Mathematics Instructional Cycle Guide

Ratio and Proportional Reasoning (6.RP.3)

Created by Ellen Meyer, 2014 Connecticut Dream Team teacher

CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, **double number line diagrams**, or equations. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

MP.1 – Make sense of problems and persevere in solving them.

- *Students can explain the meaning of the problem in the Tim/Tom task and look for entry points to the solution*
- *Students can plan and use more than one solution pathway to solve the Tim/Tom task*

MP.4 – Model with mathematics

- *Students can identify the important quantities in the Tim/Tom task and map their relationship using a diagram.*

MP.7 – Look for and make use of structure

- *Students look for patterns in the task and recognize that 7 miles every 105 minutes is the same rate as 1 mile every 15 minutes.*

WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings **(page 2)**
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint **(pages 3 - 6)**
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed **(pages 7 - 16)**
- Supporting lesson materials **(pages 17 - 30)**
- Precursory research and review of standard 6.RP.3 and assessment items that illustrate the standard **(pages 31 - 33)**

HOW TO USE THIS DOCUMENT

1) Before the lesson, administer the **(Robby Rabbit)** ***Mathematical Checkpoint*** individually to students to elicit evidence of student understanding.
2) Analyze and interpret the student work using the ***Student Response Guide***
3) Use the next steps or ***follow-up lesson plan*** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

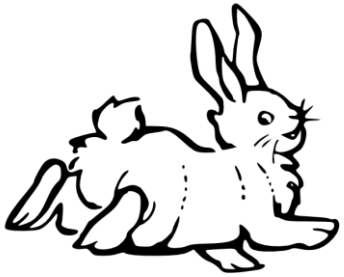
MATERIALS REQUIRED

- Projector, document reader, Smartboard
- Technology to access internet for Lesson Launch video
- Student copies of the instructional tasks, copy the Tim and Tom tasks in different colors
- Student copies of the Exit Ticket
- Student copies of the student reflections for Closure

TIME NEEDED

Robby Rabbit administration: **10 minutes**
Follow-Up Lesson Plan: **75 minute block**

Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

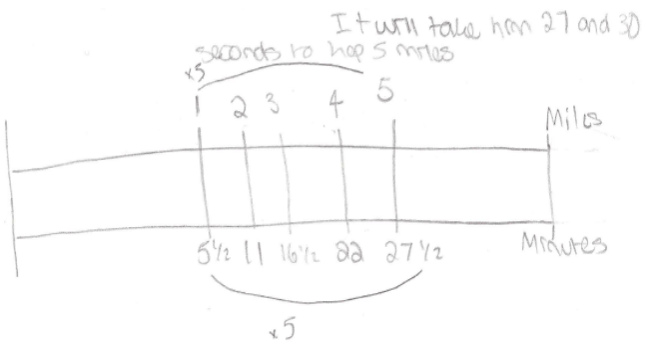
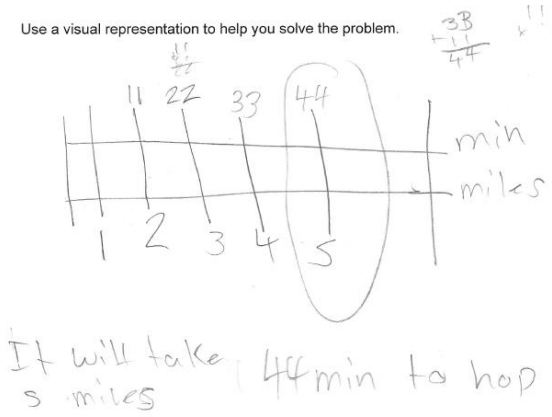
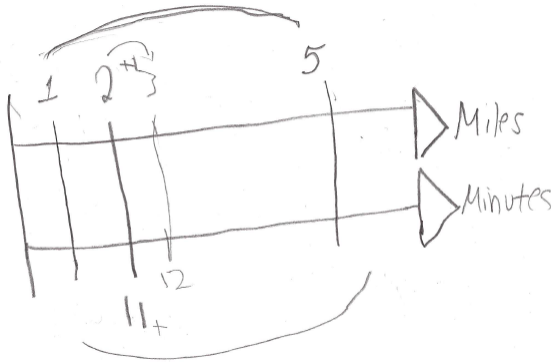
<table>
<tr><th colspan="3">Step 1: Elicit evidence of student understanding</th></tr>
<tr><th colspan="3">Mathematical Checkpoint</th></tr>
<tr><th>Question(s)</th><th colspan="2">Purpose</th></tr>
<tr><td rowspan="2"> 

Robby Rabbit is training for a race. He can hop 2 miles in 11 minutes. If he hops at the same rate, how long will it take him to hop 5 miles?

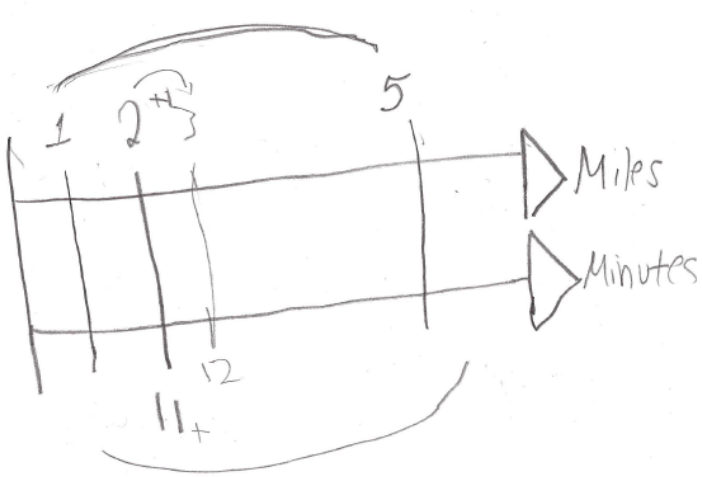
Use a visual representation to help you solve the problem.</td><td>CT Core Standard:</td><td>6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

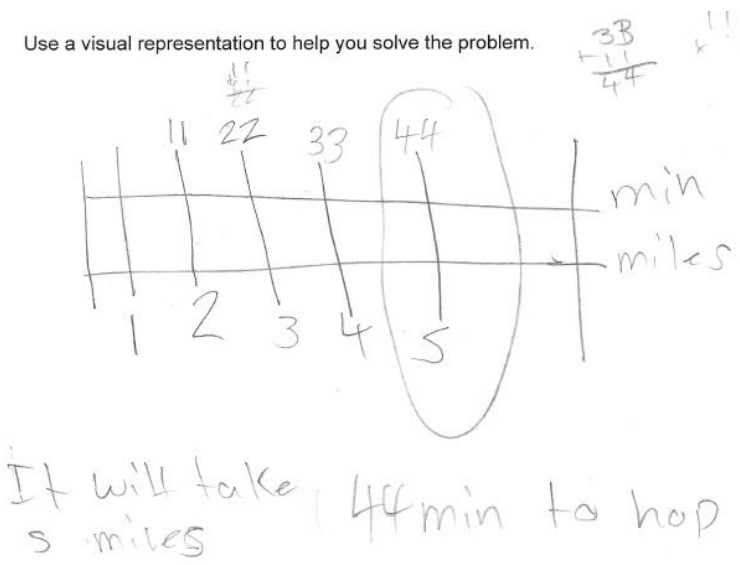
Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?</td></tr>
<tr><td>Target question addressed by this checkpoint:</td><td>How do students approach a problem that involves a comparison between 2 different quantities?

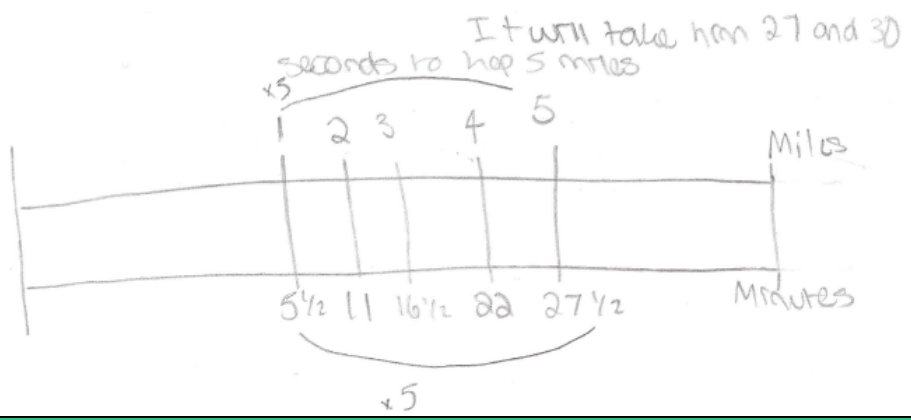
To what extent do they:
Recognize that the problem represents a proportional relationship?
Organize information from the problem so they better use the proportional relationships?
Use what they have learned about equivalence to help solve the problem?</td></tr>
</table>

Step 2: Analyze and Interpret Student Work		
Student Response Guide		
Got It	**Developing**	**Getting Started**
		

LearnZillion

Getting Started	
Student Response Example	**Indicators**
	• The student was able to create a double number line diagram and place the original ratio correctly. • The response does not show evidence that the student recognizes the proportional relationship between 2 miles and 11 minutes. • The student showed an attempt to scale 2 miles to 5 miles by going up and down by 1. • The student did not understand how to maintain the ratio and incorrectly added 1 to the 11 minutes. • The student did not use multiplicative reasoning to find the equivalent ratio for $\frac{2}{11} = \frac{1}{?}$ • The student may have avoided division because it didn't come out evenly. • The student may have thought that the solution had to be a whole number.
In the Moment Questions/Prompts	**Closing the Loop (Interventions/Extensions)**
P: Tell me about your model. P: Tell me what it means if Robby Rabbit "hops at the same rate". Q: If Robby Rabbit hopped 4 miles, how long would it take him? Q: If Robby Rabbit hopped 6 miles, how long would it take him? Q: What operation can be used to find how long it takes Robby Rabbit to hop 1 mile?	Provide student with manipulatives to show the ratio relationships of ___ for every ____. Students may also draw models of equivalent ratios. The students can record the ratio relationships using a horizontal table and then transfer the values to a double number line. Review how multiplication and division are used to find equivalent fractions. http://learnzillion.com/lessons/609-solve-missing-values-in-ratio-problems-using-a-double-number-line http://learnzillion.com/lessons/842-define-unit-rate-using-double-number-line

Developing	
Student Response Example	**Indicators**
	• The student was able to create an appropriate double number line diagram to help solve the problem. • The student was able to use addition to maintain the relationship, although for every 1 mile that was added, the student added 11 minutes instead of 5.5 minutes. • The student was able to form equivalent ratios, even though they did not find the correct solution. • The student may have thought that the solution had to be a whole number. • The student was unable to find the unit rate to help solve the problem.
In the Moment Questions/Prompts	**Closing the Loop (Interventions/Extensions)**
P: Tell me about your model. P: Tell me what it means if “Robby hops at the same rate”. Q: Is hopping 2 miles in 11 minutes the same rate as hopping 22 miles in 3 minutes? How do you know? Q: How long will it take Robby to hop 1 mile?	Provide students with situations in which they need to divide odd dollar amounts by 2. “If I had $11 and I divided it equally among 2 students, how much money would each student receive?” Review how multiplication and division are used to find equivalent fractions. http://learnzillion.com/lessons/609-solve-missing-values-in-ratio-problems-using-a-double-number-line http://learnzillion.com/lessons/842-define-unit-rate-using-double-number-line

<table>
<tr><th colspan="2">Got it</th></tr>
<tr><th>Student Response Example</th><th>Indicators</th></tr>
<tr><td> 
</td><td>

The student calculated $5\frac{1}{2}$ minutes as the time it would take Robby Rabbit to hop 1 mile, and $27\frac{1}{2}$ minutes to hop 5 miles.
The student was able to scale the ratios up and down to find correct equivalent ratios and the correct solution.
The work clearly shows the student recognized that multiplication could be used to scale the quantities up and down.

</td></tr>
<tr><th>In the Moment Questions/Prompts</th><th>Closing the Loop (Interventions/Extensions)</th></tr>
<tr><td>P: Explain the strategies you used to find the solution.

Q: What type of rate is 1 mile in $5\frac{1}{2}$ minutes?</td><td>Extension: How far could Robby Rabbit hop in 8.5 minutes?

This challenges the students to solve the problem by finding out what part of a mile Robby Rabbit can hop in 1 minute, instead of how many minutes it will take Robby to hop in 1 mile.</td></tr>
</table>

Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction	
Lesson Objective:	Use models to solve real world problems using ratio and rate reasoning.
Content Standard(s):	6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, **double number line diagrams**, or equations. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
Targeted Practice Standard :	**MP.1 Make sense of problems and persevere in solving them.** • *How do students explain the meaning of the problem in the Tim/Tom task and look for entry points to its solution?* **MP.4 Model with mathematics.** • *Are students able to identify the important quantities in the Tim/Tom task and map their relationship using a diagram?* **MP.7 Look for and make use of structure.** • *Do students look for patterns and recognize that the rates they are representing on their diagram are equivalent?*

Mathematical Goals	Success Criteria
○ Students will understand that a rate is a special ratio that compares two or more quantities that have different measurements. ○ Students will be able to represent and solve problems involving ratio and rate reasoning using diagrams.	○ Students understand how to visually represent proportional relationships using a double line diagram. ○ Students will understand that if one quantity in a ratio is multiplied or divided by a factor, then the other quantity must be multiplied or divided by the SAME factor for the proportional relationship to remain the same. ○ Students will be able to find equivalent rates using proportional reasoning

Launch (Probe and Build Background Knowledge)

Purpose: ***To have the students come to the conclusion that two measurements are needed (distance, time), and when we measure something in terms of two measurements, we are using a rate.***

https://www.youtube.com/watch?v=2CAfuv-VQwM

(This is a 4 ½ minute video that has some nice blackouts so you can choose to watch only part of the video)

- *What does it mean to run fast?*
- *How do we know if one person runs faster than another?*
- *What happens at the end of the race if one person is faster than another?*
- *What two measurements do we need to know who is the winner?*

Give students some time to think on their own, and then have students share their responses.

Some examples students may come up with:

- The person who is winning is has gone more distance in the same time as the slower runner.
- The person who is running slower has gone less distance in the same time as the faster runner.

1) Facilitate a brief discussion to introduce the double number line diagram to think about distance and time together.

- *As we saw in the video, which two measurements determine the winner of a race? (Time and distance).*
- *Explain to the students that a rate is type of ratio that compares two different quantities. Facilitate a one-minute brainstorm of rates. Examples: miles per hour, miles per gallon, cost per item, etc.*

2) Project problem to facilitate a discussion of double number line diagrams:

Ashley can run 5 meters in 2 seconds.

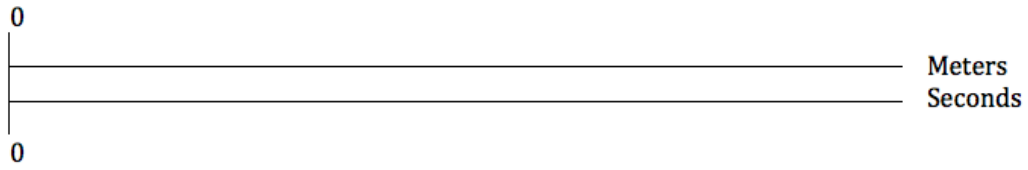
How long will it take her to run 30 meters?

- *What does is mean for Ashley to run 5 meters in 2 seconds?*
- *How can we show her progress using a diagram?*

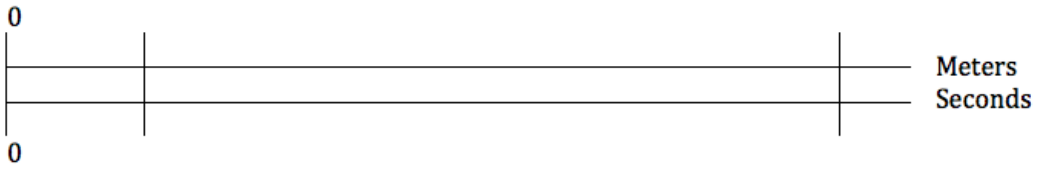
3) Explain to the students that we can show Ashley's progress using a diagram called a double number line.

- Draw a pair of parallel lines, one for each quantity that begins with 0.
- How can we use what we know about ratios and rates to label our diagram?

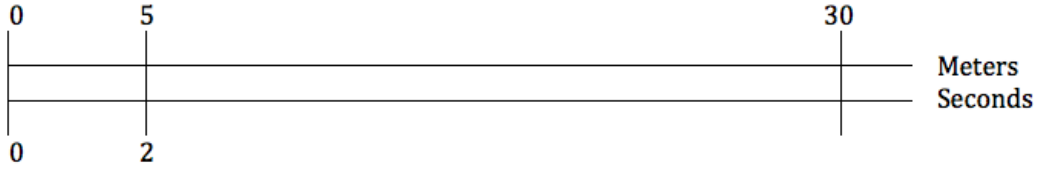
(Label each number line with one of the measurements, meters and the other with seconds.)



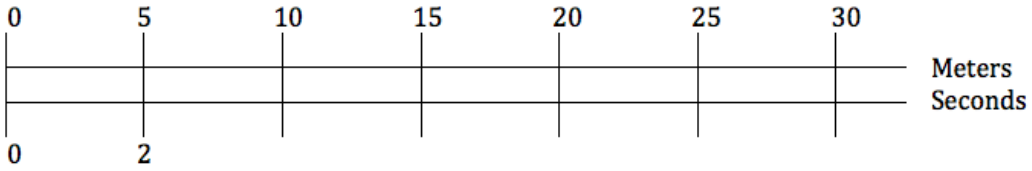
- *Let's look at the problem to see what we know. We know that Ashley can run 5 meters in 2 seconds. We want to know how many seconds it will take her to run 30 meters.*
- *How can we show this information using a double number line diagram? (We can make 2 lines to show what we know, and what we want to find out.)*



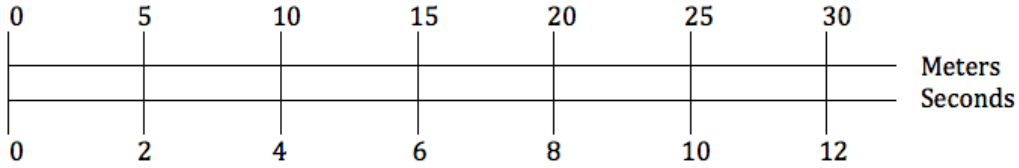
- *The first line should be labeled with 5 meters and 2 seconds, since it is closer to zero. We can label the second line with 30 meters.*



- *How can we use what we do know (5 meters and 30 meters) to help us find what we need to know (how many seconds it will take Ashley to run 30 meters)?*
- *How can we get from 5 to 30? (We can scale up on the number line by fives.)*



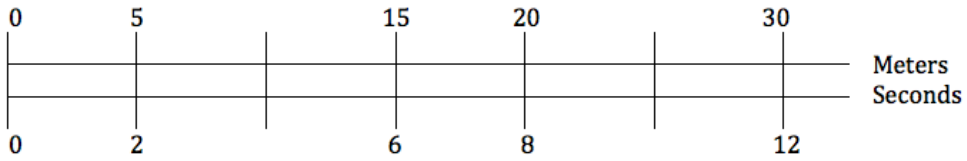
- *Now we can use what we know about ratios to fill in the number values for seconds. Every time Ashley runs 5 meters, it will take her 2 seconds. How many seconds will it take her to run 10 seconds? 15 seconds? (The students should be able see the relationships between the distance in meters and time in seconds.)*
- *It will take Ashley 12 seconds to run 30 meters.*



4) These are examples of other strategies to look for in student thinking, and not a "complete" list of the relationships students might see in the diagram:

- Some students may think " $2 \times 5 = 10$ and $2 \times 2 = 4$", then go by tens to get to 30 and fours to get to 12.
- Some students may be able to see the multiplicative relationship $5 \times 6 = 30$, and $2 \times 6 = 12$. These students are probably ready for an extension problem!
- Another way to show this is by using equivalent fractions. $\frac{5}{2} \times \frac{6}{6} = \frac{30}{12}$
- It may help to remove some of the values to help students see the multiplicative relationship.

5 x _____ = 30, 2 x _____ = 12; also 15 x _____ = 30, 6 x _____ = 12

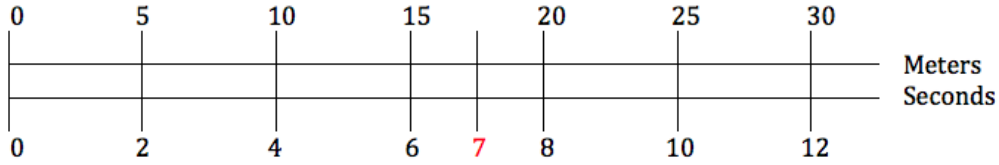


- For students who are really struggling to get started, a suggestion would be to show a single number with the numbers 5 and 30, and then ask again, "How can we get from 5 to 30?"

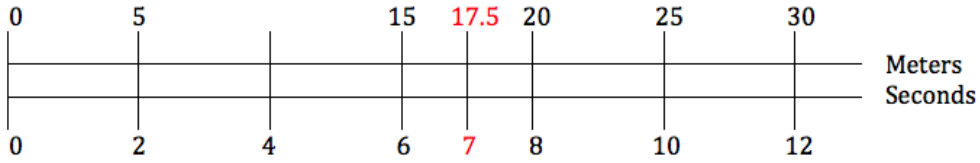
5) Project and facilitate a discussion about Part B.

- How is Part B different from Part A?

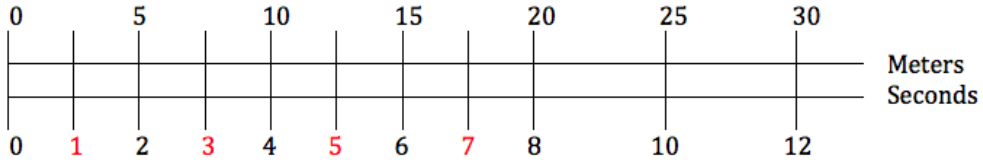
- Can we use the same diagram we used to find the solution for Part A?
- *(Some students may be able to use what they know about number lines and recognize that 7 is between 6 and 8.)*



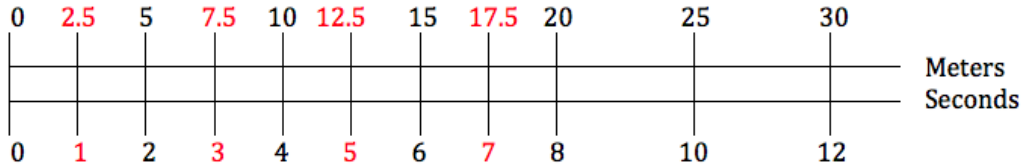
- *(Using the same reasoning, some students may then find the solution 17.5 meters because 17.5 is halfway between 15 and 20.)*



- *(Some students will need prompting to place odd numbers on the number line.)*
- What numbers are between 0 and 2, 2 and 4, 4 and 6, 6 and 8?



- *What number is between 0 and 5?*
- *How many meters can Ashley run in 1 second?*
- *What pattern(s) do you see?*
- *Double number lines make it easy to use the odd values to find the solution (Scale up and down).*
- *Note: some students may be able to figure out what values are between the even second values while others may divide 5 by 2.*
- *This is the beauty of double number lines! Values can be added to the left and in-between, as well as to the right.*



Instructional Task

Purpose: ***Introduce using double number lines by asking the students to solve a similar, but less involved, question in order to provide students a review or background of using a double number line.***

This lesson is designed to introduce students to solving rate problems using a double number line diagram. This strategy can help students represent and understand equivalences. Each line represents 2 related numbers, such as those

representing time and distance. Because of the nature of number lines, it is possible to scale the numbers up or down. This makes it easy to see the equivalences between the 2 rows of numbers at any point on the line.

Engage (Setting Up the Task)

1) *Introduce the task by projecting the problem using a document reader, smartboard, or overhead.*
Let's take a look at a problem about two friends who are planning on running a marathon this year.

> Two friends are training for a marathon:
>
> Tim left his house at 8:30 a.m. to go for a run. At 10:15 he had run a distance of 7 miles.
> Tom also left his house at 8:30 a.m. to go for a run. At 10:10 he had run a distance of 8 miles.
>
> a) At what rate of speed did Tim run, assuming he runs at a constant rate?
> b) At what rate of speed did Tom run, assuming he runs at a constant rate?
>
> c) At 10:15, Tim decides to run 5 more miles. What time will he finish running?
> d) At 10:10, Tom decides to run 4 more miles. What time will he finish running?

2) Facilitate a whole group discussion about the problem.

- *How is this problem different than the problem about Ashley?*
- *For parts a and b, what will the "rate of speed" tell us?*
- *For parts c and d, what information can you use to figure out the finish time for each friend?*
- *What time measurement do you think is best to use? (minutes / hours; This actually requires student to use another ratio: minutes per hour)*

Explore (Solving the Task)

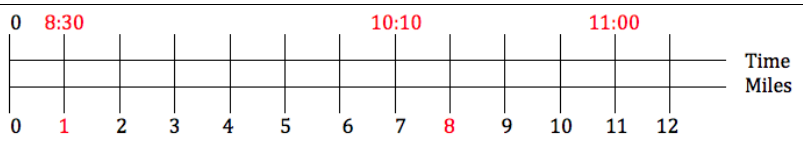
3) After the class has been introduced to the problem, explain to the students that half the class will work on the problem about Tim, and half the class will work on the problem about Tom. *(Some students may have some difficulty with the decimal numbers required for Tom's speed.)

4) Place the students in groups of 3 or 4, and hand each group enough copies of either the Tim or Tom task (copy each on a different color of copy paper). Explain that students will now work on the task with their groups.

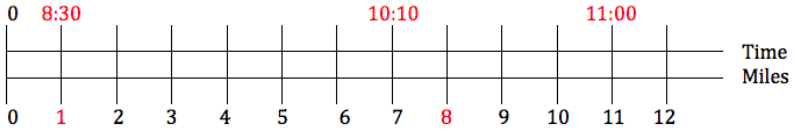
5) As students work, circulate to look for anticipated student responses / solution paths.

- *Some students may set up their diagrams according by minutes and miles, while others may use time and miles.*
- Tim: Students will need to find the rate that Tim ran one mile by calculating the elapsed time and then using division: $105 \div 7 = 15$.
- Tom: Students will need to find the rate that Tim ran one mile by calculating the elapsed time and then using division: $100 \div 8 = 12.5$.
- Some students will set up their diagrams by minutes per mile, while other many use time and miles.

- *Examples:*
  - Tim



- Tom



Possible questions/prompts to use as students work through the task:

Focusing Questions

- *What do you know, and what do you need to find out?*
- *How long did Tim/Tom run?*
- *How can hours be converted to minutes?*
- *How many MINUTES will it take Tim to run 7 miles?*
- *How many MINUTES will it take Tom to run 8 miles?*
- *How long will it take Tim/Tom to run 1 mile?*
- *Do you notice any patterns?*
- *What do you notice about the relationship between miles and minutes?*

Probing Questions

- *What have you done so far?*
- *What else can you think of to do?*
- *How did you find the (number of minutes, rate of speed, etc.)?*
- *Can you explain how you set up your diagram?*

Extending Questions

- *Can you think of another way to solve this problem?*
- *Can you think of a problem that can be solved using a double number line?*
- *Could you use a more efficient method to solve this problem (encourage multiplicative reasoning)?*

Elaborate (Discuss Task and Related Mathematical Concepts)

6) After students have finished working on the problem and most are finished, students should pair up with a student who has the other problem. This will be easy since each problem is on a handout of a different color. Encourage students who didn't finish to ask their partners how they solved their problem and then together they might use that strategy to solve the unfinished problem.

Questions to facilitate student discussions (these can be projected or written on the board):

- *What is similar about the strategies we used to solve our problems?*
- *What is different about the strategies we used to solve our problems?*
- *Do we agree or disagree with our solutions?*

Questions to answer about their solutions:

- *Which friend is the faster runner?*
- *Justify your answer using information from each partner's problem.*

7) As the pairs work together, walk around the room and select students / pairs to share their work to the class**.** Select student work that reflects different strategies.

Questions to facilitate discussion about important mathematical concepts:

- *What steps did you take to find your solution?*
- *What kinds of calculations did you make?*
- *Why did you choose to label your diagram using _____ and _____?*
-

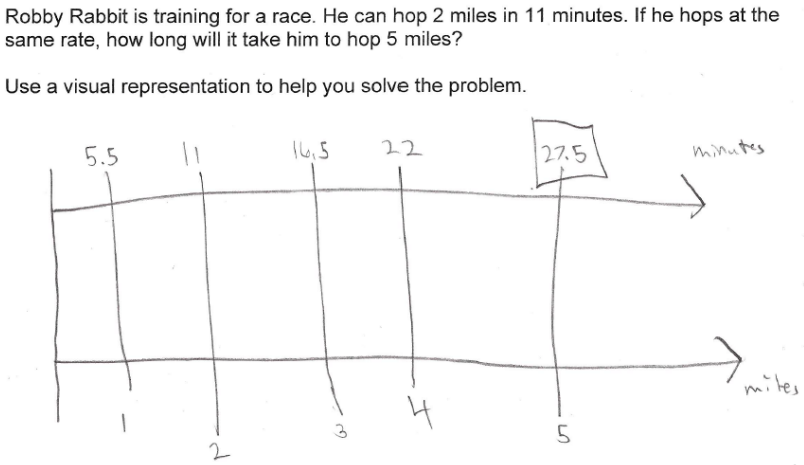
Common Misunderstanding

Purpose: ***Address a common misunderstanding students have about the nature of the comparison used in ratios.***

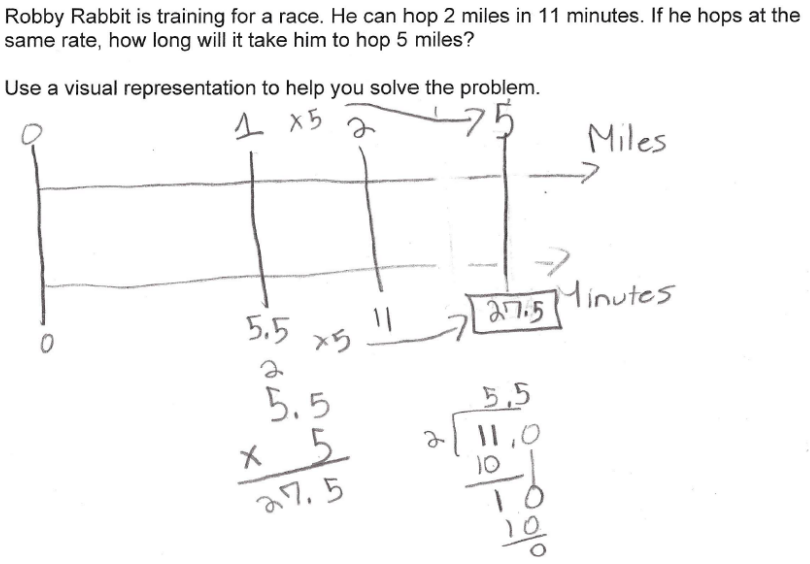
Students may use additive reasoning when reasoning with ratios. However, equivalency and reasoning with ratios must include multiplicative properties. Look for this type of reasoning as students work on the task, and also in the "Checking for Understanding" prompt.

The use of additive or multiplicative thinking is illustrated in the strategies used to solve the Robby Rabbit problem.

- This student used additive reasoning to repeatedly add 5.5 on the "minutes" number line and 1 on the "miles" number line to find the solution.



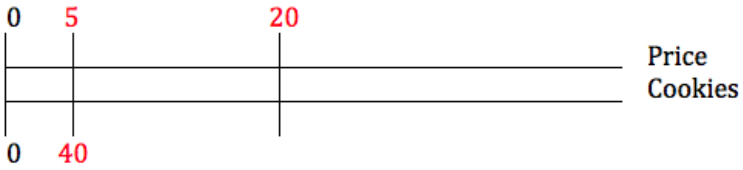
- This student clearly used division to arrive at 1 mile for every 5.5 minutes, and then used multiplicative thinking to multiply both numbers by 5 to find the solution.



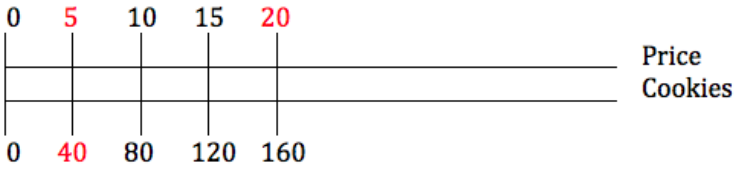
The following is a prompt that could be used to assess student understanding of the difference between additive and multiplicative reasoning.

- *What does it mean to say that a box of cookies sells for $5? (Every time I buy a box of cookies I have to pay $5)*

  1) *Set up the double number line with the information that we know, and what we want to know.*



  2) How can we find out the number of cookies? (Possible student response using additive thinking: go up by 5s for the price and by 40s for the cookies).



  - *To find the prices, we added by _____ each time.*
  - *To find the number of cookies, we added _____ each time.*
  - *Could we use multiplication instead of addition to find the solution?*
  - *Is there another way to find the solution?*
  - *If we looked at these as equivalent fractions, how could you find the missing denominator?*

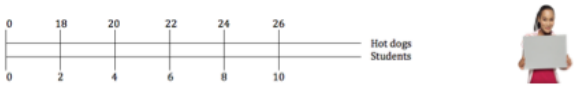
$$\frac{5}{40} = \frac{20}{\quad}$$

$$\frac{5 \times 4}{40 \times 4} = \frac{20}{160}$$

Checking for Understanding

Purpose: ***The purpose of the Exit Ticket is for students to interpret, and then agree or disagree, with how a double number line was used to solve a problem.***

Dana and Liam needed to solve the following problem for class:
24 hot dogs will feed 8 college students. How many hotdogs are needed to feed 10 college students?
Both Dana and Liam decided to use a double number line diagram to help them solve the problem.
Decide which student's answer you agree with.

Student solutions and reasoning:	Explain your choice:
Dana's Double Number Line Diagram  Dana says, "I can show that the number of students is going up by twos. So the number of hot dogs needed to feed the college students must also be going up by twos. So, 26 hotdogs are needed to feed 10 college students." Agree Disagree	
Liam's Double Number Line Diagram  Liam says, "If 24 hotdogs are needed to feed 8 students, then 3 hot dogs are needed for every student. I can multiply both the 3 hotdogs and the 1 student by 10 to find my solution. So, 30 hot dogs are needed to feed 10 college students." Agree Disagree	

Closure

Purpose: ***Students will reflect on how well they can use and interpret double number line diagrams to solve problems.***

Think about what you have learned today about using double number line diagrams to solve problems. Circle the number that matches your success with each item.

1) I can make a double number line diagram from the information given in a problem.

1	2	3	4	5
I don't get this.	I need help with this.	I need more practice on this.	I can do this all by myself.	I can show others how to do this!

2) I can find equivalent ratios using a double number line diagram.

1	2	3	4	5
I don't get this.	I need help with this.	I need more practice on this.	I can do this all by myself.	I can show others how to do this!

3) I can interpret a double number line diagram another student made.

1	2	3	4	5
I don't get this.	I need help with this.	I need more practice on this.	I can do this all by myself.	I can show others how to do this!

Extension Task

Purpose*: **Provide an extension task for those students who are ready to deepen their understanding of using double number line diagrams to reason with ratios. This task requires a measurement conversion.***

In "Toby Turtle" the students are asked to solve a problem in which they will need to convert hour to minutes, or choose to solve but have a mixed number as a solution. In the problem where Lisa is biking, the students are to use multiplicative reasoning to find distances, and her speed in miles per hour.

Extension Problems:

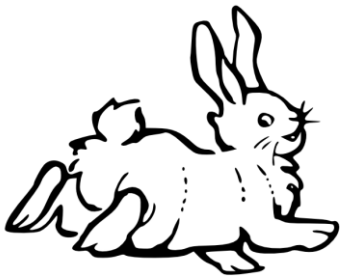
1) Star athlete Toby Turtle runs 5 meters every 2 hours.

How long does it take Toby to run 3 meters?

2) Lisa went biking on a Saturday afternoon. She rode her bike 16 miles in 80 minutes.

a) How far did she ride her bike in 10 minutes?
b) How far could she go in 2 ½ hours?
c) What is her rate of speed in mph? 12 mph

Name _____ Class _____



Robby Rabbit is training for a race. He can hop 2 miles in 11 minutes. If he hops at the same rate, how long will it take him to hop 5 miles?

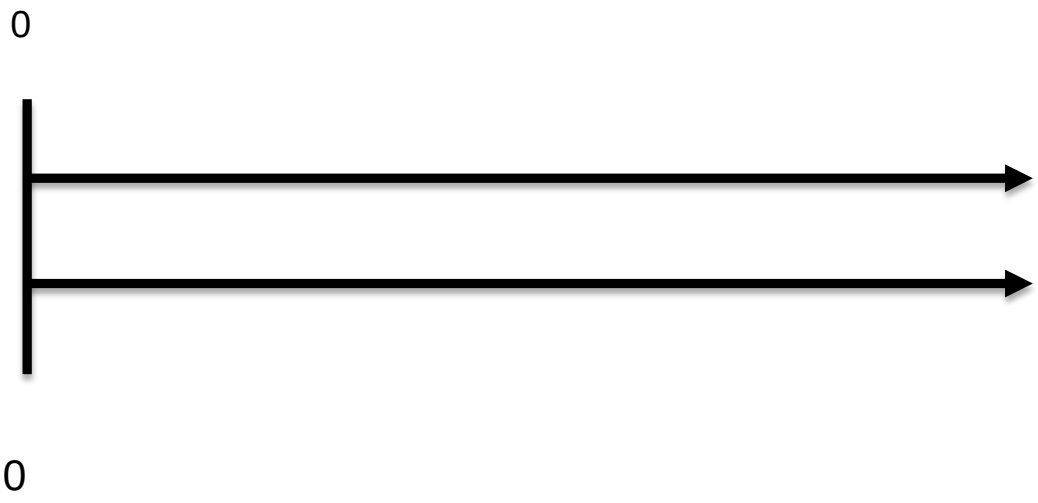
Use a visual representation to help you solve the problem.

Double Number Line Introduction

Name _____ Class _____

Ashley can run 5 meters in 2 seconds. How long will it take her to run 30 meters?

If Ashley runs for 7 seconds, how many meters did she run?



Double Number Line Introduction

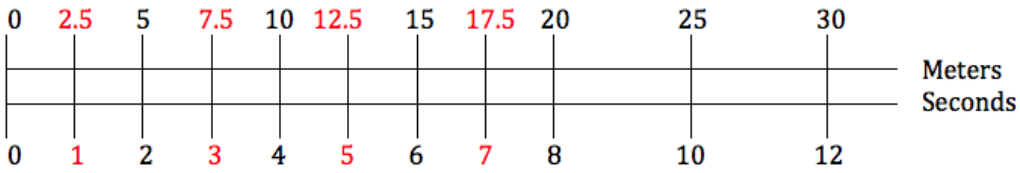
Name KEY Class _____

Ashley can run 5 meters in 2 seconds. How long will it take her to run 30 meters?

She will be able to run 30 meters in 12 seconds.

If Ashley runs for 7 seconds, how many meters did she run?

She will run 17.5 meters in 7 seconds.



LearnZillion

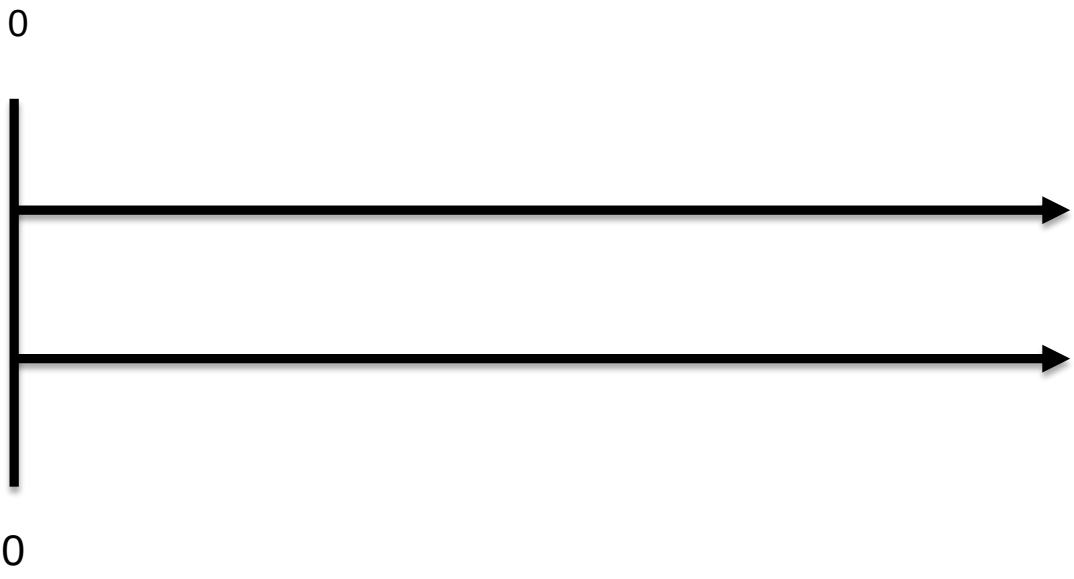
Training with Tim

Name _____ Class _____

Tim left his house at 8:30 a.m. to go for a run. At 10:15 a.m. he had run a distance of 7 miles.

At what rate of speed did Tim run, assuming he runs at a constant rate?

At 10:15, Tim decides to run 5 more miles. What time will he finish running?



LearnZillion

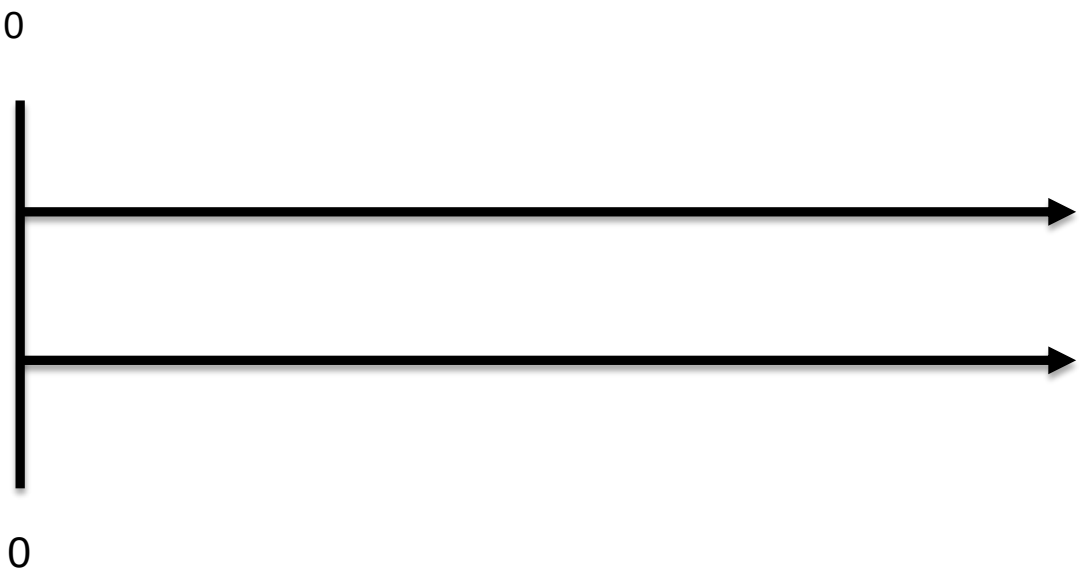
Training with Tom

Name _____ Class _____

Tom left his house at 8:30 a.m.to go for a run. At 10:10 a.m. he had run a distance of 8 miles.

At what rate of speed did Tom run, assuming he runs at a constant rate?

At 10:10, Tom decides to run 4 more miles. What time will he finish running?



KEY

Training with Tim

Name _____ Class _____

Tim left his house at 8:30 a.m. and planned to stop for a water break at 10:15 a.m. He ran a distance of 7 miles.

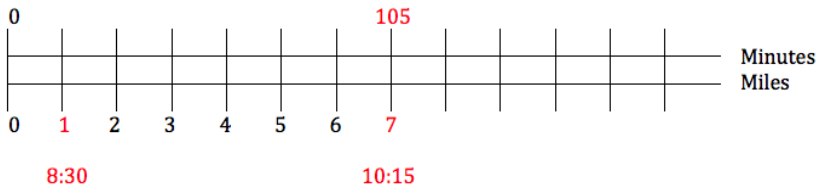
At what rate of speed did Tim run?

Tim is running at the rate of 15 minutes for each mile.

At 10:15, Tim decides to run 5 more miles. What time will he finish running?

Tim will finish running at 11:30 am.

Sample double number lines:



Students should be able to divide 105 by 7 to arrive a 15 minutes for every mile. Students who use multiplicative reasoning should be able to multiply 15 by 12 to arrive at the solution of 180 minutes, which is 3 hours.

KEY

Training with Tom

Name _____ Class _____

Tom left his house at 8:30 a.m., and planned to stop at 10:10 a.m. for a water break. He ran a distance of 8 miles.

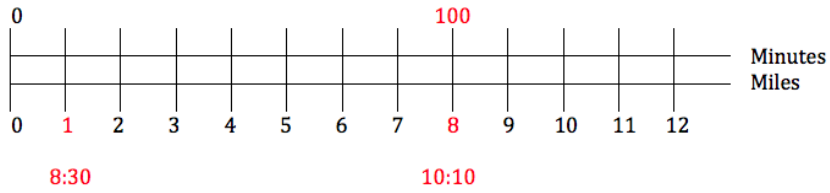
At what rate of speed did Tom run?

It takes Tom 12.5 minutes to run a mile.

At 10:10, Tom decides to run 5 more miles. What time will he finish running?

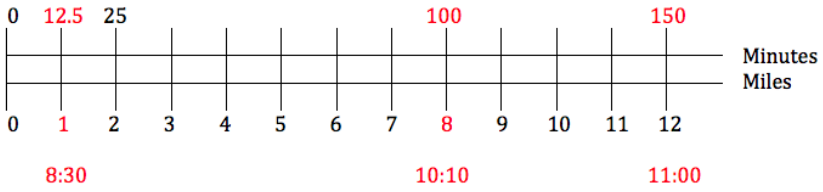
He will finish at 11:00 am.

Sample double number lines:



Students should be able to divide 100 by 8 to arrive a 12.5 minutes for every mile. However, some students may be able to divide 100 and 8 by 2 to find the minutes for 5 miles (50). They then can use the same reasoning to find 25 minutes in two miles, and then 12.5 minutes for 1 mile.

Students who use multiplicative reasoning should be able to multiply 12.5 by 12 to arrive at the solution of 150 minutes, which is 2 hours and 30 minutes. Or, some may decide to multiply 2 by 6 and 25 by 6 to arrive at the solution.



EXIT TICKET

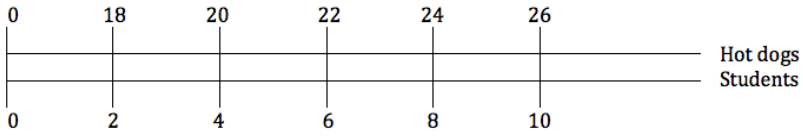
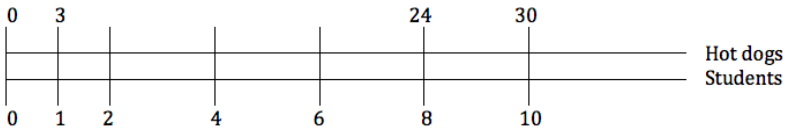
Name _____

Dana and Liam needed to solve the following problem for class: 24 hot dogs will feed 8 college students.

How many hotdogs are needed to feed 10 college students?

Both Dana and Liam decided to use a double number line diagram to help them solve the problem.

Decide which student's answer you agree with.

Student solutions and reasoning:	Explain your choice:
Dana's Double Number Line Diagram   Dana says, "I can show that the number of students is going up by twos. So the number of hot dogs needed to feed the college students must also be going up by twos. So, 26 hotdogs are needed to feed 10 college students." Agree Disagree	
Liam's Double Number Line Diagram   Liam says, "If 24 hotdogs are needed to feed 8 students, then 3 hot dogs are needed for every student. I can multiply both the 3 hotdogs and the 1 student by 10 to find my solution. So, 30 hot dogs are needed to feed 10 college students." Agree Disagree	

KEY

EXIT TICKET

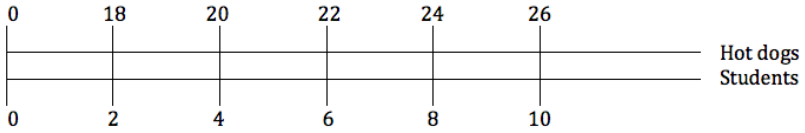
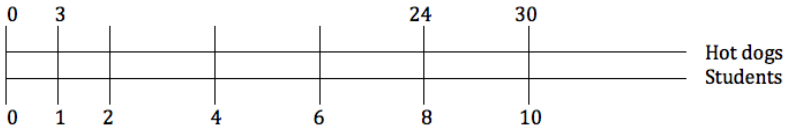
Name _____

Dana and Liam needed to solve the following problem for class:

24 hot dogs will feed 8 college students. How many hotdogs are needed to feed 10 college students?

Both Dana and Liam decided to use a double number line diagram to help them solve the problem.

Decide which student's answer you agree with.

Student solutions and reasoning:	Explain your choice:
Dana's Double Number Line Diagram   Dana says, "I can show that the number of students is going up by twos. So the number of hot dogs needed to feed the college students must also be going up by twos. So, 26 hotdogs are needed to feed 10 college students." Agree *Disagree*	I disagree with Dana. Her diagram does not show equivalent ratios. She correctly counted students by 2s, but incorrectly counted by 2s for the number of hot dogs needed. There should be 24 hot dogs for every 8 students, which is a ratio of 3 to 1.
Liam's Double Number Line Diagram   Liam says, "If 24 hotdogs are needed to feed 8 students, then 3 hot dogs are needed for every student. I can multiply both the 3 hotdogs and the 1 student by 10 to find my solution. So, 30 hot dogs are needed to feed 10 college students." *Agree* Disagree	I agree with Liam because he correctly calculated that 24 hotdogs for every 8 students is the same as 3 hotdogs for every student. He maintained equivalent ratios by multiplying both the number of hot dogs (3) and students (1) by 10. To find the solution.

LearnZillion

Student Self-Assessment

Name _____

Think about what you have learned today about using double number line diagrams to solve problems. Circle the number that matches your success with each item.

1) I can make a double number line diagram from the information given in a problem.

1	2	3	4	5
I don't get this.	I need help with this.	I need more practice on this.	I can do this all by myself.	I can show others how to do this!

2) I can find equivalent ratios using a double number line diagram.

1	2	3	4	5
I don't get this.	I need help with this.	I need more practice on this.	I can do this all by myself.	I can show others how to do this!

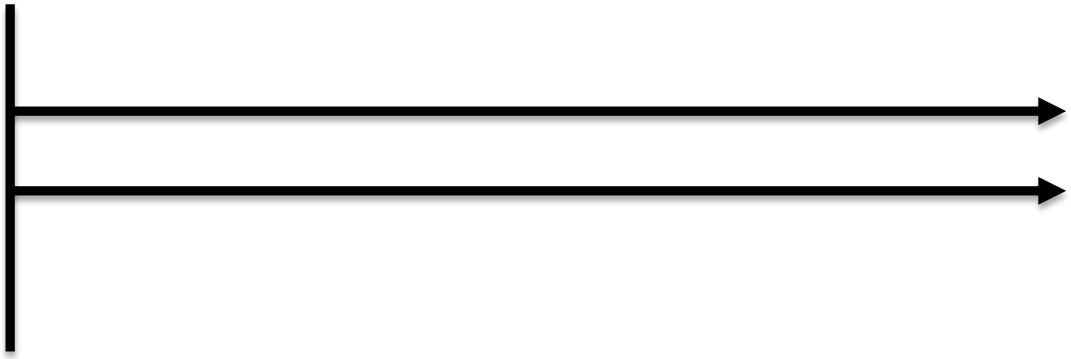
3) I can interpret a double number line diagram another student made.

1	2	3	4	5
I don't get this.	I need help with this.	I need more practice on this.	I can do this all by myself.	I can show others how to do this!

Extension Problems:

1) Star athlete Toby Turtle runs 5 meters every 2 hours.

How long does it take Toby to run 3 meters?

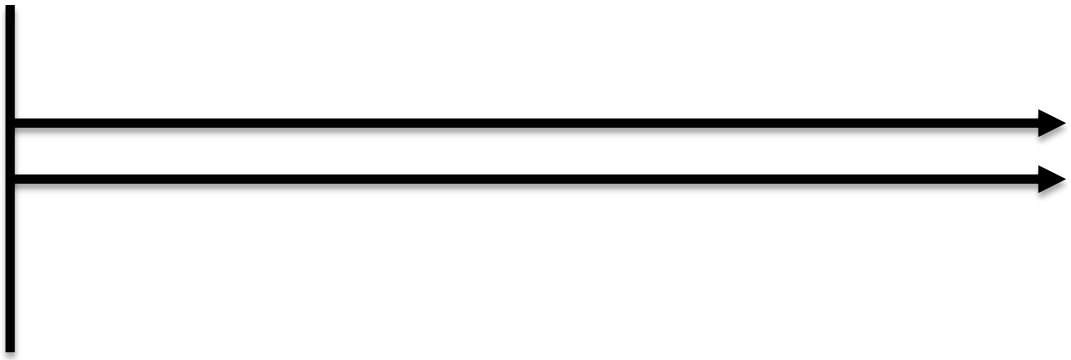


2) Lisa went biking on a Saturday afternoon. She rode her bike 16 miles in 80 minutes.

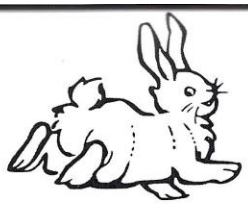
How far did she ride her bike in 10 minutes?

How far could she go in 2 ½ hours?

What is her rate of speed in mph?



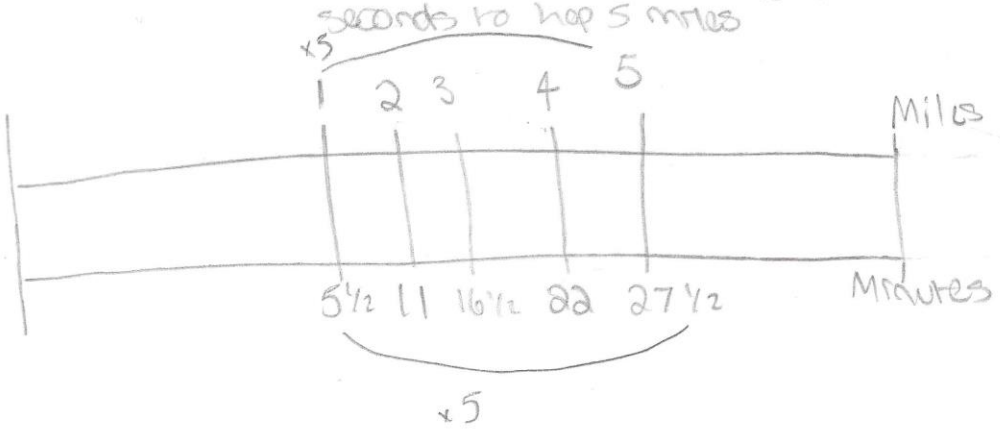
Checkpoint Work "Got It"



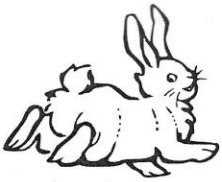
Robby Rabbit is training for a race. He can hop 2 miles in 11 minutes. If he hops at the same rate, how long will it take him to hop 5 miles?

Use a visual representation to help you solve the problem.

It will take him 27 and 30 seconds to hop 5 miles

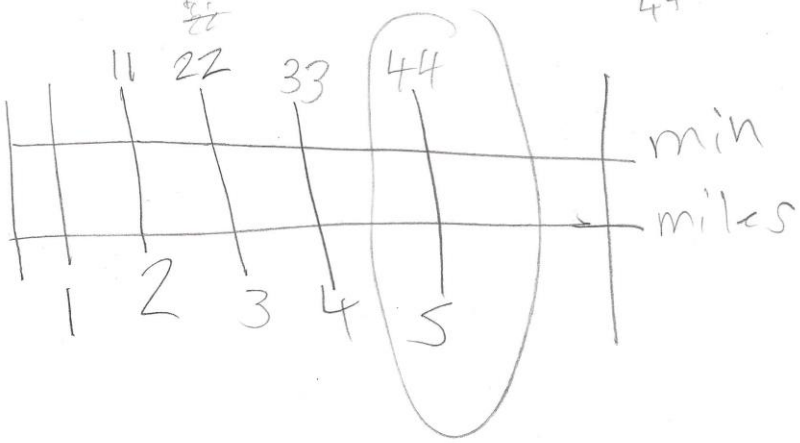


Checkpoint Work “Developing”



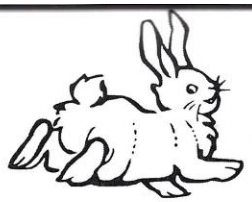
Robby Rabbit is training for a race. He can hop 2 miles in 11 minutes. If he hops at the same rate, how long will it take him to hop 5 miles?

Use a visual representation to help you solve the problem.



It will take 44 min to hop 5 miles

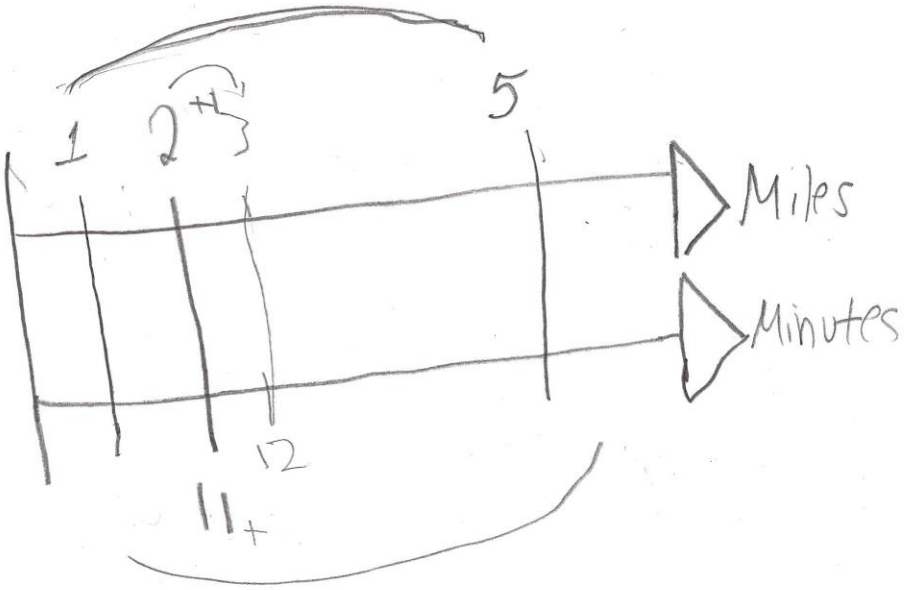
Checkpoint Work "Getting Started"



Robby Rabbit is training for a race. He can hop 2 miles in 11 minutes. If he hops at the same rate, how long will it take him to hop 5 miles?

Use a visual representation to help you solve the problem.

$2\overline{)11}$ 5, -10, 1



55
x 5

<table>
<tr><th colspan="3">Research and review of standard</th></tr>
<tr><th colspan="2">Content Standard(s):</th><th>Standard(s) for Mathematical Practice:</th></tr>
<tr><td colspan="2">6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?</td><td>MP.1 – Make sense of problems and persevere in solving them.
○ Understand the meaning of the problem and look for entry points to its solution
○ Check problems to answers and ask, “Does this make sense?”
MP.6 – Attend to Precision
○ Calculate efficiently and efficiently, expressing numerical answers with a degree of precision.</td></tr>
<tr><th colspan="2">Smarter Balanced Claim</th><th>Smarter Balanced Item</th></tr>
<tr><td colspan="2">Primary Claim: Claim 2: Problem Solving

Students can solve a range of complex, well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

Secondary Claim(s): Claim 1: Concepts and Procedures
Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.</td><td>Alia wants to buy pizza for a party.
40 to 50 people will be coming to the party.
A large pizza from Paolo’s Pizza Place serves 3 to 4 people.
Each large pizza from Paolo’s Pizza Place costs $11.50.

Part A
Alia wants to buy enough pizza so that people will not be hungry, and wants to have the least amount of pizza left over.
How many large pizzas should Alia buy?

Part B
If Alia buys the number of large pizzas that you determined in
Part A, how much money will she spend on pizza?</td></tr>
<tr><td>CPR Pre-Requisites
(Conceptual Understanding, Procedural Skills, and Representations)

Look at the Progressions documents, Learning Trajectories, LZ lesson library, unpacked standards documents from states, NCTM Essential Understandings Series, NCTM articles, and other professional resources. You’ll find links to great resources on your PLC Platform.</td><td colspan="2">Conceptual Understanding and Knowledge
• Understand that two equivalent fractions are the same size.
• Understand that a ratio is a comparison of quantities by division
• Understand that finding equivalent ratios involves a multiplicative relationship.
• Understand the language that shows a representation of quantities using a ratio

Procedural Skills
• Generate factors and multiples
• Multiply and divide rational numbers
• Recognize equivalent fractions
• Generate equivalent fractions
• Use prior knowledge of equivalent fractions to scale ratios up to down to find equivalent ratios

Representational</td></tr>
</table>

	• Representations should allow students to organize the information in problems and help reveal proportional relationships (equivalent ratio tables, double number lines). **Social knowledge** • Know the language that represents a proportional relationship: i.e. "for every 5 triangles there are 3 circles", "if a car can go 30 miles on 2 gallons of gas how many miles can it go on 6 gallons of gas", "the car traveled 55 miles per hour", "Each package serves 2".

Standards Progression		
Grade(s) below	**Target grade**	**Grade(s) above**
4.NF.A.1 Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **5.NF.5** Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a*/*b* = (*n*×*a*)/(*n*×*b*) to the effect of multiplying *a*/*b* by 1.	**6.RP.3.** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.	**7.RP.A.2B** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. **7.RP.2.C** Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn*. **7.RP.A.7** Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Common Misconceptions/Roadblocks

What characteristics of this problem may confuse students?

- Some students may not recognize that this is a multi-step task and may just multiply the price of the pizza by the number of people or by the number of servings per pizza
- Some students may be confused because 2 values are given for the number of guests (40 – 50), and the number of servings per pizza (3 – 4)
- Some students may not recognize that this problem can be solved using ratio reasoning and may just solve the problem by multiplying the number of guests by the number of servings.
- Students may not recognize that the language in the problem, “A large pizza from Paolo’s Pizza Place serves 3 to 4 people. Each large pizza from Paolo’s Pizza Place costs $11.50”, shows a unit rate.

What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?

- Students may struggle with fluency involving multiplication and division when the numbers don’t come out “evenly”.
- Students may not recognize that the language in a problem shows a proportional relationship.
- Students may not recognize that statements such as “ 55 miles per hour” or “100 calories per serving” represent unit rates because the “1” is implied.
- Students may provide an estimate that is too low or too high, showing that they did not account for the different measures of guests and servings

What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?

- If students need to compare numbers such as 7 and 4, they may default to subtraction because it is “easier” and results in a whole number answer.
- Students have used additive reasoning in earlier grades to solve problems and generate patterns and they may find it difficult to transition to multiplicative reasoning
- Students using additive reasoning may add the same number to both parts of the ratio instead of multiplying both parts of the ratio by the same number
- Some students my not understand the relationship between pizza and the number of servings, thinking of pizza only in terms of slices