

Mathematics Instructional Cycle Guide

Analyze Proportional Relationships and
Use Them to Solve Real-World Problems
(7.RP.A.2)

Created by Robin Greenwald, 2014
Connecticut Dream Team teacher

CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

Analyze proportional relationships and use them to solve real-world and mathematical problems.

CCSS.MATH.CONTENT.7.RP.A.2

Recognize and represent proportional relationships between quantities.

CCSS.MATH.CONTENT.7.RP.A.2.A

Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

CCSS.MATH.PRACTICE.MP3 Construct viable arguments and critique the reasoning of others.

CCSS.MATH.PRACTICE.MP4 Model with mathematics.

WHAT IS INCLUDED IN THIS DOCUMENT?

- A **Mathematical Checkpoint** to elicit evidence of student understanding and identify student understandings and misunderstandings (**page 2**)
- A **Student Response Guide** with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint (**pages 3-6**)
- A **Follow-up Lesson Plan** designed to use the evidence from the student work and address the student understandings and misunderstandings revealed (**pages 7-12**)
- Supporting **Lesson Materials** (**pages 13-20**)
- Precursory **Research and Review of Standard 7.RP.A.2** and assessment items that illustrate the standard (**pages 21-23**)

HOW TO USE THIS DOCUMENT

- 1) Before the lesson, administer the [Mathematical Checkpoint](#) individually to students to elicit evidence of student understanding.
- 2) Analyze and interpret the student work using the [Student Response Guide](#).
- 3) Use the next steps or **follow-up lesson plan** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint.
- 4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan.

MATERIALS REQUIRED

- Interactive white board
- Colored pencils(optional)

TIME NEEDED

Mathematical Checkpoint Proportional Reasoning administration: **15 minutes**

Follow-Up Lesson Plan: **1 to 2 instructional blocks**

Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

Step 1: Elicit evidence of student understanding

Mathematical Checkpoint

Question(s)

Purpose



Checkpoint Proportional Reasoning

The Truly Scrumptious candy company produces an 8-ounce mix of gumballs that contains 8 blue, 6 green, 4 yellow, and 2 red gumballs. The colors are proportional to this mixture for different-sized orders.

Mimi says, "I want to buy 24 ounces. That would give me 24 blue, 22 green, 20 yellow, and 18 red."

Decide if Mimi's statement is true or false, and explain or show your reasoning, using words, pictures, or numbers.

True or False?



CT Core Standard:

Analyze proportional relationships and use them to solve real-world and mathematical problems.

[CCSS.MATH.CONTENT.7.RP.A.2](#)

Recognize and represent proportional relationships between quantities.


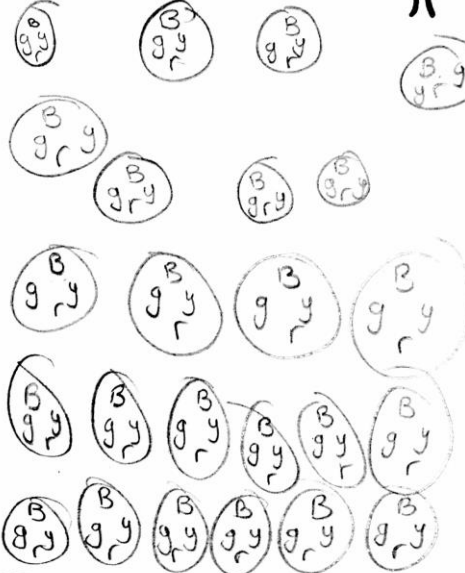

[CCSS.MATH.CONTENT.7.RP.A.2.A](#)

Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

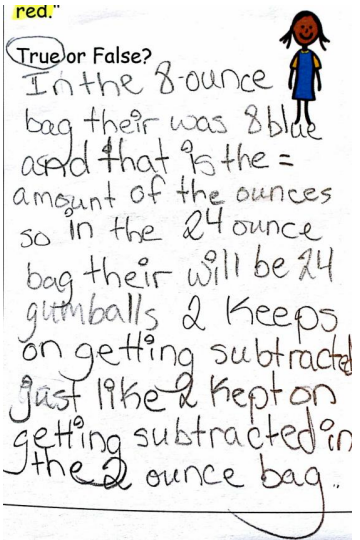
Target question addressed by this checkpoint:

Can students reason proportionally to extend the size of a sample? To what extent can they maintain proportionality in a mix when the weight of the sample is increased?

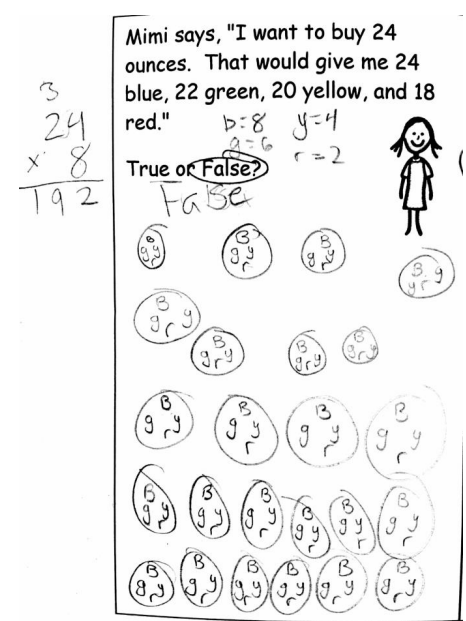
Step 2: Analyze and Interpret Student Work
Student Response Guide

Got It	Developing	Getting Started																		
<p>True or False? ¹</p> <table border="0"> <tr> <td>Regular:</td> <td></td> <td>Mimi's order</td> </tr> <tr> <td>8oz</td> <td>$\times 3 \rightarrow$</td> <td>24oz</td> </tr> <tr> <td>Blue: 8</td> <td>$\times 3 \rightarrow$</td> <td>24 ✓</td> </tr> <tr> <td>Green: 6</td> <td>$\times 3 \rightarrow$</td> <td>18 ✗</td> </tr> <tr> <td>Yellow: 4</td> <td>$\times 3 \rightarrow$</td> <td>12 ✗</td> </tr> <tr> <td>Red: 2</td> <td>$\times 3 \rightarrow$</td> <td>6 ✗</td> </tr> </table>	Regular:		Mimi's order	8oz	$\times 3 \rightarrow$	24oz	Blue: 8	$\times 3 \rightarrow$	24 ✓	Green: 6	$\times 3 \rightarrow$	18 ✗	Yellow: 4	$\times 3 \rightarrow$	12 ✗	Red: 2	$\times 3 \rightarrow$	6 ✗	<p>Mimi says, "I want to buy 24 ounces. That would give me 24 blue, 22 green, 20 yellow, and 18 red."</p> <p>$b=8$ $y=4$ $g=6$ $r=2$</p> <p>True or False? False</p>  <p>3 24 $\times 8$ 192</p> 	<p>red."</p> <p>True or False?</p> <p>In the 8-ounce bag there was 8 blue and that is the = amount of the ounces so in the 24 ounce bag there will be 24 gumballs & keeps on getting subtracted just like I kept on getting subtracted in the 2 ounce bag.</p> 
Regular:		Mimi's order																		
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Getting Started

Student Response Example	Indicators
 <p>red. True or False? In the 8-ounce bag there was 8 blue and that is the = amount of the ounces so in the 24 ounce bag there will be 24 gumballs & keeps on getting subtracted just like I kept on getting subtracted in the 2 ounce bag.</p>	<ul style="list-style-type: none"> • Student has not used a table, equivalent ratio, double number line, tape model, equation, or proportion to represent the relationship between proportional quantities. • Student has attempted to use an additive relationship between the colors without recognizing that each color forms its own individual ratio with the number of ounces. • Student has not come to a reasonable conclusion.
In the Moment Questions/Prompts	Closing the Loop (Interventions/Extensions)
<p>P: You can create a table of equivalent ratios for each colored gumball to represent a proportional relationship.</p> <p>P: Draw the 8 ounce mix of gumballs. How many of those would there be in 24 ounces?</p> <p>Q: If the mix was 4 ounces with 3 green gumballs, how many green gumballs would there be in 8 ounces? What operation did you use to find the answer?</p> <p>Q: What number sentence could you use to show how the 8 ounce mix relates to the 24 ounce mix? What operation would you use to convert the 8-ounce mix to the 24-ounce mix?</p> <p>Q: How could a double number line represent the number of greens in every 8 ounce mix?</p> <p>Q: Can you write a proportion to show how the ratios of greens to all gumballs relate for these two mix sizes?</p>	<p>The following videos highlight how additive reasoning relates to multiplicative reasoning.</p> <p>http://learnzillion.com/lessons/608-solve-missing-values-in-ratio-problems-using-a-table</p> <p>http://learnzillion.com/lessons/609-solve-missing-values-in-ratio-problems-using-a-double-number-line</p> <p>http://learnzillion.com/lessons/610-solve-missing-values-in-ratio-problems-using-multiplicative-reasoning</p>

Developing

Student Response Example	Indicators
 <p>Mimi says, "I want to buy 24 ounces. That would give me 24 blue, 22 green, 20 yellow, and 18 red."</p> <p>True or False? <u>False</u></p> <p>Key: $b=8$, $g=6$, $y=4$, $r=2$</p> <p>192 gumballs drawn in a grid.</p>	<ul style="list-style-type: none"> • Student has represented each mix of gumballs using a picture, with a key to how many gumballs of each color the letters represent. • Student has mistakenly assumed the original mix is for 1 ounce instead of 8 ounces, and therefore, drew 24 of those mixes to find the numbers for 24 ounces. • The calculation of $24 \times 8 = 192$ suggests the student mistakenly concluded there would be 192 blue gumballs. • Student correctly concludes the statement is false, but for the wrong reason. • Student does not reach a reasonable solution for the number of blue gumballs.
In the Moment Questions/Prompts	Closing the Loop (Interventions/Extensions)
<p>P: Tell me about your drawing and how it represents the problem.</p> <p>Q: How many ounces are in the original mix? How does that number of ounces relate to the new size of 24 ounces?</p> <p>Q: Can you write a number sentence or equation showing how the original number of ounces relates to the new size of 24 ounces?</p>	<p>http://learnzillion.com/lessons/3061-analyze-a-situation-using-a-rate-table</p> <p>http://learnzillion.com/lessons/869-determine-if-two-rates-are-equivalent-by-dividing</p> <p>http://learnzillion.com/lessons/875-write-an-equation-that-expresses-the-relationship-between-two-proportional-quantities</p>

Got it

Student Response Example	Indicators
<p>True or False? ¹</p> <p>Regular: 8oz $\xrightarrow{\times 3}$ 24oz</p> <p>Mimi's order</p> <p>Blue: 8 $\xrightarrow{\times 3}$ 24 ✓</p> <p>Green: 6 $\xrightarrow{\times 3}$ 18 ✗</p> <p>Yellow: 4 $\xrightarrow{\times 3}$ 12 ✗</p> <p>Red: 2 $\xrightarrow{\times 3}$ 6 ✗</p>	<ul style="list-style-type: none"> • Student identifies the multiplicative relationship between the number of ounces in the two mixes. • Student extends the function to the other quantities in the problem. • Student calculates each quantity correctly. • Student draws a reasonable conclusion.
In the Moment Questions/Prompts	Closing the Loop (Interventions/Extensions)
<p>P/Q: You concluded that Mimi's statement was false. What was her mistake? Why doesn't Mimi's method work?</p> <p>P/Q: Suppose someone asked for 30 ounces of gumballs. What issues would this present?</p>	<p>http://learnzillion.com/lessons/875-write-an-equation-that-expresses-the-relationship-between-two-proportional-quantities</p> <p>http://learnzillion.com/lessons/873-identify-the-constant-of-proportionality-in-ratio-tables</p>

Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction

Lesson Objective:	The students will be able to model a proportional relationship and reason proportionally about what happens to the quantities when one quantity changes.
Content Standard(s):	<p>Analyze proportional relationships and use them to solve real-world and mathematical problems.</p> <p><u>CCSS.MATH.CONTENT.7.RP.A.2</u> Recognize and represent proportional relationships between quantities.</p> <p><u>CCSS.MATH.CONTENT.7.RP.A.2.A</u> Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.</p>
Targeted Practice Standard :	<p><u>CCSS.MATH.PRACTICE.MP3</u> Construct viable arguments and critique the reasoning of others.</p> <p><u>CCSS.MATH.PRACTICE.MP4</u> Model with mathematics.</p>

Mathematical Goals	Success Criteria
<p>The students will understand that, when the size of a representative sample changes, the quantities that make up the sample will change proportionally.</p> <p>The students will understand that equivalent ratios have a multiplicative relationship.</p>	<p>The students will be able to model a proportional relationship using a table or a tape diagram.</p> <p>The students will be able to reason proportionally about the quantities in samples of different sizes.</p>

Launch (Probe and Build Background Knowledge)
<p>Purpose: <i>Introduce a proportional reasoning problem and get students thinking about how to approach such a problem.</i></p> <p>Begin with a warm up in which students would solve a similar, but simpler task which they are likely to understand intuitively.</p> <p>If my snack pack of 10 gummies has 6 grape gummies and 4 lemon gummies, how many gummies would there be all together in 2 snack packs? How many of them could I expect to be grape? What number sentences could I use to answer these questions? What operations does this problem require? How could we organize the information so that we could predict how many gummies would be in 3 packs, 4 packs, 5 packs, and so on?</p>
Instructional Task

Purpose: *Use an intuitive proportional reasoning problem to brainstorm problem-solving approaches, and introduce a series of tasks that will be solved using tables and tape diagrams as models.*

Engage (Setting Up the Task)

1. Introduce the task by projecting the following warm-up problem on the interactive whiteboard.

If my snack pack of 10 gummies has 6 grape gummies and 4 lemon gummies, how many gummies would there be all together in 2 snack packs? How many of them could I expect to be grape? What number sentences could I use to answer these questions? What operations does this problem require? How could we organize the information so that we could predict how many gummies would be in 3 packs, 4 packs, 5 packs, and so on?

2. Facilitate discussion about the problem, using the following prompts and questions:

-*Tell a partner what is going on in the problem.*

-*What kind of model could we use to help understand the problem?*

-*How could we organize the information in the problem?*

Students might brainstorm drawing pictures with colored pencils to model the problem. They may suggest organizing the information using a horizontal table that includes both flavors, a series of input/output tables for each flavor, or a tape diagram. They may also suggest counting by the number of gummies for each flavor, or writing equations or number sentences.

3. Explain that students will be solving problems such as this one using two different techniques. They will use a table to solve one problem, and a tape diagram to solve another. They will be able to choose either model to solve a third problem. Worksheets for the three problems are attached as *Task A*, *Task B*, and *Task C*.

Explore (Solving the Task)

Students will work in pairs to complete *Task A: Bead Kit Task* and *Task B: Piñata Task*. Assign half the pairs to work on Task A while the other half works on Task B. Once the students have solved their first task, they can become experts in that task, and can partner up with a pair that has solved the opposite task. In this way, pairs can serve as resources for each other as they proceed through the second task, and engage in discourse about their approaches to both tasks.

For the Bead Kit Task, students will model a bracelet kit using actual beads or using colored pencils to draw a kit of beads, and will use their model to fill in a table. Using their table, they will work through the task and answer the questions. The final question requires students to extend their understanding beyond their table, prompting the use of number sentences, equivalent ratios, equations, proportions, or input/output tables.

For the Piñata Task, students will model the problem using a tape diagram, and will use their model to work through the problems.

Differentiation: Because the Bead Kit Task includes extending understanding beyond the model, it might be assigned first to a pair that is high ability and independent. Since the solution to the Piñata task can be found directly in the model, it could be assigned first to a pair that may struggle. Partner pairs can then assist each other in completing the second task. By working together as a group, all students will prepare to solve *Task C: Piñata Task Extension*.

To solve the Piñata Task Extension, students may pair with a different student from their group of 4, resulting in pairs with an “expert” on each of the other two tasks. Students will discuss and choose one of the two models, a table or a tape diagram, to model and solve Task C. This task requires the students to draw their own models, increasing their independence in using the models. In addition, this task adds the element of weight to the ratio problem, as well as the conversion of units.

Focusing Questions	Probing Questions	Advancing Questions
<p>Task A</p> <ul style="list-style-type: none"> • How many of each kind of beads was in the original kit? • How did you extend the number of beads for multiple kits? • How can you use the table to find the information asked for in the problem? <p>Task B</p> <ul style="list-style-type: none"> • How can you decide how many rectangles to shade for each kind of candy in the tape diagram? • How can you decide where the 24 belongs in the tape diagram? • How can you use the tape diagram to answer the questions in the problem? <p>Task C</p> <ul style="list-style-type: none"> • How can you figure out how many ounces are in 3 pounds? • Which model will you choose, the table or the tape diagram? Why will you choose this model? 	<p>Task A</p> <ul style="list-style-type: none"> • What number sentences or operations did you use to fill in the table? • What is the problem with using your table to answer the question in which the total number of beads is 220? • What can you do if the numbers in the problem go beyond your table? <p>Task B</p> <ul style="list-style-type: none"> • What operation and number sentence did you use to decide how much each rectangle in your tape diagram was worth? • What operations and number sentences did you use to answer the questions in the problem? • How could you go about solving this problem without making a diagram? <p>Task C</p> <ul style="list-style-type: none"> • Why do you need to figure out how many ounces are in 3 pounds before solving the problem? • How does your model represent the problem? • What number sentences could you use to solve the problem? 	<ul style="list-style-type: none"> • How did modeling the first two tasks prepare you to solve the extension task C? • What methods might you use to solve Task C besides the models? • How would you explain these methods to another student? • Which method do you prefer for solving this kind of problem? Why? • How could these tasks be extended to include percents?

Elaborate (Discuss Task and Related Mathematical Concepts)

After solving their first task, pairs of students will partner with a pair that solved the opposite task. Each pair will serve as experts in the task they already solved and will offer insight and guidance to the pair that is beginning the task. Groups can discuss how their approaches to the tasks are similar or different. Before beginning the final task, groups can report on their approaches to the Bead Kit Task and Piñata Task, how they used the models, and how they might solve the problems in other ways. They may be asked which model they prefer and why.

After working with a different partner from their group to solve the Piñata Task Extension, pairs will report to the class on the model they used and how they set it up. They may share their approaches using a document camera, on Giant Post-It paper, on easel paper, or on the interactive whiteboard. They will also share the number sentences they used, and reflect on how similar problems might be solved using number sentences.

Common Misunderstanding

Purpose: *Address a common misunderstanding students have about proportional reasoning.*

Annaly bought a 6-ounce bag of gummy worms that has 5 yellow, 4 green, 3 red, and 2 orange gummy worms. She has a theory that if she gets a 12-ounce bag, she will have 11 yellow, 10 greens, 9 reds, and 8 oranges. Explain the error Annaly made.

Clarifying questions might include:

- How would you model this problem?
- How would Annaly's theory compare with your model?
- How does the number of red gummy worms in the 6-ounce bag relate to the number of ounces in the bag?
- How does the size of the 12-ounce bag compare to the original 6-ounce bag?
- What operation would be required to figure out how many red gummies should be in the 12-ounce bag? What operation did Annaly use?
- What is the mistake Annaly made, and how can we avoid making the same mistake?

Checking for Understanding

Purpose: *Pose the following question as an exit ticket to elicit evidence of students' understanding of proportional relationships. A copy of the exit ticket is attached.*

Exit Ticket

Maddie and Becca are putting together 4-ounce treat bags for a party. Each bag contains 4 pieces of bubble gum, 3 Jolly Ranchers, 2 Tootsie Rolls, and a granola bar. If they have 48 ounces of treats all together, how many granola bars are there?

Maddie says, "If we have 48 ounces of treats, there are 12 granola bars."

Becca says, "Since we have 48 ounces of treats, we have 45 granola bars."

Circle the name of the girl you agree with. Maddie Becca

Draw a model below and explain how it supports your choice.

Closure

Purpose: *Students will self-assess their learning by circling where they fall on the learning scale (attached).*

Learning Scale

Circle where you fall on the learning scale.

- 0 Even with help, I can't do any of this. I wouldn't know where to start.
- 1 With help, I can fill in a model like a table or tape diagram similar to the ones we used today. When my teacher asks me, I might be able find the information on the model to answer her.
- 2 I can make a table or tape diagram to represent the information in a problem. I can fill in most of the information. I'm not quite sure how to find the information I need to answer the question, though.
- 3 I can use a table, tape model, equivalent ratios, number sentences, or proportions to solve a problem like the one we did.
- 4 I can go beyond what we did today and solve proportional reasoning problems with all kinds of missing information. I could extend this to solving problems involving percents and quantities with different units.

Extension Task

Purpose: Learning could be extended by offering students the following tasks to extend understanding to include reasoning with percents and quantities with different units



Extension Proportional Reasoning

The Truly Scrumptious candy company produces an 8-ounce mix of gumballs that contains 8 blue, 6 green, 4 yellow, and 2 red gumballs. The colors are proportional to this mixture for different-sized orders.

Decide if each statement is true or false, and explain or show your reasoning, using words, pictures, or numbers.

Alex says, "My bag has 16 yellow gumballs. Based on the number of yellows, I can tell there are 80 gumballs in the whole bag."

True or False?



Hector says, "6% of the gumballs are green."

True or False?



Alternate extension problem:

The school cafeteria has collected data on the snacks students choose. They have concluded that, for every 20 students who purchase snacks, 10 of them will choose chips, 6 of them will choose ice cream, and 4 of them will choose fruit. The cafeteria wants to use these results to predict how many of each snack they will sell, based on the number of students. On Tuesday, they are expecting a group of 60 students. How many of each snack can they expect to sell? What percentage of the snacks the cafeteria sells will be ice cream?

Task A: Bead Kit Task

Using a Table

Problem: Mikayla is putting together kits for making beaded bracelets. Each bracelet kit will have 5 orange beads, 3 black beads, and 2 yellow beads. If Mikayla knows she has 18 black beads, how many yellow and orange beads she will need to complete the kits? How many kits will she be able to make?

Use this space to model a bead kit by making a picture or using actual beads. What would happen if you added a second bead kit to your picture? Use your modeling work to fill in the table below.

Number of kits								
Orange								
Yellow								
Black								
Total beads								

Explain the process you used for filling in the table.

Use your table to help you solve the problem.

If Mikayla knows she has 18 black beads, how many yellow and orange beads will she need to complete the kits? Answer in complete sentences, and explain how you got your answer. **Mikayla will need**

How many kits will Mikayla be able to make? Answer in complete sentences, and explain how you got your answer. **Mikayla can make**

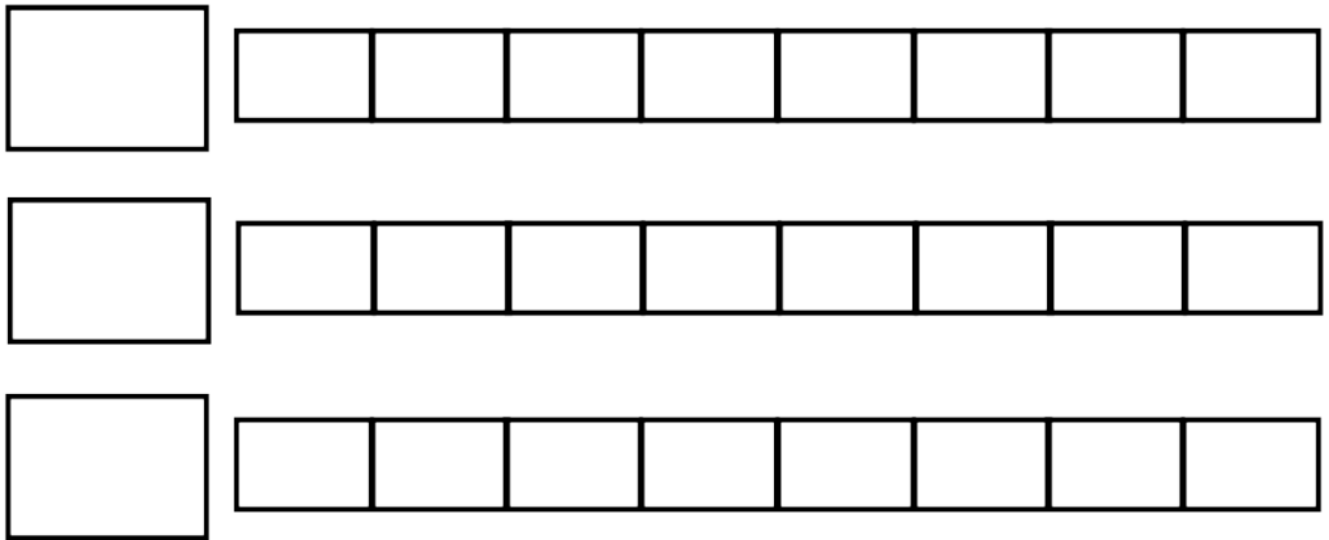
If Mikayla has a total of 220 beads, how many would she have of each color, and how many kits could she make? Explain or show how you found your answers.

Task B: Piñata Task
Using a Tape Diagram

Problem: Carlos is helping to fill a piñata for his sister's birthday party. For every 5 Tootsie Rolls Carlos uses, he will put in 8 Starbursts and 3 candy necklaces. If Carlos puts in 24 candy necklaces, how many Tootsie Rolls and Starbursts will he put in? How many pieces of candy will there be all together?

Use the tape diagram below to model the ratios in the problem by filling in a label for each kind of candy and shading the appropriate ratio.

The problem tells us that Carlos puts in _____ candy necklaces. Where does this



information fit on your tape diagram?

Using all the information you have, explain what number each rectangle represents. Complete your diagram by labeling each shaded rectangle with this number. What operation did you use? Write the number sentence that helped you figure this out, and explain how you found the answer.

Use your model to answer the questions.

How many Tootsie Rolls and Starbursts will Carlos put in if he puts in 24 candy necklaces? Answer in complete sentences, and explain how you got your answer. Show any number sentences or equations you used.

Carlos will

How many pieces of candy will there be all together? Answer in complete sentences, and explain how you got your answer, showing the number sentences or equations you used.

There will be

Task C: Piñata Task Extension

Choosing and Drawing Your Own Model

Carlos wants to make sure the piñata won't be too heavy. He weighs a sample of 5 Tootsie Rolls, 8 Starbursts, and 3 candy necklaces and finds out they weigh 6 ounces. He decides he wants to put in 3 pounds of candy. How many of each kind of candy can Carlos put in? How many pieces of candy will there be all together?

To solve this problem, we must first determine how many ounces are in 3 pounds.

(1 pound = 16 ounces). Show your number sentence and your answer here.

Number sentence:

3 pounds = _____ ounces

Now model the problem in the space using either a tape diagram or a table.

How many of each kind of candy can Carlos put in the piñata if he wants 3 pounds of candy? Answer in complete sentences, and explain how you got your answer.

How many pieces of candy will there be all together? Answer in complete sentences, and explain how you got your answer.

Exit Ticket

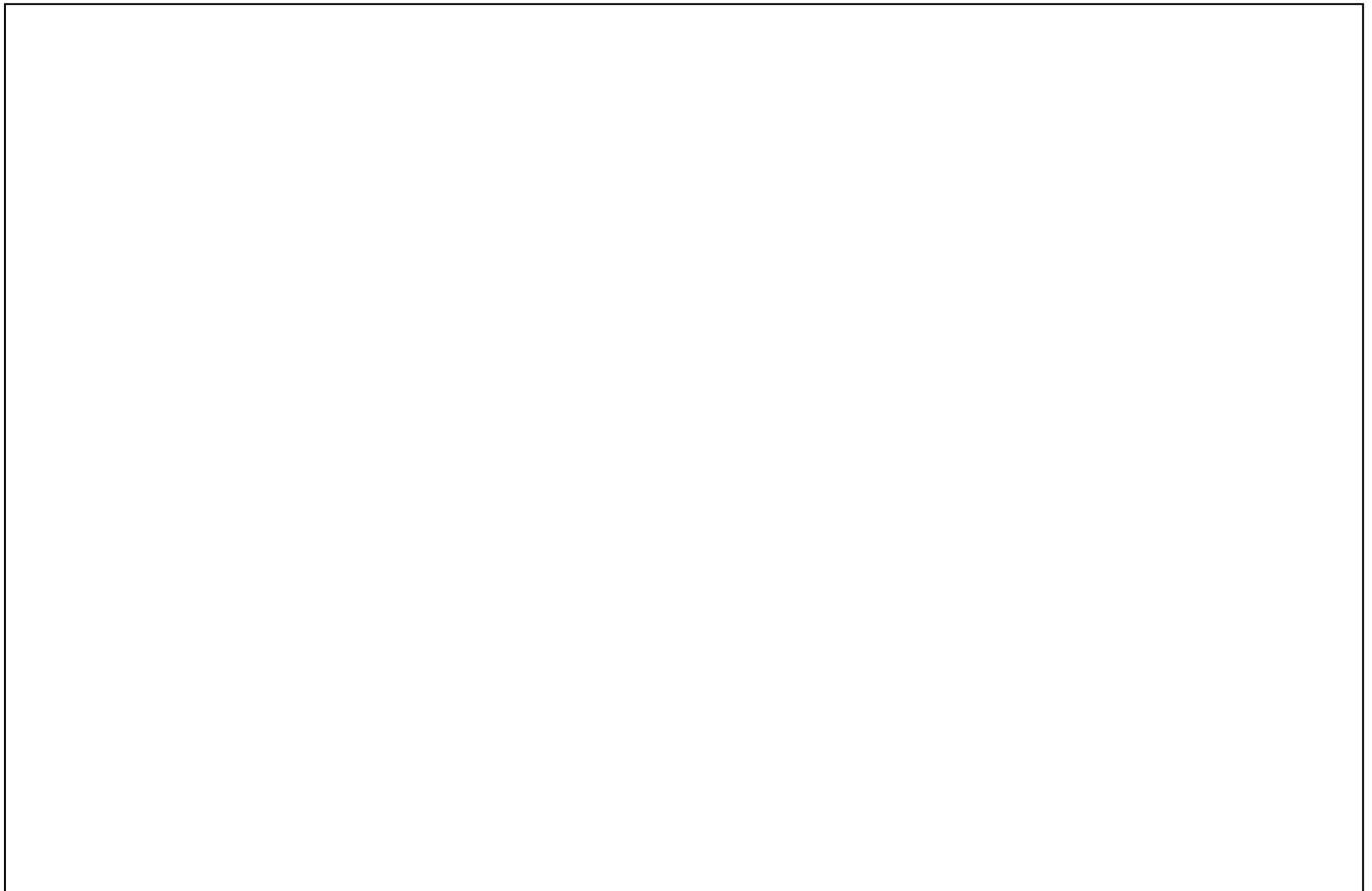
Maddie and Becca are putting together 4-ounce treat bags for a party. Each bag contains 4 pieces of bubble gum, 3 Jolly Ranchers, 2 Tootsie Rolls, and a granola bar. If they have 48 ounces of treats all together, how many granola bars are there?

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Circle the name of the girl you agree with. Maddie Becca

Draw a model below and explain how it supports your choice.



Learning Scale

Circle where you fall on the learning scale.

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1 With help, I can fill in a model like a table or tape diagram similar to the ones we used today. When my teacher asks me, I might be able find the information on the model to answer her.

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4 I can go beyond what we did today and solve proportional reasoning problems with all kinds of missing information. I could extend this to solving problems involving percents and quantities with different units.

Research and review of standard

Content Standard(s): **Standard(s) for Mathematical Practice:**

Ratios & Proportional Relationships
 Analyze proportional relationships and use them to solve real-world and mathematical problems.
CCSS.MATH.CONTENT.7.RP.A.2
Recognize and represent proportional relationships between quantities.
CCSS.MATH.CONTENT.7.RP.A.2.A
Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

CCSS.MATH.PRACTICE.MP2 Reason abstractly and quantitatively.

Smarter Balanced Claim

Smarter Balanced Item

Claim 2: Problem Solving
Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

Grade 7 Page | 1

751

A representative sample of 50 students from a high school is surveyed. Each student is asked what science Question 1 or she is taking. This table shows the responses.

Science Class	Number of Students
Physics	6
Chemistry	10
Biology	18
Earth Science	4
Health Science	12

Click in the True or False column to identify whether each statement is valid based on the survey results.

Based on the representative sample	True	False
Twice as many students are taking Health Science than are taking Physics.	<input type="checkbox"/>	<input type="checkbox"/>
20% of students at the high school are taking Chemistry.	<input type="checkbox"/>	<input type="checkbox"/>
In a group of 25 students, it is expected that 4 of the students are taking Earth Science.	<input type="checkbox"/>	<input type="checkbox"/>
In a group of 150 students, it is expected that 18 of the students are taking Physics.	<input type="checkbox"/>	<input type="checkbox"/>

For this item, a full-credit response (2 point) includes:

- true
AND
- true
AND
- false
AND
- true

For partial credit (1 point), the student correctly checks any 3 out of the 4 correct boxes.

CPR Pre-Requisites
(Conceptual Understanding, Procedural Skills, and Representations)

Conceptual Understanding and Knowledge
 -Understanding percents as a quantity per 100
 -Concept of equivalent ratios
 -Reasoning with ratios involves attending to and coordinating two quantities¹

¹Taken from [Developing Essential Understanding of Ratios, Proportions, & Proportional Reasoning](#)

-A ratio is a multiplicative comparison of two quantities¹
 -A proportion is a relationship of equality between two ratios. In a proportion, the ratio of two quantities remains constant as the corresponding values of the quantities change.¹

Procedural Skills

-Finding equivalent ratios by multiplying or dividing both numerator and denominator by the same number or multiplying or dividing the ratio by 1 in the form of n/n.
 -Setting up and solving a proportion
 -If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship.¹

Representational

-Use of a table to represent proportional relationships
 -Equivalent fractions
 -Pictorial models
 -Tape diagrams
 -Proportions
 -Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning¹

Social knowledge

-proportional relationship
 -representative sample
 -% or percent; the % sign is written after the percentage
 -Ratios are often used to make "part-part" comparisons¹
 -Ratios can often be meaningfully reinterpreted as fractions¹

Standards Progression

**Look at LearnZillion lessons and expert tutorials, the Progressions documents, learning trajectories, and the "Wiring Document" to help you with this section*

Grade(s) below	Target grade	Grade(s) above
<p>Understand ratio concepts and use ratio reasoning to solve problems.</p> <p>CCSS.MATH.CONTENT.6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. <i>For example, "The ratio of wings to</i></p>	<p>CCSS.MATH.CONTENT.7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</p>	<p><u>CCSS.MATH.CONTENT.8.F.A.1</u> Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.¹</p>

<p><i>beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."</i></p> <p>CCSS.MATH.CONTENT.6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</p> <p>CCSS.MATH.CONTENT.6.RP.A.3.A Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</p> <p>CCSS.MATH.CONTENT.6.RP.A.3.C Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.</p>	<p>CCSS.MATH.CONTENT.7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.</p>	<p>CCSS.MATH.CONTENT.8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.</i></p> <p>CCSS.MATH.CONTENT.8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</p> <p>CCSS.MATH.CONTENT.8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.</p>
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Common Misconceptions/Roadblocks

What characteristics of this problem may confuse students?

- *Students may be confused that the size of the population or sample changes for the various questions.*

What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?

- *Students often confuse the numerator or "part" in the ratio with the percentage. For example, when told 6 out of 25 gummies are yellow, they may mistakenly conclude that 6% of the gummies are yellow.*

What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?

- *Students may use addition or subtraction to describe the difference in the number of gummies, rather than multiplication or division. For example, if there are 4 red and 8 orange gummies in a bag of 25, and they calculate there are 8 reds in a bag of 50, they may mistakenly conclude there are 12 oranges, because 12 is 4 more than 8, rather than 16 oranges, because 16 is two times 8.*
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