

Mathematics Instructional Cycle Guide

Expressions and Exponents 6.EE.7

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CT CORE STANDARDS

This Instructional Cycle Guide relates to the following Standards for Mathematical Content in the CT Core Standards for Mathematics:

Insert the cluster heading and Content Standard(s) here.

<u>6.EE.7</u> Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers

This Instructional Cycle Guide also relates to the following Standards for Mathematical Practice in the CT Core Standards for Mathematics:

MP.2 Reason abstractly and quantitatively.

• Students will be reasoning through real-world problems to develop equations. They will have to provide reasons why their equation works based on key words and phrases they will identify.

MP.4 Model with mathematics.

Students will interpret a word problem and model the situation with an equation.

WHAT IS INCLUDED IN THIS DOCUMENT?

- > A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings (page 2)
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint (pages 3 6)
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed (pages 7-10)
- Supporting lesson materials (11-19)
- > Precursory research and review of standard 6.EE.7 and assessment items that illustrate the standard (20-23)

HOW TO USE THIS DOCUMENT

- 1. Before the lesson, administer the **Equations from word problems** <u>Mathematical Checkpoint</u> individually to students to elicit evidence of student understanding.
- 2. Analyze and interpret the student work using the **Student Response Guide**
- 3. Use the next steps or *follow-up lesson plan* to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint.
- 4. Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan.

MATERIALS REQUIRED

- Response boards, markers, eraser
- . Whiteboard, chalk board, or smart board
- Graphic Organizer
- Scavenger hunt activities

TIME NEEDED

Equations from word problems administration: 5-8 minutes

Follow-Up Lesson Plan: 2 instructional classes (45 minutes each)

Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class



	of student understanding al Checkpoint	
Question(s)		Purpose
Lilly was paid \$37.00 for a night of tutoring. She went to a coffee shop and bought a coffee for \$4.75. Write <i>two equations</i> that represent how much money, x, Lilly has remaining. Use a variable in your equations.	CT Core Standard:	6.EE.7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers
Explain how each of your equations represents the problem.	Target question addressed by this checkpoint:	Do students understand how to use a variable to represent an unknown value in the context of a real world problem?



	Step 2: Analyze and Interpret Student Work	
Got It	Student Response Guide Developing	Getting Started
Lilly was paid \$37.00 for a night of tutoring. She went to a coffee shop and bought a coffee for \$4.75. Write two equations that represent how much money, x, Lilly has remaining. Use a variable in your equations.	Lilly was paid \$37.00 for a night of tutoring. She went to a coffee shop and bought a coffee for \$4.75. Write two equations that represent how much money, x, Lilly has remaining. Use a variable in your equations. 37.00 4.35 33.25	Lilly was paid \$37.00 for a night of tutoring. She went to a coffee shop and bought a coffee for \$4.75. Write <i>two equations</i> that represent how much money, x, Lilly has remaining. Use a variable in your equations. \$37x00 -\$4.75 , \$37.00 - 4x.75
The first equation represents the problem because x + \$4.75 =\$37.00, so to get the amount of money Lily has remaining. You would have to subtract 137 and \$4.75 and that would equal x. The 2nd equation represents it because when you subtract 137 and x and get 44.75, you'll know that x is the amount of money will has remaining.	Explain how each of your equations represents the problem. Lilly has \$37.00. She spent \$4.75. Now She has \$33.25, that is what represents the dollar, x.	Explain how each of your equations represents the problem. I chose these 2 equations because they have the variable x in it and also the question is saying, so you have to subtract.



Getti	ng Started
Student Response Example	Indicators
Lilly was paid \$37.00 for a night of tutoring. She went to a coffee shop and bought a coffee for \$4.75. Write <i>two equations</i> that represent how much money, x, Lilly has remaining. Use a variable in your equations. \$37.00 - \$4.75 \$37.00 - 4x.75	 Student may not understand that a variable is used to represent an unknown value. Student may not write two equations. Student may not place the variable in the right location. Students' explanation may not explain why their equation represented the problem.
Explain how each of your equations represents the problem.	
I chose these 2 equations because they have the variable x in it and also the question is saying, so you have to subtract.	
In the Moment Questions/Prompts	Closing the Loop (Interventions/Extensions)
P: Tell me about your equations. Q: Explain how you know what location to put your variable. Q: Describe which operations you could use to determine the amount of money is remaining.	Provide students with a number line numbered 0 to 40. Have students locate the two given numbers. Students may be able to see that the unknown is the distance between the two numbers. They may also see that they can get there by adding or subtracting.
Q: What can a variable be used for? What is the unknown in this problem?	http://learnzillion.com/lessons/2846-understand-how-variables-are-used
Q: Does a description of what you wrote actually explain how your equations represent the problems?	http://learnzillion.com/lessons/2871-determine-whether-an-algebraic-expression-matches-a-scenario
	http://learnzillion.com/lessons/2735-write-and-solve-addition-equations-using-a-bar-model



De	veloping
Student Response Example	Indicators
Lilly was paid \$37.00 for a night of tutoring. She went to a coffee shop and bought a coffee for \$4.75. Write <i>two equations</i> that represent how much money, x, Lilly has remaining. Use a variable in your equations. 37.00 4.35	 Student may know how to set up only one of the equations. Student may have difficulty with subtraction and regrouping. Student may not use the variable appropriately in the equation. Student wrote an appropriate explanation for the equation provided. Student may write an appropriate explanation for the equation provided.
Explain how each of your equations represents the problem.	
Lilly has \$37.00. She spent \$4.75. Now She has \$33.25, that is what represents the dollar, x.	
In the Moment Questions/Prompts	Closing the Loop (Interventions/Extensions)
Q: What are the key parts that make up an equation?	Provide students with 37 blocks. Have students practice taking away 4 blocks.
P: Look at your work, did you solve the problem?	Then discuss what happens if you give students money. Provide students with 8 quarters. Start with \$2.00 and take away \$0.75. Have student practice multiple
Q: What is the purpose of having a variable in an equation?P: Did you use it in that way?	subtractions and determine how to "regroup."
Q: How could you have used regrouping in your answer?	http://learnzillion.com/lessons/2735-write-and-solve-addition-equations-using-a-bar-model
	http://learnzillion.com/lessons/2882-write-and-solve-subtraction-equations-using-a-bar-model



	Got it
Student Response Example	Indicators
Lilly was paid \$37.00 for a night of tutoring. She went to a coffee shop and bought a coffee for \$4.75. Write two equations that represent how much money, x, Lilly has remaining. Use a variable in your equations.	 Student may provide two different equations, one with addition and one with subtraction. Student may have used the variable in both equations. Student may have written an explanation that described what each of the equations represented.
Explain how each of your equations represents the problem.	
The first equation represents the problem because x + \$4.75 =37.00, so to get the amount of money Lily has remaining. You would have to subtract 137 and \$4.75 and that would equal x. The 2nd equation represents it because when you subtract 137 and x and get 44.75, you'll know that x is the amount of money will has remaining.	
In the Moment Questions/Prompts	Closing the Loop (Interventions/Extensions)
P: Tell me about your two equations. Q: How could you prove you that your equations will work?	Have students write a tutoring/coffee scenario for the equation: $5(37) = x + 3(4.75)$.
Q: Is there another equivalent equation you would write to represent the scenario? Q: How would the equation change if she bought "n" number of coffees for her and her family?	Student can then work with multiplication and division word problems. Attempt to find multiple equations for each as well. http://learnzillion.com/lessons/3548-relate-variables-using-an-equation



	Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction
Lesson Objective:	Develop and solve equations from real-world problems.
Content Standard(s):	6.EE.7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.
Targeted Practice Standard	 MP.2 Reason abstractly and quantitatively. Students will be reasoning through real-world problems to develop equations. They will have to provide reasons why their equation works based on key words and phrases they will identify. MP.4 Model with mathematics. Students will interpret a word problem and model the situation with an equation.
Mathematical Goals	Success Criteria
 Understand how to determine that occur in a real-wor 	 Write equations using addition or multiplication to represent real world problems. Determine a solution for the problem. Analyze a word problem to pick out key information.

Launch (Probe and Build Background Knowledge)

Purpose: Assess and get students thinking in terms of writing equations.

Provide students with response boards and markers. Put the following two word problems up on the board. Instruct students to do the following with each problem:

- Create an equation for the problem using a variable, x.
- Determine a solution and label using the correct units for the problem.
- Discuss with a partner how you formed an equation and how you solved.
- ***Note that some students may create subtraction and division equations to represent the word-problems. Acknowledge that these are also correct but encourage students to find another form of representation that uses addition and multiplication.***
- 1) Christina has a certain number of markers. Javier has 20 markers. Combined, Christina and Javier have 400 markers. How many markers does Christina have? (20 + x = 400, x = 180)
- 2) There are 20 students in each classroom in a school. The school has a total of 400 students. How many classrooms are there at the school? (20x = 400; x = 20 classrooms)



Instructional Task

Purpose: Provide students with a graphic organizer of key words for analyzing a real-word problem and practice creating and solving equations.

Engage (Setting Up the Task)

- 1) Discuss with students how to analyze a real-word problem. Students will pick out key words and phrases and discuss what key words were in the warm up problems.
- 2) Provide students with a list of words that are pre-cut and placed in a bag or envelope. Also, give students a graphic organizer (see below) that has 4 sections of each operation symbol. Allow students time to place the key words in what they believe to be the correct operation symbol with a partner.
- 3) Review with students by turning the board into a larger scaled organizer. Have students come to the board and fill in the "board organizer." After each student comes to the board, discuss as a class if they agree with the placement. If the placement is incorrect, have the student explain why they placed it there and allow another student to explain where the correct placement is located.

Explore (Solving the Task)

- Introduce the Scavenger hunt activity.
 - The activities will be set up in stations (6).
 - Set up students in groups and explain how they should rotate through the activity based on classroom rules and expectations.
 - Provide students with the flow chart to use to guide them through the guestions (included at end).
- 5) During the activity, there will be one extra station where the teacher will be located. This station will be called the "Conversation Station." Here, the teacher will have a copy of each question or will show each on their computer. The teacher will review the stations that the groups have answered before coming to the station. Allow students time to re-read the word problem and then allow them to provide the answers they determined on their flow charts. This is the time where the teacher will discuss what key words are in the problems and what the equation will be. Note if groups have interesting or insightful feedback they can share in the whole class discussion. Also, provide students will helpful hints to continue through the stations.
- 6) Once each group has traveled to each station, bring the class back together as a group. Discuss each problem. Be sure to call on any groups that prepared unique or especially insightful answers. Go through the questions that caused the groups the most trouble.

Elaborate (Discuss Task and Related Mathematical Concepts)

7) Once the discussion is over, project the word problems on the board. Allow the students to now solve for the variables and determine an answer. Review this as a class.



Checking for Understanding

Purpose: Provide students with this questions that will check their understanding of writing equations from real-world problems.

A group of 7 friends went to the movies. They spent a total of \$42.00 on their tickets. How much did each ticket cost?

Molly believes the equation to be

Ben believes the equation to be

7 + x = 42

7x = 42

Who wrote the equation correctly: Molly, Ben, or neither?

Explain why:

Common Misunderstanding



Purpose: Address common misunderstandings students have when forming an equation. Word problems do not require that the variable is always isolated; it is often imbedded in the equation

Provide students with bar model.

James read some books during summer vacation.
 His sister, Avery read 7 books. Together they read
 19 books. How many did James read?

Let \boldsymbol{b} represent the number of books James read

An expression for the number of books James and his sister read is: b + 7

b	7
19	

$$b + 7 = 19$$

$$b = 12$$



Checking for Understanding, Exit Tick	Checking	for U	Inders	tanding.	, Exit	Ticke
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Purpose: To check that students who had a misunderstanding can now write an equation from a real-world problem.

Dariel gave his brother \$15.00. His brother now has \$36.00. How much money did Dariel's brother start with?

Let x represent the amount of money Dariel's brother started with.

Write an equation to represent the situation.

Closure

Purpose: Provide an opportunity for students to self-asses and reflect on their own learning with a 3 questions that will be handed out to students.	This is based on
a "My Learning Scale." Students can be anywhere from a 4, 3, 2, or 1.	

4 = I completely understand. I can even Teach It!

3 = I am getting it and I can show you.

2 = I am starting to get it with help, but could use some more practice.

1 = Even with help, I still don't get it.

1) I can read a real-world problem and understand what I am trying to solve for.

3

1 2

4

2) I can write an equation using a variable based on a real-world problem and explain what each part of the equation represents.

1

2

3) I can solve an equation for the variable.

2

4

4) Something I still need help with is _____

Extension Task

Purpose: Provide an extension task for students who are ready to extend their understanding of writing equations. The extension will allow students to develop their own real-world problems to then exchange with another student.

Please write two real-world problems that can be solved using an addition and a multiplication equation. Your problems must include an unknown that the reader will solve for using an equation. Use the key words that we used at the beginning of class to assist in using the correct mathematical language.

Once the students have created their word problems, they are to swap their problem to a classmate and solve the new one.



Check Point

Lilly was paid \$37.00 for a night of tutoring.

She went to a coffee shop and bought a coffee for \$4.75. Write *two equations* that represent how much money, x, Lilly has remaining. Use a variable in your equations.

Explain how each	of your equations represents
the problem.	



Operation Key Words

Addition	Subtraction
Multiplication	Division
	1



List of Operation Words:

Sum
Shared equally
Difference
Total
Times
Take away
Out of
In all
Each
Gave
Altogether
Per
Remain/remaining
More than
In all
Combined
Equally
Per
Fewer
Left
And
Together



Michael and Sara were given the following problem:

There are 25 soccer balls in a store. The total number of soccer balls and basketballs in the store is 325. Which equation can be used to find *x*, the number of basketballs in the store?

Michael chose the first equation and Sara chose the second.

Michael

$$25 + x = 325$$

Sara

$$25x = 325$$



Aleah and Tyler were given the following problem:

Cleo has a certain number of seashells. Pete has 25 seashells. Together Cleo and Pete have 325 seashells. Which equation can be used to find *x*, the total number of seashells that Cleo has?

Aleah chose the first equation and Tyler chose the second.

Aleah

25 + x = 325

Tyler

25x = 325



Jaden and Julia were given the following problem:

There are 25 cans of soup in a case. The manager of a grocery store needs to order 325 cans of soup. Which equation can be used to find *x*, the total number of cases the manager needs to order?

Jaden chose the first equation and Julia chose the second.

Jaden

$$25 + x = 325$$

Julia

$$25x = 325$$



Aidan and Rachel were given the following problem:

Marissa had 25 marbles in a bag. She gave some to her brother. Her brother now has 325 marbles. Which equation can be used to find *x*, the number of marbles that Marissa gave her brother?

Aidan chose the first equation and Rachel chose the second.

Aidan

$$25 + x = 325$$

Rachel

$$25x = 325$$



Kim and Joe were given the following problem:

The school auditorium can seat 325 students. In the auditorium there are 25 rows with the same number of seats in each row. Which equation can be used to find *x*, the number of seats in each row in the school auditorium?

Kim chose the first equation and Joe chose the second.

Kim

25 + x = 325

Joe

$$25x = 325$$



Flow Chart

		Who is correct?	Explain what the error in reasoning is for the other student.
Is EITHER student			
correct?		Explain: why he/she is correct.	
Yes – one is correct.	Yes		
res one is correct.	103		
		Explain error in reasoning for	Explain error in reasoning for student #2.
		student #1.	·
No –both are incorrect.			
both are moonest.	No		



Research and review of standard					
Content Standard(s):	Standard(s) for Mathematical Practice:				
Standard: 6.EE.7 . Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers	MP.2 Reason Abstractly and Quantitatively				
Smarter Balanced Claim	Smarter Balanced Item				
Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.	Read each of the following problem situations. Label each situation according to the equation that would answer the question. If neither equation works, select "Neither." The labels may be used more than one time. The school auditorium can seat 325 students. In the auditorium there are 25 rows with the same number of seats in each row. Which equation can be used to find x, the number of seats in each row in the school auditorium? There are 25 soccer balls in a store. The total number of soccer balls and basketballs in the store is 325. Which equation can be used to find x, the number of basketballs in the store? Marissa had 25 marbles in a bag. She gave some to her brother. Her brother now has 325 marbles. Which equation can be used to find x, the number of marbles that Marissa gave her brother? Neither There are 25 cans of soup in a case. The manager of a grocery store needs to order 325 cans of soup. Which equation can be used to find x, the total number of cases the manager needs to order? Cleo has a certain number of seashells. Pete has 25 seashells. Together Cleo and Pete have 325 seashells. Which equation can be used to find x, the total number of				



CPR Pre-Requisites

(Conceptual Understanding, Procedural Skills, and Representations)

Conceptual Understanding and Knowledge

- Understand equivalent expressions
- Understand equivalent equations
- Understand variables are particulars and unknowns
- Understand that variables have different meanings depending on context and purpose
- Understand using variables permits writing expressions whose values are not known or vary under different circumstances
- Understand that the solutions of an equation are the values of the variables that make the equation true when substituted back in as the variable.
- Understand that they can use the properties of operations to rewrite expressions in equivalent forms.
- Understand that a variable can be in any position of an equation.
- Understand that equations can represent real world or mathematical problem.
- Understand that the expressions on each side of the equal sign are equivalent.

Procedural Skills

- · Add, subtract, multiply, and divide numbers
- How to deconstruct a real-world problem

Representational

- Write algebraic equations from real world problems
- Use variables to represent unknown values in a world problem context.

Social knowledge

- Identify key vocabulary words based on operations.
- A coefficient should be in front of a variable
- If there is no symbol between a number and variable, it is multiplication. (Notation for multiplication)



Standards Progression *Look at LearnZillion lessons and expert tutorials, the Progressions documents, learning trajectories, and the "Wiring Document" to help you with this section						
Grade(s) below	Target grade	Grade(s) above				
CCSS.Math.Content.5.NF.A.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.) CCSS.Math.Content.5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. CCSS.Math.Content.5.NF.B.4.a Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.) CCSS.Math.Content.5.NF.B.4.b Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	CCSS.Math.Content.6.NS.A.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. CCSS.Math.Content.7.EE.B.4.a Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?				



Common Misconceptions/Roadblocks

What characteristics of this problem may confuse students?

- Picking out key words through content vocabulary
- The value "x" represents something different in each problem

What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?

- Students have difficult with solving equations with variables
- Students can not create algebraic equations from verbal models
- Students will assume if there is not a coefficient in front of a variable, there is not actually a number there. They do not see that y=1y
- Students do not know that 2w means 2 times w

What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?

- Students believe variables will mean the same thing for each problem
- Students may generalize equivalence and not see how an equation can be represented by an infinite amount of word problems or verbal models
- Students believe that each word problem only has one equation to represent it
- Students may write an expression vertically instead of horizontally