah makes 6 cups of cake batter. She pours and levels all the batter into a rectangular cake pan with a length of 11 inches, a width of 7 inches, and a depth of 2 inches.<br><br><br>One cubic inch is approximately equal to 0.069 cup.<br>What is the depth of the batter in the pan when it is completely poured in? Round your answer to the nearest $\frac{1}{8}$ of an inch. |
| **UCPR Pre-Requisites**<br>*(Conceptual Understanding, Procedural Skills, and Representations)*<br><br>*Look at the Progressions documents, Learning Trajectories, LZ lesson library, unpacked standards documents from states, NCTM Essential Understandings Series, NCTM articles, and other professional resources. You'll find links to great resources on your PLC Platform.* | **Conceptual Understanding and Knowledge**<br>• Understand that the volume of a material can be formed into different shapes and that shapes can define volume.<br>• Understand that different units describe the same amount of material when using unit conversation<br>• Understand that equation can be written to represent real-life situations, and numbers and variables in an equation have contextual reference.<br>• Interpret that quotients can have same or different units.<br>• Understand the need to change units<br><br>**Procedural Skills**<br>• Divide two units and interpret the result.<br>• Solve a multi-step equation<br><br>**Representational**<br>• Write equations to represent word problems, using a variable to represent unknown quantities |

| | |
|---|---|
| | **Social knowledge**<br>• Different units of measure<br>• Volume of Prism Formula<br>• How to convert units<br>• Rounding decimals and fractions<br>• Vertical and Horizontal Lengths |

| **Standards Progression**<br>**Look at LearnZillion lessons and expert tutorials, the Progressions documents, learning trajectories, and the "Wiring Document" to help you with this section* | | |
|---|---|---|
| **Pre-Requisites** | **Co-Requisites** | **Post-Requisites** |
| 5.MD.C.5<br>Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.<br>5.MD.C.5.B<br>Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. | HSN.Q.A.2<br>Define appropriate quantities for the purpose of descriptive modeling.<br>HSN.Q.A.3<br>Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | HSG.GMD.A.3<br>Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*<br>HSG.GMD.A.1<br>Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.<br>HSG.MG.A.3<br>Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).* |

**Common Misconceptions/Roadblocks**

**What characteristics of this problem may confuse students?**

- *Students may not understand that it is not necessary to fill to top.*
- *Student may not understand baking terms.*
- Student may not know the measure "cup"

**What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?**

- *Students cannot apply Volume formula backwards to find length of a side.*
- *Students cannot round from a decimal answer to fractional answer*
- *Students multiply instead of divide to convert measurement*
- *Students do not realize that they have to convert units*
- Students cannot visualize the size of pan from diagram

**What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?**

- *Depth refers to going downward like an ocean or lake and height is going upward like a building*
- *Volume problems only involve whole number measurements*