Mathematics Instructional Cycle Guide

One Variable Equations (6.EE.7)

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CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

One Variable Equations (6.EE.7)

6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in p, q, and x are all nonnegative rational numbers.

This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

MP 2 Reason abstractly and quantitatively

- Represent a real-world problem using an algebraic equation
- Flexibly use properties of operations

WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings **(page 2)**
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint **(page 3-6)**
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed **(pages 7-11)**
- Supporting lesson materials **(pages 12-16)**
- Precursory research and review of standard **6.EE.7** and assessment items that illustrate the standard **(pages 17-19)**

HOW TO USE THIS DOCUMENT

1) Before the lesson, administer the **Writing Algebraic Equations** ***Mathematical Checkpoint*** individually to students to elicit evidence of student understanding.
2) Analyze and interpret the student work using the ***Student Response Guide***
3) Use the next steps or ***follow-up lesson plan*** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

MATERIALS REQUIRED

- **White boards**
- **Markers and erasers**
- **Student copies of activity cards and exit slip**

TIME NEEDED

Writing Algebraic Expressions administration: **10 minutes**
Follow-Up Lesson Plan: **60 minutes**

Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

<table>
<tr><th colspan="3">Step 1: Elicit evidence of student understanding</th></tr>
<tr><th colspan="3">Mathematical Checkpoint</th></tr>
<tr><th>Question(s)</th><th colspan="2">Purpose</th></tr>
<tr><td rowspan="2">The teacher needs to order more pencils. The pencils come in packs of 30. How many packs of pencils does the teacher need to buy to get a total of 240 pencils? Which equation can be used to find x, the number of boxes the teacher needs to order?

Dan says the correct equation is:
30 + n = 240

Hannah says the correct equation is:
30n = 240

John says that neither Dan nor Hannah is correct.

Circle the name of the student you agree with: Dan Hannah John

Justify your answer:</td><td>CT Core Standard:</td><td>6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in p, q, and x are all nonnegative rational numbers.</td></tr>
<tr><td>Target question addressed by this checkpoint:</td><td>How do students approach the problem with an unknown variable? To what extent do they…
Understand a variable is used to represent an unknown quantity
Understand that 3p indicates multiplication e.g. 3p = 3 x p
Recognize that an equation that represents the structure of the problem may not necessarily include the operation necessary to solve the problem</td></tr>
</table>

Step 2: Analyze and Interpret Student Work		
Student Response Guide		
Got It	**Developing**	**Getting Started**
Hannah is right because you need to multiply the number of pencils in a box (30) by the number of boxes you need to equal 240.	John is correct. I think this because if you divide the total (240) by 30 you will get the number of boxes you will need to buy. Although you can do it Hannah's way and you would have to guess and check until you get the correct answer.	I agree with John. You would need to do: 240 divided by 30 to figure out how many packs you need to buy.

Getting Started	
Student Response Example	**Indicators**
I agree with John. You would need to do: 240 divided by 30 to figure out how many packs you need to buy.	• The student defers to the operation she/he would use to solve the problem, rather than the operation that represents the situation.
In the Moment Questions/Prompts	**Closing the Loop (Interventions/Extensions)**
• Explain the question to me in your own words? • What do you already know? • Where can you start? • Can you write an equation to represent the situation? • How could you check your solution? • Hannah wrote an equation with multiplication. Is there anything in the problems that sounds like multiplication?	Teach students the bar method for writing and solving a multiplication equation **http://learnzillion.com/lessons/2814-write-and-solve-multiplication-equations-using-a-bar-model** **https://docs.google.com/a/stoningtonschools.org/file/d/0B_2_-NMZ5KYqcTNTSnVndlpZeXM/edit**

Developing	
Student Response Example	**Indicators**
John is correct. I think this because if you divide the total (240) by 30 you will get the number of boxes you will need to buy. Although you can do it Hannah's way and you would have to guess and check until you get the correct answer.	• Student recognizes the operation that could be used to solve the problem, and rejects the operation that describes the situation even though it is the inverse operation. • Student understands multiplication can be used, but does not choose Hannah's answer.
In the Moment Questions/Prompts	**Closing the Loop (Interventions/Extensions)**
• What other equations are in the same fact family as 3 x 5 = 15 • Is there another equation you could write to represent this situation? • What does the variable represent? • How can you check your answer for accuracy?	• Student needs to determine when to multiply and when to divide **http://learnzillion.com/lessons/1505-write-and-solve-an-algebraic-equation-by-determining-when-to-use-multiplication-and-division**

Got it	
Student Response Example	**Indicators**
Hannah is right because you need to multiply the number of pencils in a box (30) by the number of boxes you need to equal 240.	• Student can represent situation in an equation. • Student understands that the situation described is a multiplication situation. • Student describes the situation using an unknown value in a position other than as the answer.
In the Moment Questions/Prompts	**Closing the Loop (Interventions/Extensions)**
• Explain to me your thinking in your response. • In your own words, what does the final equation mean? • How can check if the final answer makes sense? • In this problem, explain the relationship between multiplying and dividing. • Why do you think some students would choose John?	• Introduce the term coefficient. **https://www.illustrativemathematics.org/illustrations/425** **https://www.illustrativemathematics.org/illustrations/1107**

Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction	
Lesson Objective:	Students will write equations to represent the structure of real world problems.
Content Standard(s):	6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in p, q, and x are all nonnegative rational numbers.
Targeted Practice Standard :	MP 2 Reason abstractly and quantitatively • Represent a real-world problem using an algebraic equation • Flexibly use properties of operations MP 3 Construct viable arguments and critique the reasoning of others • Listen to the reasoning of others in matching equations to real-world problems • Decide if the arguments of others make sense and ask probing questions to clarify the arguments MP 4 Model with mathematics • Apply prior knowledge to solve real-world problems • Check to see if an answer makes sense within the context of a real-world situation

Mathematical Goals	Success Criteria
• Understand the operation that represents the problem in an algebraic equation. • Understand the difference between solving the problem using computation and representing the structure of the problem in an algebraic equation.	• Write an algebraic equation that represents the real world problem.

Launch (Probe and Build Background Knowledge)

Purpose: Assess and activate knowledge about writing simple algebraic equations.

Students will be paired with a partner for this warm-up activity. Each partner group will have a small white board, marker and eraser. The teacher will show two examples of real-world problems.

Student will be instructed to:

- Discuss the two problems with their partner
- Compare and contrast the two problems for similarities and differences
- Record their similarities and differences on the graphic organizer

Problem 1:

Jennifer collected 24 insects for her science project. She arranged them in 3 shadow boxes with the same number of insects in each box.

Problem 2:

Jennifer collected some insects for her science project. She arranged them in three shadow boxes with 8 insects in each box.

Possible Student Responses:

Similar	Different
• Total number of insects • Same number of boxes • Same number of insects in each box	• In problem #1 we know the number of insects • In problem #2 we do not know the number of insects • In problem #1 we do not know how many insects are in each box • In problem #2 we know how many insects are in each box

Teacher Notes:

- In problem #1 students will want to use computation and would divide to find out how many bugs are in each box. We're looking to match the structure of the problem statement, which is 24 = 3x.
- In problem #2 students will want to multiply to find out how many total insects. y = 3 * 8
- However, both problems can be represented using multiplication in an algebraic equation:
  Problem #1 24 = 3x
  Problem #2 y = 3(8)

Instructional Task

Purpose: Students will be matching algebraic equations to real-world problems.

Engage (Setting Up the Task)

1) Students will be working with a partner to complete the activity

2) Students will be given two sets of cards. The cards are formatted so that the original document acts as the answer key. The teacher should cut the cards BEFORE giving them to students.
   - Real-world story problems
   - Algebraic equations

3) Students will be instructed to:
   - Place all of the cards face up on the table
   - Use the Think-Pair format for partner work
   - Read each real-world problem individually and match it to the correct algebraic equation
   - Discuss the reasoning for the selected equation

Explore (Solving the Task)

Provide students time to work and discuss the matching of the cards. Circulate to observe, question, and prompt students. Take note to listen to student's response to their partner's representation in their own words.

4) Discussion prompts/questions:
   - Discuss with your partner what the real-world story is representing in your own words.
   - How will you solve the problem?
   - How do the operations and the terms in the equation match the story?
   - What does the variable represent in the story?

Elaborate (Discuss Task and Related Mathematical Concepts)

As a whole class, summarize the lesson.

- Show each real-world story on the projector. Ask for student volunteers to share their interpretation of the story in their own words.
- The students will then explain how the algebraic equation they matched to the story correctly represents the story.

Discussion prompts/questions:

- Which equations did find the easiest to match to the story? The hardest to match to the story?
- Explain your reasoning.

Checking for Understanding

Purpose:

- How is the way you would solve the problem using a computational strategy different from the way you would represent the problem in an algebraic equation?

Possible teacher response: *Sometimes it's different, and sometimes it's the same. It depends on where the missing part is. Back up in Jen's problems we would need to use division to solve the first one, and multiplication to solve the second one, even though they both have the same number of bugs and boxes*

- Why do we take the time to represent a problem using an algebraic equation rather than just solve the problem without an equation and just using our computational skills?

Possible teacher response*: For some problems, it's not clear what we might do to calculate the answer. It might help to represent the stuff that's going on in the problem in an equation, then we can decide.*

Common Misunderstanding

Purpose: Address a common misunderstanding students often have when solving the equation. Students often want to solve the equation or "figure out the solution" using computation rather than solving using an algebraic approach.

Using the algebraic equations from your cards, create a t-chart to compare/contrast solving the equation by using computation and solving the equation using an algebraic approach. Students are not expected to solve the equation at this point in the lesson. The students may solve the problem using a computational approach.

Equation: _____

Algebraic Approach	Computation Approach

Rather than use an algebraic approach to solve the problem, students may use the following non-algebraic routes:

- Random: Guessing answers in no particular order and hoping to guess the correct answer sooner or later by chance.
- Sequential: Trialing numbers in sequence, as in a spreadsheet, hoping that the correct answer will be reached.
- Guess-check-improve: Carrying out one or more trials, using the result of each trial to choose the next one.

Discussion questions and prompts:

- What does the variable in the problem represent?
- How do the computations and algebraic equations relate to one another?
- If you want to use subtraction to find your answer, which operation would you use for the algebraic equation?
- Is there more than one correct equation?

Checking for Understanding

Purpose: Pose the following the problem to elicit evidence of student's understanding of writing an algebraic equation to represent a real-world problem.

Jamie ran the same number of miles each month for 6 months. She ran a total of 234 miles.

Write an algebraic equation to represent the situation. Solve the equation using an algebraic approach.

Solution: 6 x m = 234

Closure

Purpose: Provide students with an opportunity to self-assess their own learning related to the success criteria.

Think about your learning and your current level of understanding. Circle the number that best matches your level of success with each item.

1) I can write an algebraic equation to represent a real-world problem.

Not at all		I still have questions/confusion		I got it!
1	2	3	4	5

2) I can solve an algebraic equation using an algebraic approach rather than just computation.

Not at all		I still have questions/confusion		I got it!
1	2	3	4	5

Extension Task

Purpose: In this task, students will be asked to deepen their understanding of representing real-world problems with an algebraic equation.

Students will be given a set of equations. They will be instructed to write a real-world problem to match the equation.

Write Your Own Story:

Algebraic Equations
3n = 15
n + 4 = 20
17 – n = 11
21 = 3n

There are 30 pencils in a box. The teacher needs 240 pencils. Which equation can be used to find x, the number of boxes the teacher needs to order?

Dan says the correct equation is:

$30 + x = 240$

Hannah says the correct equation is:

$30x = 240$

John says that neither Dan nor Hannah are correct.

Circle the name of the student you agree with: Dan Hannah John

Justify your answer:

Name _____Date _____

Warm-Up

- Discuss the two problems with your partner
- Compare and contrast the two problems for similarities and differences
- Record your similarities and differences on the graphic organizer
- During the discussion, if a similarity or difference is noted that is NOT on your graphic organizer, add it to yours

Problem 1:

Jennifer collected 24 insects for her science project. She arranged them in 3 shadow boxes with the same number of insects in each box.

Problem 2:

Jennifer collected some insects for her science project. She arranged them in three shadow boxes with 8 insects in each box.

Similarities	Differences

Algebraic Equations

Cut out each set of cards. Match the real-world problem with the correct algebraic equation.

Together, Joe and Bob have 48 baseball cards. Joe has three times as many baseball cards as Bob. How many baseball cards does Joe have?	$3n + n = 48$
Lauren gets paid $8.00 per hour to babysit. She made $48. How many hours did she babysit?	$8n = 48$
I think of a number, double it, and add 12. My answer is 48. What number am I thinking of?	$2n + 12 = 48$
Gumballs cost 4 cents more than a licorice. John pays 48 cents for two gumballs and four licorices. What is the price of a licorice?	$2(n + 4) = 48$
Karen spends $48 on three hats and a pair of gloves. The gloves cost $12. How much does each hat cost?	$3n + 12 = 48$
Pam walks her dog twice a day. Her evening walk takes twice as long as her morning walk. If she walks her dog for 48 minutes a day, how long is her morning walk?	$2n + n = 48$

Algebraic Equations

EXIT Slip

Jamie ran a total of 234 miles in six months. How many miles did she average each month?

- Write an algebraic equation to represent the situation.
- Solve the equation using an algebraic approach.

Think about your learning and your current level of understanding. Circle the number that best matches your level of success with each item.

3) I can write an algebraic equation to represent a real-world problem.

Not at all		I still have questions/confusion		I got it!
1	2	3	4	5

2) I can solve an algebraic equation using an algebraic approach rather than just using computation.

Not at all		I still have questions/confusion		I got it!
1	2	3	4	5

Name_____Date_____

Write Your Own Story

Write a real-world story for each of the algebraic equations. Be sure the equation correctly represents the story.

3n = 15

N + 4 = 20

17 - n = 11

21 = 3n

<table>
<tr><th colspan="2">Research and review of standard</th></tr>
<tr><th>Content Standard(s):</th><th>Standard(s) for Mathematical Practice:</th></tr>
<tr><td>6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in p, q, and x are all nonnegative rational numbers.)</td><td>What Standard(s) for Mathematical Practice are implicit in this item or content standard?

MP2 Attend to Precision
Connect abstract symbols to their numerical referents.

MP7 Look for and Make Use of Structure

Looking for structure in expressions by parsing them into a sequence of operations; making use of the structure to interpret the expression's meaning in terms of the quantities represented by the variables.</td></tr>
<tr><th>Smarter Balanced Claim</th><th>Smarter Balanced Item</th></tr>
<tr><td>Claim 1: Concepts and Procedures

Students can explain and apply mathematical concepts and
Carry- out mathematical procedures with precision and fluency.</td><td>Read each of the following problem situations. Label each situation according to the equation that would answer the question. If neither equation works, select "Neither." The labels may be used more than one time.

The school auditorium can seat 325 students. In the auditorium there are 25 rows with the same number of seats in each row. Which equation can be used to find x, the number of seats in each row in the school auditorium?

There are 25 soccer balls in a store. The total number of soccer balls and basketballs in the store is 325. Which equation can be used to find x, the number of basketballs in the store?

25 + x = 325

Marissa had 25 marbles in a bag. She gave some to her brother. Her brother now has 325 marbles. Which equation can be used to find x, the number of marbles that Marissa gave her brother?

25x = 325

Neither

There are 25 cans of soup in a case. The manager of a grocery store needs to order 325 cans of soup. Which equation can be used to find x, the total number of cases the manager needs to order?

Cleo has a certain number of seashells. Pete has 25 seashells. Together Cleo and Pete have 325 seashells. Which equation can be used to find x, the total number of seashells that Cleo has?</td></tr>
<tr><td>CPR Pre-Requisites
(Conceptual Understanding, Procedural Skills, and Representations)</td><td>Conceptual Understanding and Knowledge
Understand a variable is used to represent an unknown quantity
Understand that two or more terms written together indicate multiplication.
Understand expressions written as 2 x (8+7) express the calculation "add 8 and 7, then multiply by 2."</td></tr>
</table>

	Procedural Skills • Add whole numbers • Subtract whole numbers • Multiply whole numbers • Divide whole numbers • Identify the variable **Representational** • Write equations to represent word problems using a variable to represent the unknown quantity **Social knowledge** • Understand that 3p indicates multiplication e.g. 3p = 3 x p

Standards Progression		
Grade(s) below	**Target grade**	**Grade(s) above**
What previous grade level standards build up to the grade level standard this item assesses? 5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.	*What other grade level standards are connected to the standard this item assesses?* 6.EE.1 Write and evaluate numerical expressions involving whole-number exponents. 6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers. 6.EE.2a Write expressions that record operations with numbers and with letters standing for numbers. 6.EE.4 Identify when two expressions are equivalent (i.e. when the two expressions name the same number regardless of which value is substituted into them).	*What subsequent grade level standards build off of the grade level standard this item assesses?* 7.EE.3 Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. 7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

	6.EE.6 use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	

Common Misconceptions/Roadblocks
What characteristics of this problem may confuse students? • Using the correct algebraic equation to represent the real-world problem. • When writing algebraic equations, knowing what the variable represents. **What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?** • A common misconception involving variables is that students interpret them as labels. • Students will not correctly represent the problem using an algebraic approach, they will use computation. **What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?** • Student may not see the need for writing an equation with a variable, they may just compute.