Mathematics Instructional Cycle Guide

Concept (HSF.IF.A.1)

Created by Alicia Loesche, 2014 Connecticut Dream Team teacher

CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

HSF.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

MP.4 Model with Mathematics: Identify important quantities an analyze relations mathematically to draw conclusions.

MP. 6 Attend to precision: Use clear definitions in discussion with others and in their own reasoning.

WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings **(page 2)**
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint **(pages 3-6)**
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed **(page 7)**
- Supporting lesson materials **(pages 7-12)**
- Precursory research and review of standard **insert standard code** and assessment items that illustrate the standard (**page 13-18)**

HOW TO USE THIS DOCUMENT

1) Before the lesson, administer the ***Mathematical Checkpoint*** individually to students to elicit evidence of student understanding.
2) Analyze and interpret the student work using the ***Student Response Guide***
3) Use the next steps or ***follow-up lesson plan*** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

MATERIALS REQUIRED

- **Worksheets**

TIME NEEDED

Functionality administration: **1 Days**
Follow-Up Lesson Plan: **2-3 Days**

Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

<table>
<tr><th colspan="3">Step 1: Elicit evidence of student understanding</th></tr>
<tr><th colspan="3">Mathematical Checkpoint</th></tr>
<tr><th>Question(s)</th><th colspan="2">Purpose</th></tr>
<tr><td rowspan="2">Amanda and Chris are taking Algebra II. Their task is to determine whether or not the equation $y^2 = x + 2$ is a function. Amanda believes that this IS a function. Chris disagrees and believes that this is NOT a function. Who is correct, Amanda or Chris?
1. Complete the table of values for the relation.

<table><tr><td>X</td><td>Y</td></tr><tr><td>0</td><td></td></tr><tr><td>2</td><td></td></tr></table>
2. Who is right, Amanda or Chris? EXPLAIN HOW YOU KNOW</td><td>CT Core Standard:</td><td>HSF.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).</td></tr>
<tr><td>Target question addressed by this checkpoint:</td><td>• How do students interpret an equation?
• How do students apply their knowledge of functions to equations?
• Are students able to determine if an equation is a function?
• How do students interpret positive and negative components of the equation?
• Can we tell if the relation is a function without a graph or with an ordered pair?</td></tr>
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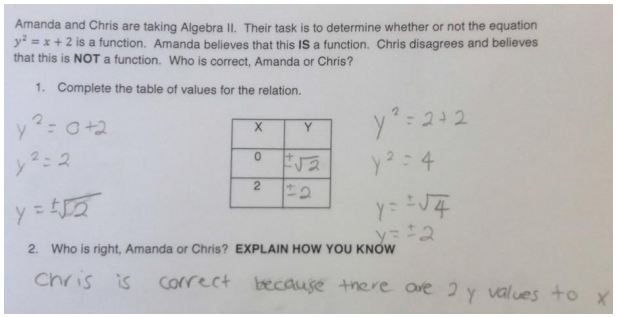
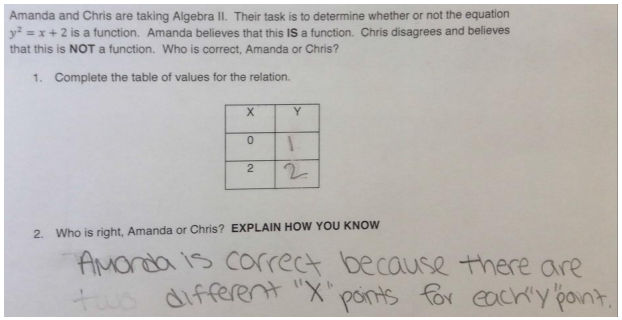
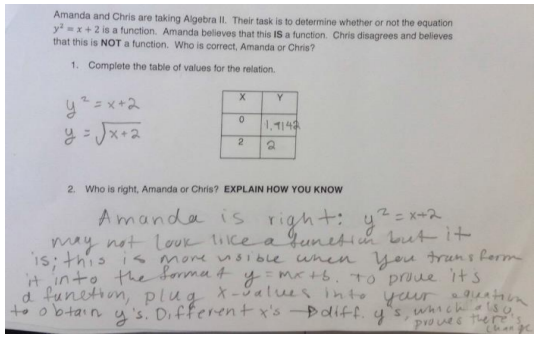
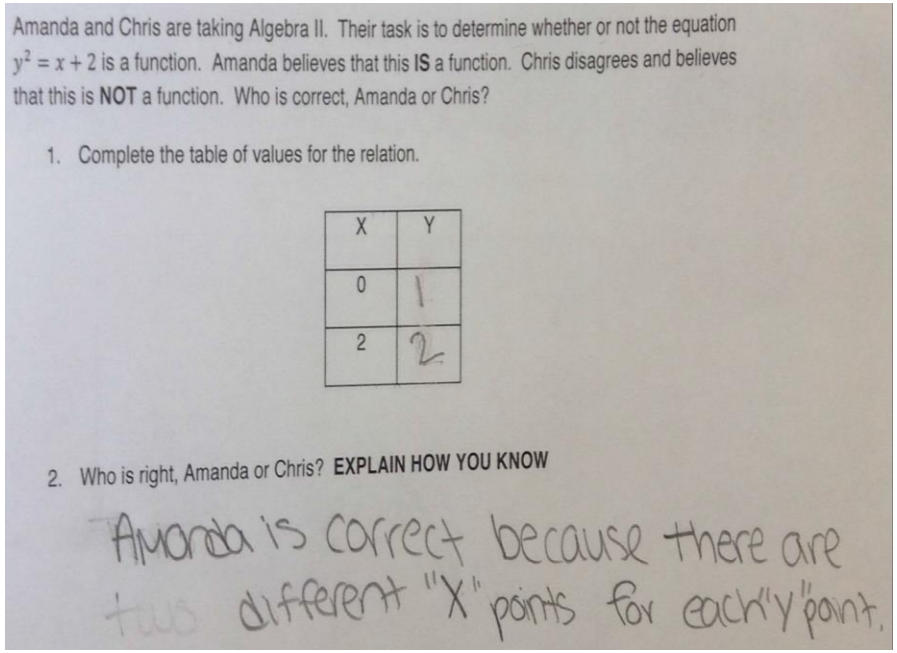
Step 2: Analyze and Interpret Student Work		
Student Response Guide		
Got It	**Developing**	**Getting Started**
		

<table>
<tr><th colspan="2">Getting Started</th></tr>
<tr><th>Student Response Example</th><th>Indicators</th></tr>
<tr><td> 
</td><td>

What possible indicators may be included in a student response who has demonstrated minimal understanding of the standard?
Students may have a basic understanding of the relationship between domain and criteria of a function (each element of the domain must map to a unique element of the range)
Students may not understand the operation square root.
Students do not understand the concept of square root therefore not resulting in two “y” values.
Students may not understand that a function from one set to another set assigns each element of the domain to exactly one element of the range.

What strategies, and representations will or will not be used? What understandings or procedural fluency does the student response reveal?
I will review with these students that in a functional relationship, each element in the domain has a unique output.
I would remediate students to basic examples of squaring integers and taking the square root.

What undeveloped understandings, misconceptions, and common mistakes may be revealed in the student response to this item?
Students are not able to make the connection between taking the square root of a function and the output
Students are not able to connect the procedure of evaluation a relationship over a domain to the concept of a function

</td></tr>
<tr><th>In the Moment Questions/Prompts</th><th>Closing the Loop (Interventions/Extensions)</th></tr>
<tr><td>What questions could you ask, or feedback could you provide in the moment to develop student understanding, create disequilibrium, or advance student thinking?
What does it mean to take the square root of a number?
What does it mean for a relation or an equation to be a function? What does this look like graphically?
Explain how you got your numbers in the table</td><td>https://learnzillion.com/lessons/2759-identify-functions-in-various-forms#quickcode-modal</td></tr>
</table>

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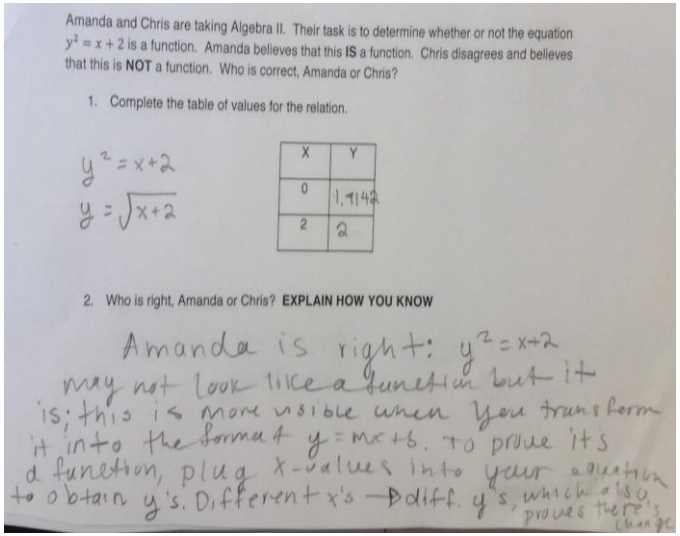
Amanda and Chris are taking Algebra II. Their task is to determine whether or not the equation $y^2 = x + 2$ is a function. Amanda believes that this IS a function. Chris disagrees and believes that this is NOT a function. Who is correct, Amanda or Chris?
1. Complete the table of values for the relation.
$y^2 = x+2$
$y = \sqrt{x+2}$
<table><tr><th>X</th><th>Y</th></tr><tr><td>0</td><td>1.4142</td></tr><tr><td>2</td><td>2</td></tr></table>
2. Who is right, Amanda or Chris? EXPLAIN HOW YOU KNOW
Amanda is right: $y^2 = x+2$ may not look like a function but it is; this is more visible when you transform it into the format $y = mx+b$. To prove it's a function, plug x-values into your equation to obtain y's. Different x's → diff. y's, which also proves there's
</td><td>

What possible indicators may be included in a student response who has demonstrated some understanding of the standard?
Student can rearrange the equation to solve for y
Students understand that If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x.
Students understand that the graph of f is the graph of the equation y = f(x).
Although students cannot evaluate for a given value, they are able to accurately determine if an equation is a function based on the notion that each element of the domain has a unique corresponding y value.
Students are able to accurately apply the concept of functionality.
What strategies, and representations will or will not be used? What understandings or procedural fluency does the student response reveal?
Students should graph the function
What undeveloped understandings, misconceptions, and common mistakes may be revealed in the student response to this item?
Students are able to accurately connect the concept of domain and functionality
Students do not understand the concept of square root function.

</td></tr>
<tr><th>In the Moment Questions/Prompts</th><th>Closing the Loop (Interventions/Extensions)</th></tr>
<tr><td>What questions could you ask, or feedback could you provide in the moment to develop student understanding, create disequilibrium, or advance student thinking?

Explain another way you could solve this problem
Explain what you mean by “different x’s” → “different Y’s</td><td>https://learnzillion.com/lessons/2759-identify-functions-in-various-forms#quickcode-modal</td></tr>
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<table>
<tr><th colspan="2">Got it</th></tr>
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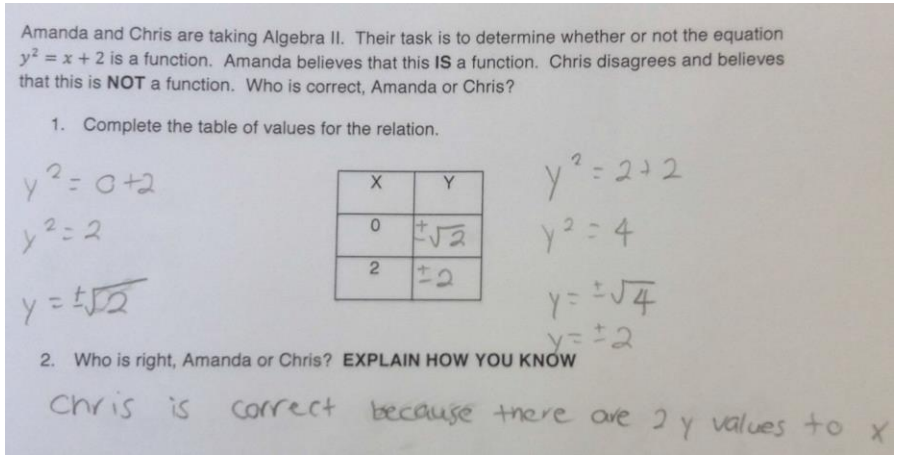
<tr><td> 
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What indicators must be included in an exemplar student response
The student was able to correctly evaluate the function for the given values including noting of the positive and negative version
Student is able to use the proper definition of domain to determine in the equation is a function
Students simplify the radical and do not get a decimal approximation
Students understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
Students is able to determine that If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x.
What strategies, and representations will or will not be used? What understandings or procedural fluency does the student response reveal?
Students are able to accurately evaluate the equation for a given value without using a calculator to obtain a decimal approximation.
Students know there are both positive and negative aspects to the output meaning the equation is not a function.
What undeveloped understandings, misconceptions, and common mistakes may be revealed in the student response to this item?
This student is on target

</td></tr>
<tr><th>In the Moment Questions/Prompts</th><th>Closing the Loop (Interventions/Extensions)</th></tr>
<tr><td>What questions could you ask, or feedback could you provide in the moment to extend or push student understanding, create disequilibrium, or advance student thinking?
Why are there two out puts or “y” values for each input?
Why can’t a function have two “y” values for a given input?
Can this equation be made a function? Explain
Write a real life situation that models a function. Explain</td><td>https://learnzillion.com/lessons/2759-identify-functions-in-various-forms#quickcode-modal</td></tr>
</table>

Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction	
Lesson Objective:	Students will be able to determine if a relation is a function given a set of values, a graph, and algebraic equation
Content Standard(s):	HSF.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).
Targeted Practice Standard :	**MP.4 Model with Mathematics:** Identify important quantities and analyze relations mathematically to draw conclusions. **MP. 6 Attend to precision:** Use clear definitions in discussion with others and in their own reasoning.

Mathematical Goals	Success Criteria
Students will be able to demonstrate understanding of functionality, by evaluating an equation and applying understanding of domain.	Students will justify and explain why it is not a function if domain is not restricted.

Launch (Probe and Build Background Knowledge)

Purpose:

Amanda and Chris are taking Algebra II. Their task is to determine whether or not the equation $y^2 = x + 2$ is a function. Amanda believes that this **IS** a function. Chris disagrees and believes that this is **NOT** a function. Who is correct, Amanda or Chris?

1. Complete the table of values for the relation.

X	Y
0	
2	

2. Who is right, Amanda or Chris? **EXPLAIN HOW YOU KNOW**

Instructional Task

Purpose: Begin by equating domain and range to something students can relate to. For example a gumball machine: the input is your quarter your output is the gumball.

1) Use the picture to answer the followings

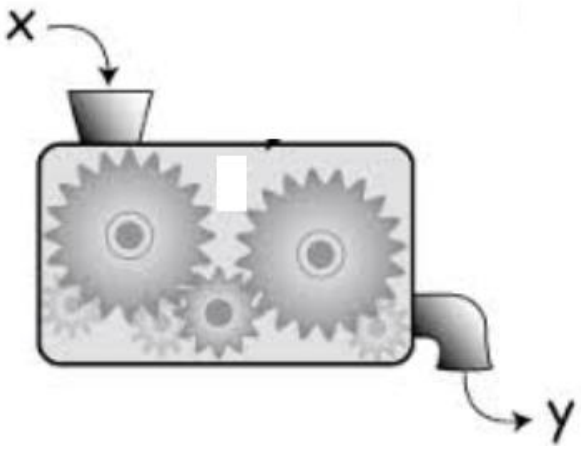
   a) Describe how does the Gumball machine work?

   b) What type of money (bills or coins) does this machine take? Explain

   c) What is the algebraic meaning of the type of money you found above?

   d) Little Johnnie is crying because he put in his only toy coin into the machine and didn't get a gumball. Explain to him what is happening.

Engage (Setting Up the Task)



Ask students to describe a real life situation for x and y.

Describe what is going on with x as it travels through the machine?

FUNCTION: a relationship between domain and range. There is exactly one element of the range for each element of the domain**.**

DOMAIN: the set of all inputs

Range: the set of all outputs

After the definitions, loop back to Gumball Machine

1) When does the Gumball machine represent a function?
2) Describe a situation when Gumball machine does not represent a function.

   Have students pair up and discuss their answers followed by whole class sharing.
3) The Gumball machine above was monitored for three instances and the results are in the tables below.

   The machine takes only quarters (One quarter at a time, and one quarter for one gumball).
   Explain in each case if the gumball machine represents a function or not.

A)

Quarters	Gumballs
John (1Q)	1
Tony (2Q)	2
Briana (3Q)	3
Amanda (4Q)	4

B)

Quarters	Gumballs
Tom (2Q)	1
Alicia (3Q)	3
Christ (2Q)	2
Amanda (1Q)	1

C)

Quarters	Gumballs
Sue(2Q)	0
Jen (3Q)	3
Christ (2Q)	2
Sean(1Q)	1

Explore (Solving the Task)

Example Set 1: Coordinate Pairs

State the domain and range then determine if the ordered pairs represent a function and how you know

1.$\{(0,2),(3,-1),(9,5),(-4,8)\}$

Domain:

Range:

Function: _____

How do you know?

2.$\{(12,1),(3,0),(2,4),(5,1)\}$

Domain:

Range:

Function: _____

How do you know?

3. Given {(x, 3), (2, -5), (r, 7), (-1, 6)}

a) Find values of x and r for the ordered pairs to represent a function

b) Find values of x and r for the ordered pairs not to represent a function

c) Do the ordered pairs represent a function if x = r? Explain your answers.

Example Set 2: Tables of Values
State the domain and range then determine if the table represents a function and how you know.

1.

X	Y
0	2
3	-1
9	5
-4	8

Domain:

Range:

Function: _____

How do you know?

2.

X	Y
12	1
3	0
2	4
5	1

Domain:

Range:

Function: _____

How do you know?

3.

X	Y
0	2
-4	-1
9	5
-4	8

Domain:

Range:

Function: _____

How do you know?

4. Complete the table below so that you have a function.

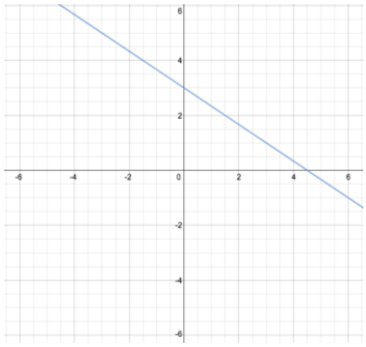
Input	-1	0	1	?	5
Output	1	3	5	9	?

Describe in your words the function. _____

5) Create a function with 5 input elements and 3 output elements. Explain your choice.

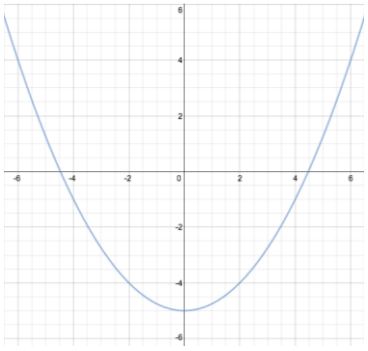
Elaborate (Discuss Task and Related Mathematical Concepts)

Example Set 3: Graphs
Determine if the graph represents a function and how you know.

1. 

Function: _____

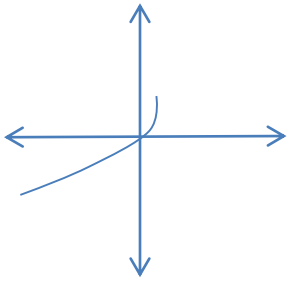
How do you know?

2. 

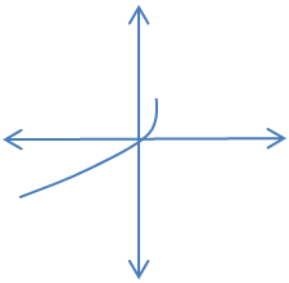
Function: _____

How do you know?

3. Complete the graph (from the right) so that you have a function



4. Complete the graph so that you don't have a function



Purpose: A brief description of what questions or prompts you will use to elicit evidence of student understanding and the strategy you will use to elicit the evidence during the lesson

Example Set 4: Equations

Complete the table of values then determine if the equation is a function and state how you know.

1. $y = \frac{-2}{3}x + 3$

X	Y
-3	
0	
3	

Function: _____

How do you know?

2. $y = \frac{1}{4}x^2 - 5$

X	Y
-2	
0	
2	

Function: _____

How do you know?

3. $y^2 - 4 = x$

X	Y
-4	
0	
2	

Function: _____

How do you know?

Example Set 5: Applications

1. The independent variable is the time you ride in a car at 55 mph (using cruise control) and the dependent variable is the distance traveled. Is this relationship a function? Explain your response.

2. The independent variable is a student's age and the dependent variable is the number of siblings the student has. Is this relationship a function? Explain your response.

3. As a scuba diver dives deeper and deeper into the ocean, the pressure of the water on his body steadily increases. The pressure at the surface of the water is 14.7 pounds per square inch (psi). The pressure increases at a rate of 0.445 psi for each foot you descend.

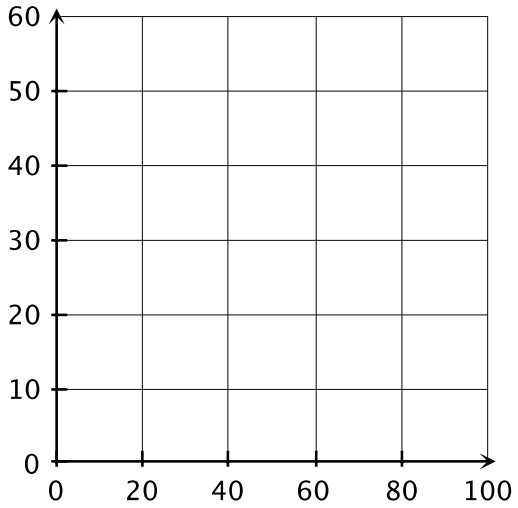
(a) Write an equation that represents the pressure *P* as a function of the depth *d*.

(b) Complete the table below. Show all of your work in the space below.

Depth (# of feet)	**Pressure (psi)**
0	
20	

40	
60	
80	
100	

(c) Make a graph of the function. Make sure you label your axes with the correct units.



(d) How far is the scuba diver if the pressure is 34.7 psi

(e) Describe in words why this is a function or not.

Common Misunderstanding

Purpose: Functions apply to a vast variety of situations that are modeled through a regular pattern

- Students sometimes interpret the parenthesis as multiplication
- Students often try to use the distributive property as if when working with binomials to distribute the value instead of using substitution
- Students have difficulty interpreting graphs- inputs and outputs
- Students often confuse vertical versus horizontal lines for the vertical line test.

Checking for Understanding

Purpose: Prove students with the opportunity to verbalize their understanding of functionality

Provide students with the following open ended question:

Explain why all functions are relations, but not all relations are functions

Closure

Purpose: Provide students with the opportunity to self-assess their own learning.

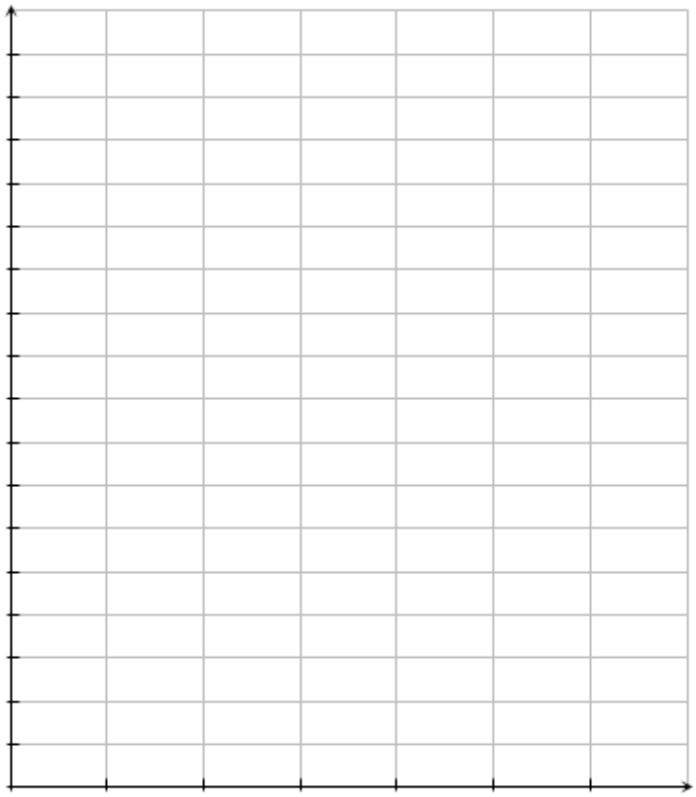
How many handshakes are possible between *n* people? In this activity, you will explore how the total number of handshakes that take place in a group of people is related to the number of people in the group. Separate into teams of six students to complete the following exercises.

1. Complete the table below by counting the total number of handshakes that take place between the numbers of people given in the table. In each case, perform the handshakes in your group.

# of People	# of Handshakes
2	
3	
4	
5	
6	

2. If x the # of people is the input variable, and y the # of handshakes is the output variable, identify the ordered pairs (x,y) from the table above.

3. Plot the ordered pairs on the graph below. Scale and label the axes appropriately.



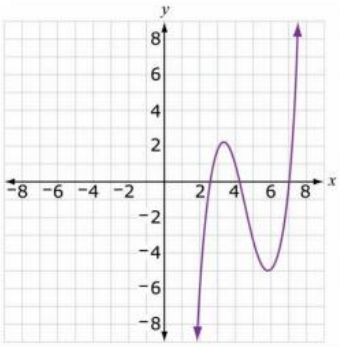
4. Is this a function? Explain.

5. Let x represent the number of people in a group, and $f(x)$ represent the number of total possible handshakes that take place between members in the group. Write a recursive rule to find $f(x)$.

Extension Task

Purpose: A brief description of how you will extend the learning for students who are ready to go deeper

After study functions and relations, John and Christy are discussing the relevance in real life. Christy cannot see the importance of functions. Write an argument or rational as to why functions are relevant to real life. Be sure to include examples!

Research and review of standard	
Content Standard(s):	**Standard(s) for Mathematical Practice:**
HSF.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).	**MP.4 Model with Mathematics:** Identify important quantities an analyze relations mathematically to draw conclusions. **MP. 6 Attend to precision:** Use clear definitions in discussion with others and in their own reasoning.
Smarter Balanced Claim	***Smarter Balanced Item***
Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.	For numbers 1a – 1d, determine whether each relation is a function. 1a. $\{(0,1),(1,2),(3,1),(4,2)\}$ ○ Yes ○ No 1b. $y = \pm\sqrt{4 - x^2}$ ○ Yes ○ No  1c. ○ Yes ○ No 1d. $\{(5,3),(2,4),(5,2)\}$ ○ Yes ○ No

CPR Pre-Requisites *(Conceptual Understanding, Procedural Skills, and Representations)* *Look at the Progressions documents, Learning Trajectories, LZ lesson library, unpacked standards documents from states, NCTM Essential Understandings Series,*	**Conceptual Understanding and Knowledge** • Understand representations of relations including ordered pairs, graph, and verbal descriptions. • Determine if a relation is a function via ordered pairs, graphical representation, and verbal descriptions. • Understand that all functions are relations, but not all relations are functions • Distinguish between a function and a relation. • Understand what domain and range of a relation are • Understand how to interpret input and output values given a graph **Procedural Skills**

NCTM articles, and other professional resources. You'll find links to great resources on your PLC Platform.	• Students should have an understanding of the coordinate plane • Students should have an understanding of coordinate points • Students should know how to evaluate an equation for a given value • Students should be able to graph an equation **Representational** • Students should be able to precisely graph • Student should be able to interpret and create graphical representations of given equations • Students should be able to interpret word problems to create graphic and algebraic representations. **Social knowledge** • $f(x)$ is read "f of x" • Function notation • Order of coordinates in order pairs • Location of the x- axis and y-axis on a coordinate plane • $\pm$ is read "plus or minus" x^n is read "x to the n" or "x to the nth power"

Standards Progression **Look at LearnZillion lessons and expert tutorials, the Progressions documents, learning trajectories, and the "Wiring Document" to help you with this section*		
Pre-Requisite Standards	**Co-Requisite Standards**	**Future Standards**
8.F.2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change* **8.F.4.** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	CCSS.MATH.CONTENT.HSF.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	**F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

Common Misconceptions/Roadblocks
What characteristics of this problem may confuse students? • Students may have difficulty interpreting that when considering both the positive and negative aspects of a square root, the relation is no longer a function

What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?

- Students sometimes interpret the parenthesis as multiplication
- Students often try to use the distributive property as if when working with binomials to distribute the value instead of using substitution
- Students have difficulty interpreting graphs- inputs and outputs
- Students often confuse vertical versus horizontal lines for the vertical line test.

What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?

Students may assume that since all functions are relations, all relations are functions, which is false.