



# Mathematics Instructional Cycle Guide

Concept 8.F.4

Created by Michelle Combs, 2014  
Connecticut Dream Team teacher

## CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

**Standard: 8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

MP1: Make sense of problems and persevere in solving them.

Students will explain the correspondences in the equation, table and graph forms of a function.

MP2: Reason abstractly and quantitatively

Students will make sense quantities and their relationships in given situations.

MP3: Construct viable arguments and critique the reasoning of others.

Students will find the error in problems that have been solved by others.

## WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings (**page 2**)
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint (**pages 3-6**)
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed (**pages 7-11**)
- Supporting lesson materials (**page 12-17**)
- Precursory research and review of standard **8.F.4** and assessment items that illustrate the standard (**pages 18-20**)

## HOW TO USE THIS DOCUMENT

- 1) Before the lesson, administer the 8.F.4 [Mathematical Checkpoint](#) individually to students to elicit evidence of student understanding.
- 2) Analyze and interpret the student work using the [Student Response Guide](#)
- 3) Use the next steps or **follow-up lesson plan** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
- 4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

## MATERIALS REQUIRED

- Document Camera
- Chart Paper and Markers
- Post-it notes

## TIME NEEDED

**8.F.4** administration: **20 minutes**

Follow-Up Lesson Plan: **3 days**

***Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.***

Step 1: Elicit evidence of student understanding

Mathematical Checkpoint

Question(s)

Purpose

**Bill solved the following problem:**

A band will be paid a flat fee for playing a concert. Additionally, the band will receive a fixed amount for every ticket sold. If 40 tickets are sold, the band will be paid \$200. If 70 tickets are sold, the band will be paid \$260.

- Determine the rate of change.

**Answer: Bill used the two points (40, 200) and (70, 260) and**

**determined**  $\frac{260 - 200}{70 - 40} = \frac{60}{30} = 2.$

- How much will the band receive for each ticket sold?

**Answer: The band will receive \$2 for each ticket sold.**

- Let x represent the number of tickets sold and y represent the amount the band will be paid. Construct a linear function to represent the relationship between the number of tickets sold and the amount the band will be paid.

**Answer:  $y = 2x$**

**Find the error: Write a response to Bill explaining what parts he completed correctly and where he made the error in his solution.**

**CT Core Standard:**

**Use functions to model relationships between quantities.**

**Standard: 8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

**Target question addressed by this checkpoint:**

*How do students approach a problem in which there are two unknown quantities? To what extent do they...*

- interpret the constant as an initial value and input this as the y-intercept in the slope-intercept form of a linear equation?*
- apply previous learning of expressions and equations to help them arrive at a solution?*

**Step 2: Analyze and Interpret Student Work**  
**Student Response Guide**

**Got It**

Demonstrated conceptual understanding and mastery.

8.F.4 – Mathematical Checkpoint



Bill solved the following problem:

A band will be paid a flat fee for playing a concert. Additionally, the band will receive a fixed amount for every ticket sold. If 40 tickets are sold, the band will be paid \$200. If 70 tickets are sold, the band will be paid \$260.

- Determine the rate of change.

**Answer:** Bill used the two points (40, 200) and (70, 260) and determined

$$\frac{260 - 200}{70 - 40} = \frac{60}{30} = 2. \quad \frac{260 - 200}{70 - 40} = \frac{60}{30} = 2$$

- How much will the band receive for each ticket sold?

**Answer:** The band will receive \$2 for each ticket sold.

- Let  $x$  represent the number of tickets sold and  $y$  represent the amount the band will be paid. Construct a linear function to represent the relationship between the number of tickets sold and the amount the band will be paid.

**Answer:**  $y = 2x$

$$y = 2x + 130$$

**Find the error:** Write a response to Bill explaining what parts he completed correctly and where he made the error in his solution.

In the problem Bill started out right. He found the rate of change for how much the band will be paid. His mistake was when he wrote out his equation  $y = 2x$ . This equation is wrong because he forgot to add the flat fee that the band is paid. So the correct equation would be  $y = 2x + 130$ . The flat fee is 130 because when you use the equation  $y = 2x$  you have to do  $2 \cdot 40$  (tickets) which will give you 80. Then you have to subtract 80 from 200 to get the 120 flat fee.

**Developing**

Demonstrated some understanding and possibly some misunderstandings or undeveloped understanding.

8.F.4 – Mathematical Checkpoint



Bill solved the following problem:

A band will be paid a flat fee for playing a concert. Additionally, the band will receive a fixed amount for every ticket sold. If 40 tickets are sold, the band will be paid \$200. If 70 tickets are sold, the band will be paid \$260.

- Determine the rate of change.

**Answer:** Bill used the two points (40, 200) and (70, 260) and determined

$$\frac{260 - 200}{70 - 40} = \frac{60}{30} = 2.$$

- How much will the band receive for each ticket sold?

**Answer:** The band will receive \$2 for each ticket sold.

- Let  $x$  represent the number of tickets sold and  $y$  represent the amount the band will be paid. Construct a linear function to represent the relationship between the number of tickets sold and the amount the band will be paid.

**Answer:**  $y = 2x + 40 + 80$

**Find the error:** Write a response to Bill explaining what parts he completed correctly and where he made the error in his solution.

---

---

---

---

---

---

---

---

---

---

---

---

**Getting Started**

Demonstrated minimal understanding and possibly misconceptions.

8.F.4 – Mathematical Checkpoint



Bill solved the following problem:

A band will be paid a flat fee for playing a concert. Additionally, the band will receive a fixed amount for every ticket sold. If 40 tickets are sold, the band will be paid \$200. If 70 tickets are sold, the band will be paid \$260.

- Determine the rate of change.

**Answer:** Bill used the two points (40, 200) and (70, 260) and determined

$$\frac{260 - 200}{70 - 40} = \frac{60}{30} = 2.$$

- How much will the band receive for each ticket sold?

**Answer:** The band will receive \$2 for each ticket sold.

- Let  $x$  represent the number of tickets sold and  $y$  represent the amount the band will be paid. Construct a linear function to represent the relationship between the number of tickets sold and the amount the band will be paid.

**Answer:**  $y = 2x - 5x$

**Find the error:** Write a response to Bill explaining what parts he completed correctly and where he made the error in his solution.

Bill got up the problem correctly but instead of the 130 being paid each they could be \$50 each which you multiply 40 by 2 you get 80. Find the problem says \$200 instead of \$180.  $200 - 80 = 120$ .

---

---

---

---

---

---

---

---

---

---

---

---

Getting Started

Student Response Example

Indicators

8.F.4 – Mathematical Checkpoint



Bill solved the following problem:

A band will be paid a flat fee for playing a concert. Additionally, the band will receive a fixed amount for every ticket sold. If 40 tickets are sold, the band will be paid \$200. If 70 tickets are sold, the band will be paid \$260.

1. Determine the rate of change.

Answer: Bill used the two points (40, 200) and (70, 260) and determined  $\frac{260 - 200}{70 - 40} = \frac{60}{30} = 2$ .

2. How much will the band receive for each ticket sold?

Answer: The band will receive \$2 for each ticket sold.

3. Let  $x$  represent the number of tickets sold and  $y$  represent the amount the band will be paid. Construct a linear function to represent the relationship between the number of tickets sold and the amount the band will be paid.

Answer:  $y = 2x + 5x$

Find the error: Write a response to Bill explaining what parts he completed correctly and where he made the error in his solution.

Bill got up the problem correctly but instead of the tickets being \$2 each they could be \$5 each. When you multiply 40 by 5 you get 200 and the problem says \$200 instead of \$20.  
 $2 \times 40 + 5 = 200$

- Response does not take into account the flat fee as an amount being added to the tickets sold.
- The student connects that the first ordered pair is not a solution to the linear function that Bill created and therefore changed the equation, but they did not verify that their new equation was also true for the other ordered pair.
- The student shows that the equation is where the error is, but does not understand the process for correcting this error because they only change the equation in part three without thinking how it affects the process of getting there in the first two parts of the problem.

In the Moment Questions/Prompts

Closing the Loop (Interventions/Extensions)

Tell me why you changed the equation from  $2x$  to  $5x$ .

I see from your explanation that this works for the first ordered pair, does it work for the second ordered pair as well?

Is there any other information from the problem that might be important?

Is there another method you could use to solve this problem? (table/graph)

Graph the two points on a coordinate grid and interpret the line of the graph.


<https://learnzillion.com/lessons/1836-create-equation-table-and-graph-from-a-situation>

Developing

Student Response Example

Indicators

8.F.4 – Mathematical Checkpoint



Bill solved the following problem:

A band will be paid a flat fee for playing a concert. Additionally, the band will receive a fixed amount for every ticket sold. If 40 tickets are sold, the band will be paid \$200. If 70 tickets are sold, the band will be paid \$260.

1. Determine the rate of change.

Answer: Bill used the two points (40, 200) and (70, 260) and determined

$$\frac{260 - 200}{70 - 40} = \frac{60}{30} = 2.$$

2. How much will the band receive for each ticket sold?

Answer: The band will receive \$2 for each ticket sold.

3. Let x represent the number of tickets sold and y represent the amount the band will be paid. Construct a linear function to represent the relationship between the number of tickets sold and the amount the band will be paid.

Answer:  $y = 2x + 40 + 50$

Find the error: Write a response to Bill explaining what parts he completed correctly and where he made the error in his solution.

---

---

---

---

---

---

- Response has notes made showing some understanding of process needed to find the error in this problem, but there is no explanation of notes made. (Flat fee is circled, equation is noted in form  $y = mx + b$ , notes are made in part three showing understanding that is where the error occurs, but correction made is not right and there is no explanation of any work shown in the space below so it is unclear of student's exact thinking)
- The student started to make a table and then erased it. It seems that they knew that this was a strategy that could have helped them reach a conclusion, but they did not know how to use it to help them reach a solution.
- The student has written a new equation, which is where the error occurs in this problem, but the new equation is not accurate.

In the Moment Questions/Prompts

Closing the Loop (Interventions/Extensions)

Explain to me the notes you made on your paper. (Further specific prompts regarding why the flat fee was circled, and what the equation in part three represents if needed)

You started to make a table and then erased it-why was that not helpful to you?

What is another method we have used to find the equation of a line when we are given the points and the slope? Do you think we could apply that method to this problem?

Graph the two points on a coordinate grid and interpret the line of the graph.

<https://learnzillion.com/lessons/1835-determining-the-yintercept>

<https://learnzillion.com/lessons/1836-create-equation-table-and-graph-from-a-situation>



Student Response Example

Indicators



Bill solved the following problem:

A band will be paid a flat fee for playing a concert. Additionally, the band will receive a fixed amount for every ticket sold. If 40 tickets are sold, the band will be paid \$200. If 70 tickets are sold, the band will be paid \$260.

1. Determine the rate of change.

Answer: Bill used the two points (40, 200) and (70, 260) and determined

$$\frac{260-200}{70-40} = \frac{60}{30} = 2. \quad \frac{260-200}{70-40} = \frac{60}{30} = 2$$

2. How much will the band receive for each ticket sold?

Answer: The band will receive \$2 for each ticket sold.

3. Let  $x$  represent the number of tickets sold and  $y$  represent the amount the band will be paid. Construct a linear function to represent the relationship between the number of tickets sold and the amount the band will be paid.

Answer:  $y = 2x$

$$y = 2x + 130$$

Find the error: Write a response to Bill explaining what parts he completed correctly and where he made the error in his solution.

In the problem Bill started out right. He found the rate of change for how much the band will be paid. His mistake was when he wrote out his equation  $y = 2x$ . This equation is wrong because he forgot to add the flat fee that the band is paid. So the correct equation would be  $y = 2x + 130$ . The flat fee is 130 because when you use the equation  $y = 2x$  you have to do  $2 \cdot 40$  (tickets) which will give you 80. Then you have to subtract 30 from 200 to get the 130 flat fee.

- The student interpreted the rate of change as the slope of a linear function and worked to find the flat fee of \$120 by inputting a given point into the equation.
- The student was able to explain that the rate of change was determined correctly and that is also the amount of money that the band will get from each ticket.
- The student explains that the flat fee of \$120 is a flat fee (additional amount) that the band will receive, no matter how many tickets are sold.

In the Moment Questions/Prompts

Closing the Loop (Interventions/Extensions)

Tell me where Bill made the error.

What does 120 mean in the equation you wrote?

How could you check to make sure your equation is correct?

Be able to represent this situation as a table, graph, and equation.

<https://learnzillion.com/lessons/1837-interpreting-slope-and-y-intercept-in-context>

**Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction**

<b>Lesson Objective:</b>	Determine the rate of change and initial value of a function.
<b>Content Standard(s):</b>	<p align="center"><b>Use functions to model relationships between quantities.</b></p> <p><b>Standard: 8.F.4</b> Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</p>
<b>Targeted Practice Standard :</b>	<p>MP1: Make sense of problems and persevere in solving them.</p> <p>MP2: Reason abstractly and quantitatively</p> <p>MP3: Construct viable arguments and critique the reasoning of others.</p>

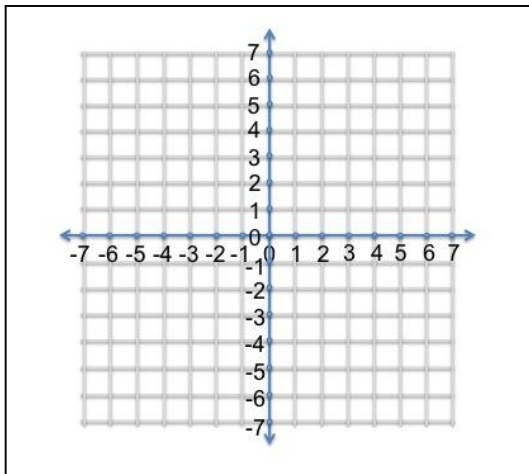
Mathematical Goals	Success Criteria
<p>In terms of a situation:</p> <p>Understand that the rate of change in a function is the slope of a graphed line.</p> <p>Understand that the initial value of a function is the y-intercept of a graphed line.</p> <p>Understand that the y-intercept of a line can be identified from a table as the value of y when x is equal to zero.</p>	<p>Represent a given situation as a table, graph or an equation written in slope-intercept form.</p> <p>Interpret the slope of a line as the rate of change in a given situation.</p> <p>Interpret the y-intercept of a line as a constant that represents an initial value in a given situation.</p>

**Launch (Probe and Build Background Knowledge)**

**Purpose:** Assess and Activate prior knowledge of Functions (Focus of 8.F.3)

Have students represent the linear equation  $y = 2x + 1$ . Have half of the room complete the representation as a table of data and have the other half of the room complete the representation of the graphed line. Upon completion pair the tables up with the graphs and have them compare the similarities and differences in their representations. Have them share how each can identify the slope and initial value from the other's representation.

x	y





## Instructional Task

**Purpose:** Introduce the new shoes task and allow students time to reason and problem solve.

### Engage (Setting Up the Task)

- 1.) Introduce the task by projecting a scenario: **You desperately want to buy the latest sneakers that are going to be released soon at Shoe Locker. Your parents refuse to spend that much money on any pair of sneakers. You decide you're going to put away any money you receive from your upcoming birthday and mow as many lawns as you need to in order to have enough money to purchase the sneakers.**
- 2.) Facilitate discussion about the scenario using the following prompts/questions:

Talk with a partner about how this scenario might relate to a linear function.

On the post it notes provided brainstorm questions you have about this scenario, and note pieces of information that you need in order to solve this problem. Consider the following in your brainstorm:

- What part of this scenario will represent an initial value?
- What part of this scenario will represent a rate of change?
- What numerical information do you need provided to you to help you know how long it will take you to be able to purchase the sneakers?

When students are done brainstorming organize their post it notes into categories.

- 3.) Provide the same scenario with numerical data inputted:

**You desperately want to buy the latest sneakers that are going to be released soon at Shoe Locker. Your parents refuse to spend that much money on any pair of sneakers. You decide you're going to put away \$100 that you received from your birthday and mow as many lawns as you need to in order to have enough money to purchase the sneakers. You plan to charge \$8 per lawn to save for the sneakers, which will have a total cost of \$240.**

**Represent your answer with an equation, graph, and table of values to model the scenario.**

**Extension-Your best friend wants the same sneakers, but his birthday was last month and he only has \$30 of birthday money left. He is going to mow lawns in a different neighborhood and charge \$10 per lawn. How many lawns will he have to mow in order to be able to buy the sneakers?**

**Represent your answer with an equation, graph, and table of values to model the scenario.**

- 4.) Encourage students to discuss this scenario with a partner before diving in to finding the solution. Remind them to use the information from the group discussion about the scenario to guide them when solving the problem.
- 5.) Explain to students that they will now work on the task individually for a specified amount of time. (20 minutes)
- 6.) After the allotted individual work time students will compare their answers and discuss any differences they may have. This can be done with a partner, or as a gallery walk in table groups. (2 minutes talk time per person)

### Explore (Solving the Task)

7.) During this time students will create a real world scenario that can be expressed as an equation, graph and table of values. Provide students time to collaborate with their groups on their own unique scenario and allow for them to show their collaboration by compiling their data on chart paper. While they are working, circulate the room to observe and question student groups. Possible questions/prompts as students are engaged in the task:

#### Focusing Questions

- What does each part of an equation represent?
- What are the unknown variables in your scenario?
- What part of your scenario will represent an initial value?
- What part of your scenario will represent a rate of change?

#### Probing Questions

- How can you use the different representations of your scenarios to verify that each representation is correct?
- Which representation is easiest to start with?
- How can you verify that the equation is correctly represented?
- In what way does your scenario represent a function?

#### Advancing Questions

- How would your scenario be different if there was no initial value?
- How can you change your scenario so that it resulted in a line parallel to the one you graphed?
- How could you change your scenario so that the line of your graph is steeper?

### Elaborate (Discuss Task and Related Mathematical Concepts)

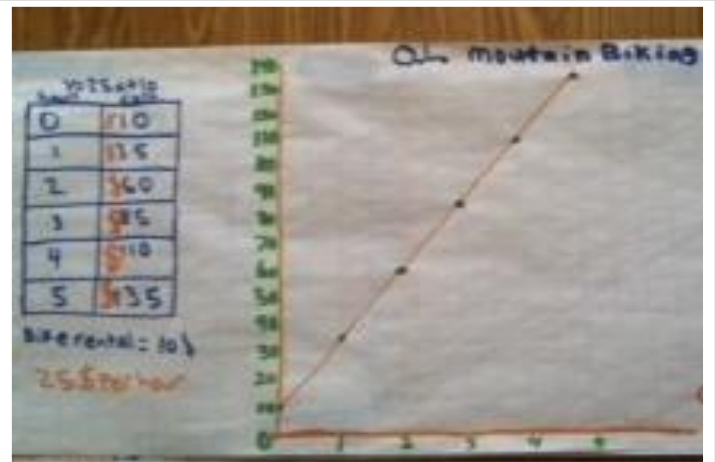
8.) Call the class back together and allow for groups to present their charts.

Samples of student work:

Scuba World

and

Outdoor Mountain Biking



### Common Misunderstanding

**Purpose:** Address a common misunderstanding students often have about interpreting a function's initial value.

Students will often forget that the starting amount is the amount at zero weeks and have a hard time showing that as the y-intercept of the graph.

- **Remind students that they often need to look for key words that represent a starting value in problems like these: fixed amount, initial fee, flat fee, amount saved.**
- **Remind students that not all graphs pass through the graph's origin.**

Display pictures of graphed data without the scenarios and have students identify the function's initial value.

### Checking for Understanding

**Purpose:** Pose the following question as an exit ticket to elicit evidence of students' understanding of the initial value of a function:

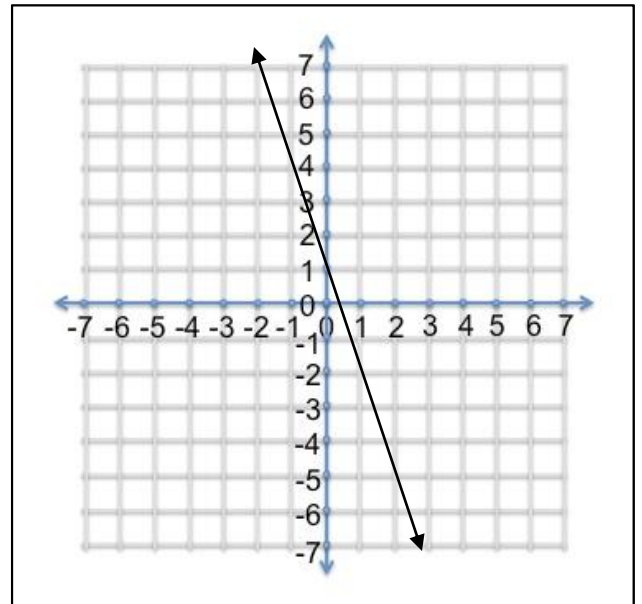
Rachel uses the equation  $y = 3x + 2$  to create the table and graph below.

Make any necessary corrections to Rachel's work.

Explain why you made the corrections you did and make conjectures about why Rachel made the mistakes she did.

$$y = 3x + 2$$

x	y
0	0
1	5
2	8
-1	1
-2	4



## Closure

**Purpose:** Provide students an opportunity to self-assess their own learning related to the success criteria by providing students with the following self-assessment:

Name: \_\_\_\_\_

Date: \_\_\_\_\_

	Novice	Apprentice	Practitioner	Expert
Self-Assessment on Functions Standard 8.F.4	I am just starting to learn this concept and I don't really understand it yet.	I am starting to understand this concept, but I still need someone to help me through it.	I can mostly do problems on this concept by myself, but I sometimes get stuck.	I understand this concept well, and I feel I could thoroughly teach it to someone else.
I can construct a function to model a relationship between two quantities.				
I can determine the rate of change of a function.				
I can determine the initial value of a function.				
I can represent a linear relationship as a table, graph and equation.				

After this lesson, I feel I could really use more time learning \_\_\_\_\_

## Extension Task

**Purpose:** Provide an extension task for those students who are ready to deepen their understanding of functions.

**Extension-Your best friend wants the same sneakers, but his birthday was last month and he only has \$30 of birthday money left. He is going to mow lawns in a different neighborhood and charge \$10 per lawn. How many lawns will he have to mow in order to be able to buy the sneakers?**

**Represent your answer with an equation, graph, and table of values to model the scenario.**

Who will have to mow less lawns in order to earn enough money for the sneakers? Explain.

**8.F.4 – Mathematical Checkpoint**



**Bill solved the following problem:**

A band will be paid a flat fee for playing a concert. Additionally, the band will receive a fixed amount for every ticket sold. If 40 tickets are sold, the band will be paid \$200. If 70 tickets are sold, the band will be paid \$260.

1. Determine the rate of change.

**Answer: Bill used the two points (40, 200) and (70, 260) and determined**

$$\frac{260 - 200}{70 - 40} = \frac{60}{30} = 2$$

2. How much will the band receive for each ticket sold?

**Answer: The band will receive \$2 for each ticket sold.**

3. Let  $x$  represent the number of tickets sold and  $y$  represent the amount the band will be paid. Construct a linear function to represent the relationship between the number of tickets sold and the amount the band will be paid.

**Answer:  $y = 2x$**

**Find the error: Write a response to Bill explaining what parts he completed correctly and where he made the error in his solution.**

---

---

---

---

---

---

## New Shoes Task-part one



**You desperately want to buy the latest sneakers that are going to be released soon at Shoe Locker. Your parents refuse to spend that much money on any pair of sneakers. You decide you're going to put away any money you receive from your upcoming birthday and mow as many lawns as you need to in order to have enough money to purchase the sneakers.**

Talk with a partner about how this scenario might relate to a linear function.

On the post it notes provided brainstorm questions you have about this scenario, and note pieces of information that you need in order to solve this problem. Consider the following in your brainstorm:

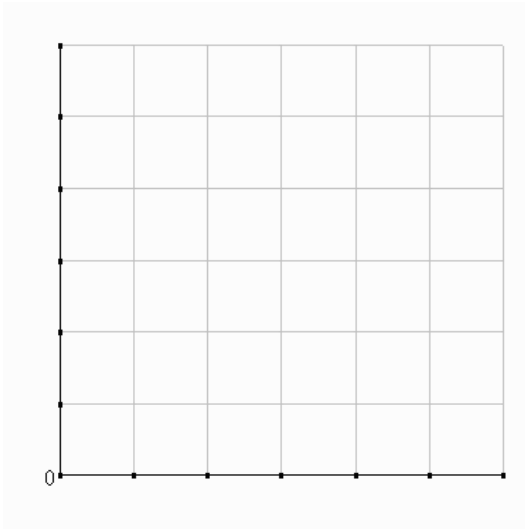
- What part of this scenario will represent an initial value?
- What part of this scenario will represent a rate of change?
- What numerical information do you need provided to you to help you know how long it will take you to be able to purchase the sneakers?

## New Shoes Task-part two



You desperately want to buy the latest sneakers that are going to be released soon at Shoe Locker. Your parents refuse to spend that much money on any pair of sneakers. You decide you're going to put away \$100 that you received from your birthday and mow as many lawns as you need to in order to have enough money to purchase the sneakers. You plan to charge \$8 per lawn to save for the sneakers, which will have a total cost of \$240.

Represent your answer with an equation, graph, and table of values to model the scenario.



Table


Equation: \_\_\_\_\_

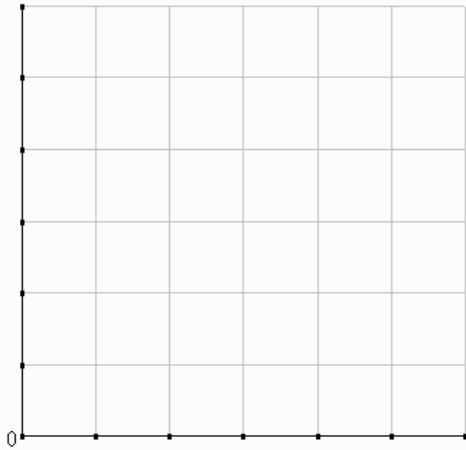


## New Shoes Task-part two

### Extension

Extension-Your best friend wants the same sneakers, but his birthday was last month and he only has \$30 of birthday money left. He is going to mow lawns in a different neighborhood and charge \$10 per lawn. How many lawns will he have to mow in order to be able to buy the sneakers?

Represent your answer with an equation, graph, and table of values to model the scenario.



Table


Equation: \_\_\_\_\_

Who will have to mow less lawns in order to earn enough money for the sneakers? Explain.

---

---

---

---

---

## Exit Ticket

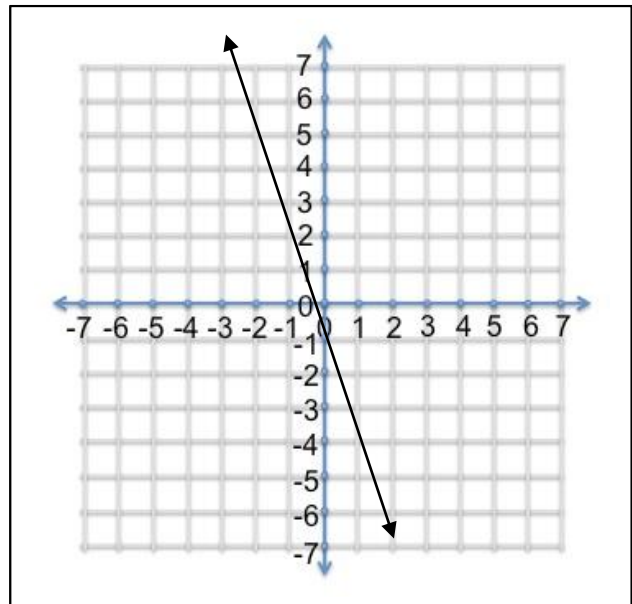
Rachel uses the equation  $y = 3x + 2$  to create the table and graph below.

Make any necessary corrections to Rachel's work.

Explain why you made the corrections you did and  
make conjectures about why Rachel made the mistakes she did.

$$y = 3x + 2$$

x	y
0	0
1	5
2	8
-1	1
-2	4



## Self-Assessment

Name: \_\_\_\_\_

Date: \_\_\_\_\_

	<b>Novice</b>	<b>Apprentice</b>	<b>Practitioner</b>	<b>Expert</b>
Self-Assessment on Functions Standard 8.F.4	I am just starting to learn this concept and I don't really understand it yet.	I am starting to understand this concept, but I still need someone to help me through it.	I can mostly do problems on this concept by myself, but I sometimes get stuck.	I understand this concept well, and I feel I could thoroughly teach it to someone else.
I can construct a function to model a relationship between two quantities.				
I can determine the rate of change of a function.				
I can determine the initial value of a function.				
I can represent a linear relationship as a table, graph and equation.				

After this lesson, I feel I could really use more time learning \_\_\_\_\_

**Research and review of standard**

Research and review of standard	
Content Standard(s):	Standard(s) for Mathematical Practice:
<p><b>Use functions to model relationships between quantities.</b></p> <p><b>Standard: 8.F.4</b> Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</p>	<p>MP2 Reason abstractly and quantitatively.</p> <ul style="list-style-type: none"> <li>Make sense of quantities and relationships in problem situations.</li> </ul> <p>MP4 Model with mathematics.</p> <ul style="list-style-type: none"> <li>Identify important quantities and map their relationship using such tools as diagrams, two-way tables, graphs, flowcharts, and/or formulas.</li> </ul>
Smarter Balanced Claim	Smarter Balanced Item
<p><b>Claim 2: Problem Solving</b>  <i>Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.</i></p>	<p>The total cost of an order of shirts from a company consists of the cost of each shirt plus a one-time design fee. The cost of each shirt is the same regardless of how many shirts are ordered. The company provides the following examples to customers to help them estimate the total cost of an order of shirts:</p> <ul style="list-style-type: none"> <li>50 shirts cost \$349.50</li> <li>500 shirts cost \$2370</li> </ul> <p>Part A  <i>Based on the examples, what is the cost of each shirt, not including the one-time design fee?</i>                      Explain how you found your answer.</p> <p>Part B                      What is the cost of the one-time design fee?                      Explain how you found your answer.</p>
<p><b>CPR Pre-Requisites</b>  <i>(Conceptual Understanding, Procedural Skills, and Representations)</i></p>	<p><b>Conceptual Understanding and Knowledge</b></p> <ul style="list-style-type: none"> <li>Understand that an equation can be written and solved in order to answer a real world problem.</li> <li>Understand that a variable can be used to represent one or more unknowns in a given scenario.</li> </ul> <p><b>Procedural Skills</b></p> <ul style="list-style-type: none"> <li>Apply inverse operations to solve equations.</li> </ul> <p><b>Representational</b></p> <ul style="list-style-type: none"> <li>Write an equation to represent real world scenarios.</li> <li>Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph and a table of values.</li> </ul> <p><b>Social knowledge</b></p> <ul style="list-style-type: none"> <li>Know that an equation can have a constant and a variable</li> <li>Know that only like terms can be combined</li> </ul>

**Standards Progression**

*\*Look at LearnZillion lessons and expert tutorials, the Progressions documents, learning trajectories, and the "Wiring Document" to help you with this section*

Grade(s) below	Target grade	Grade(s) above
<p>6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true.</p> <p>6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form <math>x + p = q</math> and <math>px = q</math> for cases in which <math>p</math>, <math>q</math> and <math>x</math> are all nonnegative rational numbers.</p> <p>6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.</p> <p>7.EE.4.a Solve word problems leading to equations of the form <math>px + q = r</math> and <math>p(x + q) = r</math>, where <math>p</math>, <math>q</math>, and <math>r</math> are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.</p>	<p>8.EE.7.a Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form <math>x = a</math>, <math>a = a</math>, or <math>a = b</math> results (where <math>a</math> and <math>b</math> are different numbers).</p> <p>8.EE.7.b Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.</p> <p>8.EE.8.b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.</p> <p>8.EE.8.c Solve real-world and mathematical problems leading to two linear equations in two variables.</p> <p>8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.</p> <p>8.F.3 Interpret the equation <math>y = mx + b</math> as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.</p>	<p>HSF.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.</p> <p>HSF.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</p>

## Common Misconceptions/Roadblocks

### What characteristics of this problem may confuse students?

- Students might not interpret the “one-time” fee as a constant, separate from the cost of each shirt.
- Students might not set up an equation correctly, which would result in an incorrect answer.

### What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?

- Students may just jump in and attempt to divide the numbers given instead of working out a strategy for attacking the problem
- Students who have created an equation might not have a correct equation.
- Students might have correct equations but do not know how to combine them in order to solve.
- Students often do not go back and check to be sure that their answer is reasonable.

### What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?

- Students may think there is only one unknown in a given scenario because they focus on some attributes of a situation and ignore others.
- Students may try to substitute numbers in order to solve, which is not the best strategy and may cause frustration leading to not being able to quickly solve the problem.