

Mathematics Instructional Cycle Guide

Evaluating Expressions (6.EE.A.1)

Created by Steve Gionfriddo, 2014 Connecticut Dream Team teacher

CT CORE STANDARDS

This Instructional Cycle Guide relates to the following Standards for Mathematical Content in the CT Core Standards for Mathematics:

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.A.1 Write and evaluate numerical expressions involving whole-number exponents.

This Instructional Cycle Guide also relates to the following Standards for Mathematical Practice in the CT Core Standards for Mathematics:

SMP.2 Reason abstractly and quantitatively.

- Analyze givens, constraints, relationships, and goals.
- Explain correspondences between equations, verbal descriptions, tables, and graphs.
- Ask themselves, "Does this make sense?"

SMP.7 Look for and make use of structure.

- Look closely to discern a pattern or structure.
- See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings (page 2)
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint (pages 3-6)
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed (pages 7-11)
- > Supporting lesson materials (pages 12-30)
- Precursory research and review of standard 6.EE.A.1 and assessment items that illustrate the standard (pages 31-32)

HOW TO USE THIS DOCUMENT

- 1) Before the lesson, administer the **Checkpoint 1** <u>Mathematical Checkpoint</u> individually to students to elicit evidence of student understanding.
- 2) Analyze and interpret the student work using the Student Response Guide
- 3) Use the next steps or *follow-up lesson plan* to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
- 4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

MATERIALS REQUIRED

- · Classroom display or interactive board
- Student response boards or notebooks
- Appropriate writing tools (pencils, dry-erase markers, etc.)
- Six-sided number cube (dice) or single-digit number spinner
- Print-ready worksheets from this packet (pages 12-30)

TIME NEEDED

Checkpoint 1 administration: 5-10 minutes Follow-Up Lesson Plan: 45-60 minutes

Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

Step 1: Elicit Evidence of Student Understanding			
Mathematical Checkpoint			
Question(s)	Purpose		
Evaluate each expression and circle all of the expressions that are equivalent. A. $(3 \div 3 + 1)^4$ B. $(3 + 2 \div 1)^4$	CT Core Standard:	6.EE.A.1 : Write and evaluate numerical expressions involving whole-number exponents.	
C. $(2 \div 1)^4$ D. $(1 + 1)^4$ E. None of the expressions are equivalent. Describe how you determined your choices by explaining your mathematical reasoning.	Target question addressed by this checkpoint:	To what extent do students understand how to evaluate numerical expressions involving wholenumber exponents?	

Step 2: Analyze and Interpret Student Work			
Student Response Guide			
Got It	Developing	Getting Started	
1. Evaluate each expression and circle all of the expressions that are equivalent. (A) (3+3+1)^4 = 16 (3+2+1)^4 = 625 (C) (2+1)^4 = 16 (C) (1+1)^4 = 16 (C) (1+1)^4 = 16 (C) (2+1)^4 = 16 (C)	1. Evaluate each expression and circle all of the expressions that are equivalent. (A) (3+3+1) ⁴ (B) (3+2+1) ⁴ (C) (2+1) ⁷ (C) (2+1) ⁷ (E) None of the expressions are equivalent. (A) 3 (3 + 3 + 1) ⁴ (C) (2+1) ⁷ (E) None of the expressions are equivalent. (B) 3 (3+2+1) ⁴ (C) (2+1) ⁷ (C)	1. Evaluate each expression and circle all of the expressions that are equivalent. A. (3+3+1) ⁴ B. (3+2+1) ⁴ C. (2+1) ⁴ D. (1+1) ⁴ E) None of the expressions are equivalent. 2. Describe how you determined your choices by explaining your mathematical reasoning. T. choose (None of the expressions are equivalent E.) because the side number which is number 4 problemmean that's what it has to equal to AB Cland D don't equal to the side number.	

Getting Started Student Response Example Indicators Student response may indicate a misconception about exponents 1. Evaluate each expression and circle all of the expressions that are equivalent. A. $(3 \div 3 + 1)^4$ Student response may indicate a misunderstanding about procedural B. $(3 + 2 \div 1)^4$ fluency for simplifying expressions C. $(2 \div 1)^4$ D. $(1+1)^4$ (E) None of the expressions are equivalent. Student response may indicate a deficiency with some mathematical language 2. Describe how you determined your choices by explaining your mathematical reasoning. In the Moment Questions/Prompts Closing the Loop (Interventions/Extensions) Q: What do you know about the relationship between a base and an Students may benefit from order of operation instruction. Students may benefit from modeling the steps needed to perform this task. Students may benefit from exponent? Q: What do you know about powers of 10? using lined paper or graph paper to organize their simplifying. Students may **Q**: What do you know about order of operations? benefit from highlighting each step or having each step highlighted for them by a **Q**: What operation do you perform first inside the parentheses? teacher. Provide students with question/prompt answering frames to support Q: What steps would you perform to simplify an expression with parentheses? their explanations. **Q**: What is the value for the steps of the expression that are inside the parentheses? http://learnzillion.com/lessons/463-evaluate-a-numerical-expression-using-order-Q: How does the idea of repeated multiplication (exponents) make you look at of-operations this problem differently? http://learnzillion.com/lessons/460-write-numerical-expressions-involvingwholenumber-exponents

wholenumber-exponents

http://learnzillion.com/lessons/461-evaluate-numerical-expressions-by-using-

Developing Student Response Example Indicators Student response may indicate some misconceptions about equivalent 1. Evaluate each expression and circle all of the expressions that are equivalent. expressions $\begin{array}{c} \text{A.} \\ \text{B.} & (3 \div 3 + 1)^4 \\ \text{B.} & (3 + 2 \div 1)^4 \end{array}$ Student response may indicate some misconceptions about exponents C. $(2 \div 1)^4$ (D.) $(1+1)^4$ Student response may indicate some misunderstandings about E. None of the expressions are equivalent. procedural fluency for simplifying expressions Student response may indicate some difficulty with explaining conceptual and procedural understanding Student response may indicate some deficiency with some mathematical language 2. Describe how you determined your choices by explaining your mathematical reasoning. In the Moment Questions/Prompts Closing the Loop (Interventions/Extensions) Q: How did you determine whether or not the expressions were equivalent? Students may benefit from exponent/powers instruction. Students may benefit **P**: Describe what "equivalent expressions" means. from modeling the repeated multiplication procedure for this task. Students may **Q**: How do you know what to do first? benefit from using lined paper or graph paper to organize their simplifying. P: Tell me about exponents. Provide students with question/prompt answering frames to support their Q: What do you remember about the relationship between a base and an explanations. Q: How does the idea of repeated multiplication (exponents) make you look at http://learnzillion.com/lessons/463-evaluate-a-numerical-expression-using-orderthis problem differently? of-operations http://learnzillion.com/lessons/460-write-numerical-expressions-involvingwholenumber-exponents

wholenumber-exponents

http://learnzillion.com/lessons/461-evaluate-numerical-expressions-by-using-

Got It			
Student Response Example	Indicators		
1. Evaluate each expression and circle all of the expressions that are equivalent. (a) (3+3+1)*=16 (b) (3+2+1)*=605 (c) (2+1)*=16 (c) (1+1)*-16 (c) (1+1)*-16 (c) (3+5×5×5×5=625 (c) 2×2×2×16 (d) (3+5×5×5×6=625 (e) 2×2×2×16 (e) (3+5×5×6=625 (e) (3+5×5×6=625 (e) (3+5×6×6=625 (e)	 Student response may indicate a minor misconception about order of operation procedures Student response may indicate correct simplification, however, the response needs some procedural understanding evidence Student response may indicate correct understanding that the exponent means repeated multiplication 		
In the Moment Questions/Prompts	Closing the Loop (Interventions/Extensions)		
P: Tell me how you found the numbers where you used repeated multiplication.Q: What are the parts of a power? Did you use any of those or could you have used them?	Remind students to write all of their procedural thinking as they complete this task. Provide expressions that use the fraction line as division (ex. $\frac{4+5}{2+1}$) and challenge students to rewrite the expression a different way and simplify (ex. (4 + 5) \div (2 + 1), simplifies to a value of 3).		
P : Using the same format – operations inside parentheses raised to the fourth power – write another equivalent expression.	http://learnzillion.com/lessons/463-evaluate-a-numerical-expression-using-order-of-operations		

Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction			
Lesson Objective:	Students will be able to arrange numbers, symbols, and a whole-number exponent to		
	represent an expression that simplifies to a designated value.		
Content Standard(s):	6.EE.A.1 Write and evaluate numerical expressions involving whole-number exponents.		
Targeted Practice	SMP.2 Reason abstractly and	d quantitatively.	
Standard :	 Analyze givens, constr 	raints, relationships, and goals.	
	I	ces between equations, verbal descriptions, tables, and	
	graphs.		
	 Ask themselves, "Does 		
	SMP.3 Construct viable arguments and critique the reasoning of others.		
	Justify their conclusions, communicate them to others, and respond to the arguments of others.		
	Distinguish correct logic or reasoning from that which is flawed		
	Listen or read the arguments of others, decide whether they make sense, and ask		
	useful questions.		
	SMP.7 Look for and make use of structure.		
	Look closely to discern a pattern or structure.		
	See complicated things, such as some algebraic expressions, as single objects or		
	as being composed of several objects.		
Mathematical Goals		Success Criteria	
· · · · · · · · · · · · · · · · · · ·		 Arrange numbers, symbols, and a whole-number exponent to represent an expression that simplifies 	

Launch (Probe and Build Background Knowledge)

simplifying expressions

Understand use of exponents as it relates to

Purpose: Assess and activate background knowledge about expressions and order of operations.

Provide students with response boards or notebooks. Project/write the following information. Instruct students to work with a partner to analyze the information and respond to the questions.

to a designated value

Explain mathematical reasoning

Write an expression that simplifies to a designated

Analyze the following expression and information:

 $15 - 3 \cdot 4 + 2$

Josh says the value is 1.

Hillary says the value is 50.

Mariliz says the value is 5.

Trevor says the value is 72.

Turn and talk with a partner.

Can you determine how Josh, Hillary, Mariliz, and Trevor simplified the expression?

Who do you think is correct?

(Note for teacher – Mariliz is correct. 3 • 4 is 12; 15 - 12 is 3; 3 + 2 is 5)

Instructional Task

Purpose: Introduce the index card task and provide students time to reason and problem-solve.

Engage (Setting Up the Task)

1. Introduce the task by projecting or writing the following numbers and symbols.

 $5 \ 5 \ 5 \ + \div ^{2} \ ()$

- 2. Facilitate discussion about the numbers and symbols using the following prompts/guestions:
 - How might these numbers and symbols work together to find a value?
 - What does the smaller-font, raised number two suggest?
 - What mathematical question could we ask about these numbers and symbols?
- 3. Project or write the same numbers and symbols with the prompt. Instruct students to use two minutes to write how they may approach solving the prompt using words on their response boards or notebooks. Instruct students NOT to solve or attempt to solve the problem. Elicit the process from students.

Write an expression that is equivalent to 36 using each of the following numbers and symbols once in the expression.

 $5 5 5 + \div ^2 ()$

- 4. Facilitate a pair-share to have students share their thinking with a partner. Allow for 1-2 minutes of discussion, ensuring that both partners shared their approach.
- 5. Explain that students will now work on the prompt with their math group. Distribute pre-made 3x5 cards or index cards with each number and symbol separately printed on them. You should have eight cards in each pack. Provide copies of the task to each student and specify the time students will have to work on task.

Explore (Solving the Task)

6. Provide time for students to work on the task in groups. Circulate to observe, question, and note students who are strategic candidates to share out responses. Possible questions/prompts to engage students in task:

Focusing Questions

- What information do you know that can help you make sense of this task?
- What does each part of the expression tell you?
- What numbers and symbols can you group to represent a value that you may be able to use?
- What is the best way to use the parentheses in this task?
- What does the exponent tell you to do?
- What is the best way to use the exponent in this task?

Probing Questions

- Why did you decide to arrange/write the addition symbol first and the division symbol second inside the parentheses?
- Why did you decide to arrange/write the division symbol first and the addition symbol second inside the parentheses?
- Do you always add first from left to right when the addition symbol appears first in the expression?
- Can you explain more about how you used the exponent?
- How does your arrangement/expression relate to order of operation?

Advancing Questions

- Your solution, $(5 \div 5 + 5)^2$, is a correct response. How could you change this expression so that a different arrangement is also equivalent to 36?
- Why are $(5 \div 5 + 5)^2$ and $(5 + 5 \div 5)^2$ solutions to this task?
- Is this statement true or false: 5 + 25 ÷ 5 ≠ 60 ÷ 12 + 5? Explain your mathematical reasoning.
- Is this statement true or false: $3^2 \cdot (3^2 + 4 \cdot 5) = (2 \cdot 10^2) + (1 + 2^2 \cdot 15)$? Explain your mathematical reasoning.

Elaborate (Discuss Task and Related Mathematical Concepts)

- 7. Gather the attention of the class for a whole-group discussion about the task. Project or write the following questions for students to consider as others share their work.
 - How is this approach the same as the approach we used?
 - How is this approach different than the approach we used?
 - Does this approach make sense to me?
 - What follow-up questions do I have about this approach?
 - Do we agree or disagree with this solution? Why?
 - Did we discover another solution to this task? How?
- 8. Facilitate a whole-group discussion. Allow students to share their work and reasoning. Use volunteers with the most common approach used by students in the class to share first, allowing access to the approach and bringing out common misunderstandings. Select a more novel approach to be shared second and third. Some possible questions to help facilitate discussion and bring out important mathematical ideas include:
 - Why did you decide to arrange/write the addition symbol first and the division symbol second inside the parentheses?
 - Why did you decide to arrange/write the division symbol first and the addition symbol second inside the parentheses?
 - Do you always add first from left to right when the addition symbol appears first in the expression?
 - How does your arrangement/expression relate to order of operation?
 - Why are $(5 \div 5 + 5)^2$ and $(5 + 5 \div 5)^2$ solutions to this task?

Checking for Understanding

Purpose: Pose the following prompt to elicit evidence of students' understanding of order of operations and powers.

The following expression and solution are correct:

 $(5 + 21 \div 7)^2$ simplifies to 64

Explain why this number sentence is true.

Common Misunderstanding

Purpose: Address a common misunderstanding students often have about order of operations and using exponents.

Remind students that we worked with addition, division, and an exponent in a specific order during the task. Project or write the following problem.

Steve says that $4 + 4^2 \div 4$ simplifies to 6.

When he was told that his first solution was incorrect, Steve reviews the expression again and concludes that the solution should be 5.

What did Steve do wrong in his first attempt?

What can you suggest to Steve to help him correctly simplify these types of expressions in the future?

Is Steve's second attempt correct? If so, explain why. If not, explain why.

- Instruct students to use a minute or two to consider how Steve arrived at his solutions.
- Allow students to Turn & Talk with a partner to discuss how they think Steve arrived at his solutions.
- Designate one side of the room correct and the other side incorrect. Instruct students to move to the side of the room that they feel answers the second question (Is Steve's second attempt correct?).
- Facilitate discussion to share mathematical reasoning of students from each side.
- Can we offer Steve some guided questions to help clarify his misunderstandings?
- How does Steve's understanding of exponents affect his conclusion?
- How does Steve's understanding of order of operations affect his conclusion?
- How could we help Steve understand why 4 + 4² ÷ 4 simplifies to a value of 8?

Checking for Understanding

Purpose: Pose the following question as an exit card to elicit evidence of students' understanding of using numbers, symbols, and a whole-number exponent to represent an expression that simplifies to a designated value.

Jalla and Becca are making an expression puzzle.

They have the following numbers and symbols on separate puzzle pieces:

7 7 7 - ÷

They need to create a piece of the puzzle to represent the solution, but they arrived at different conclusions.

Jalla claims, " $7^2 - 7 \div 7$ simplifies to the value 48." Becca claims, " $7^2 - 7 \div 7$ simplifies to the value 6."

Do you agree or disagree with Jalla and Becca? Support your claim by explaining your mathematical reasoning.

Closure

Purpose: Provide students an opportunity to self-assess their learning related to the success criteria by projecting, writing, or providing the question below to complete.

Think about your learning. Circle the number you feel best matches your level of success with each item.

1. I can use order of operations to simplify expressions.

 Not at all
 Absolutely

 1
 2
 3
 4
 5

2. I can use exponents as they relate to simplifying expressions.

 Not at all
 Absolutely

 1
 2
 3
 4
 5

3. I can arrange numbers, symbols, and a whole-number exponent to represent an expression that simplifies to a designated value.

 Not at all
 Absolutely

 1
 2
 3
 4
 5

4. I can write an expression that simplifies to a designated value.

 Not at all
 Absolutely

 1
 2
 3
 4
 5

5. I can explain my mathematical reasoning in detail.

 Not at all
 Absolutely

 1
 2
 3
 4
 5

After this lesson, I feel Like I need more time learning ...

Extension Task

Purpose: Provide an extension task for those students who are ready to deepen their understanding of evaluating and writing expressions. This extension task adds rigor and critical thinking skills by challenging students to use a top-down approach to determining a solution.

Determine components of an expression that are equivalent to a given value. When simplified, the expression needs to have the value 144.

Components should include: three numbers, two operation symbols, a whole-number exponent, and a pair of parenthesis.

----- and/or -----

Purpose: Provide an extension task for those students who are ready to deepen their understanding of evaluating and writing expressions. This extension task adds rigor and critical thinking skills by challenging students to grapple with evaluating multiple combinations in an effort to determine a solution.

"24"

Using a spinner or number cube, select four random single-digit numbers.

Then, formulate an expression with the four numbers and any operations you would need to use to write an expression that simplifies to the value 24.

You decide if an operation is used more than once or not used at all.

You may also use parentheses.

Note - There are some RARE occurrences when the four-number combination does not simplify to 24, but most combinations do.

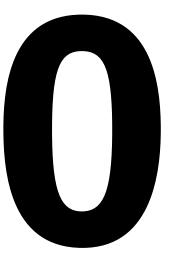
✓ Checkpoint 1

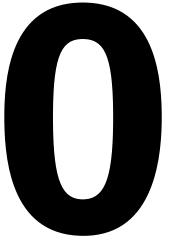
- 1. Evaluate each expression and circle all of the expressions that are equivalent.
 - A. $(3 \div 3 + 1)^4$
 - B. $(3 + 2 \div 1)^4$
 - C. $(2 \div 1)^4$
 - D. $(1 + 1)^4$
 - E. None of the expressions are equivalent.

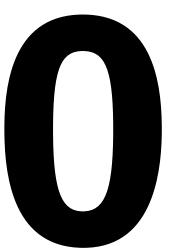
2. Describe how you determined your choices by explaining your mathematical reasoning.

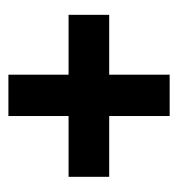


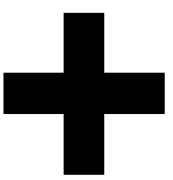


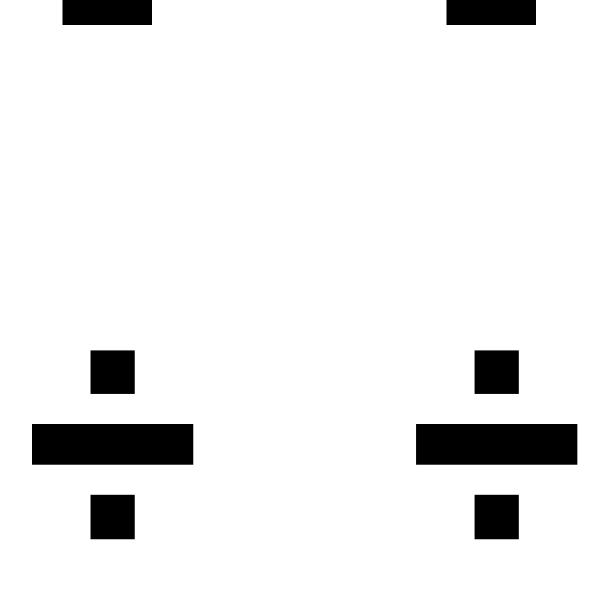


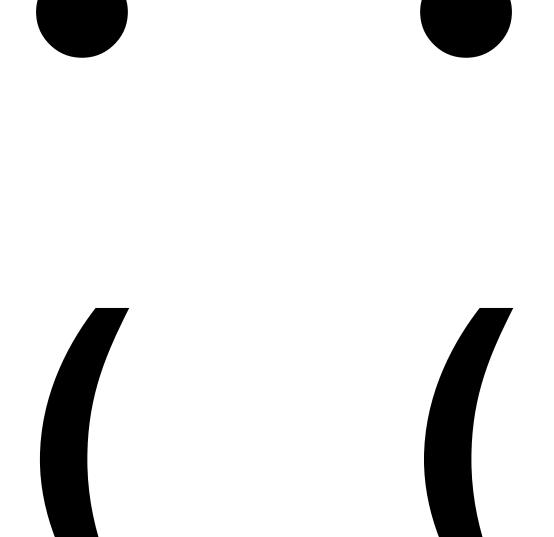


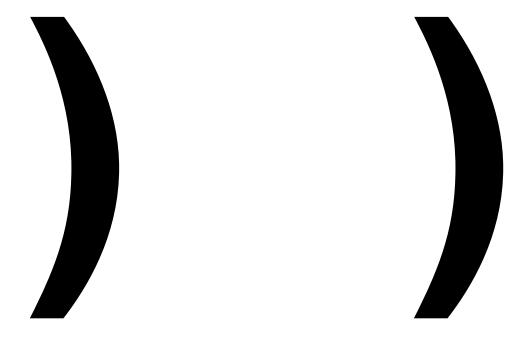












Name	Date	#
Checkpoint 2		
The following expression and solution a	re correct:	
$(5 + 21 \div 7)^2$ simplifies to 64		
Explain why this number sentence is true.		

Name	Date	#
? Common Misunderstanding		
Steve says that 4 + 4 ² ÷ 4 simplif When he was told that his first solution was incorrect, Steve concludes that the solution shou	e reviews the expression	on again and
What did Steve do wrong in his firs What can you suggest to Steve to help him correctly simplify the Is Steve's second attempt correct? If so, explain	ese types of expression	

Name	Date	#
Checkpoint 3		
Jalla and Becca are making an expression They have the following numbers and symbols on sepa		es:
7 7 7 - ÷	2	
They need to create a piece of the puzzle to represent the solution conclusions.	n, but they arrive	d at different
Jalla claims, "7² - 7 ÷ 7 simplifies to the value 48." Becca claims, "7² - 7 ÷ 7 simplifies to the value 6."		
Do you agree or disagree with Jalla and Becca? Support your claim by explaining your mathematical reasoning.		

Not at all

5. I can explain my mathematical reasoning in detail.

Absolutely

^ Extension Task 1

Determine components of an expression that are equivalent to a given value. When simplified, the expression needs to have the value 144.

Components should include: three numbers, two operation symbols, a whole-number exponent, and a pair of parenthesis.



"24"

Using a spinner or number cube, select four random single-digit numbers.

Then, formulate an expression with the four numbers and any operations you would need to use to write an expression that simplifies to the value 24.

You decide if an operation is used more than once or not used at all.

You may also use parentheses.

Note - There are some RARE occurrences when the four-number combination does not simplify to 24, but most combinations do.

Re	esearch and Review of Standard
Content Standard(s):	Standard(s) for Mathematical Practice:
6.EE.A.1 : Write and evaluate numerical expressions involving whole-number expone	 Analyze givens, constraints, relationships, and goals. Explain correspondences between equations, verbal descriptions, tables, and graphs. Ask themselves, "Does this make sense?" SMP 7: Look for and make use of structure. Mathematically proficient students: Look closely to discern a pattern or structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.
Smarter Balanced Claim	Smarter Balanced Item
Claim 1: Concepts and Procedures Students can explain and apply mathematic concepts and carry out mathematical proced with precision and fluency.	7 7 7 2 (exponent of 2) + ÷ ()
CPR Pre-Requisites (Conceptual Understanding, Procedural Skills, and Representations) Look at the Progressions documents, Learning Trajectories, LZ lesson library, unpacked standards documents from states, NCTM Essential Understandings Series, NCTM articles, and other professional resources. You'll find links to great resources on your PLC Platform.	 Conceptual Understanding and Knowledge Understand the proper order of operations to simply expressions Understand the exponent means multiply the base a set number of times Understand expressions are equivalent when they are identical once simplified Procedural Skills Add whole numbers Multiply whole numbers Divide whole numbers Use parenthesis to group operations Represent, analyze, and generalize equivalent expressions Simplify numeric expressions Representational
	 Write an expression using whole numbers and symbols Represent a whole number using a base and exponent Social knowledge
	Know the phrase Order of Operations means simplify a mathematical expression in a specific sequence

Stand	arde	Droo	ression
Stariu	aı uə	FIUG	C 331011

*Look at LearnZillion lessons and expert tutorials, the Progressions documents, learning trajectories, and the "Wiring Document" to help you with this section

Document" to help you with this section				
Grade(s) below	Target grade	Grade(s) above		
4.OA.B.4: Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.	6.EE.A.2b : Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of	8.EE.A.1 : Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.		
5.NBT.A.2: Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	two terms.			

Common Misconceptions/Roadblocks

What characteristics of this problem may confuse students?

Students may confuse using each number 7 once in their expression with using the number 7 one time overall

What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?

- Students may not understand the relationship between a base and an exponent
- Students may not understand how to simplify numerical expressions
- Students may not understand how to apply the Distributed Property of Multiplication

What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?

• Students may believe that they have to always simplify a numerical expression from left to right