Mathematics Instructional Cycle Guide

Solving addition and subtraction fraction word problems (5.NF.A.2)

Created by Fallon Wagner, 2014 Connecticut Dream Team teacher

CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

Use equivalent fractions as a strategy to add and subtract fractions.

5.NF.A.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2*.

This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

MP2 Reason abstractly and quantitatively.

MP7 Look for and make use of structure.

WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings **page 2, 12**
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint **pages 3-8**
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed **pages 9-11**
- Supporting lesson materials **pages 12-20**
- Precursory research and review of standard **5.NF.A.2** and assessment items that illustrate the standard **pages 21-23**

HOW TO USE THIS DOCUMENT

1) Before the lesson, administer the **The Bakery** ***Mathematical Checkpoint*** individually to students to elicit evidence of student understanding.
2) Analyze and interpret the student work using the ***Student Response Guide***
3) Use the next steps or ***follow-up lesson plan*** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

MATERIALS REQUIRED

- **Copies of Mathematical Checkpoint and worksheets (see pages 12-20), fraction strips or circles (see page 20), scrap paper or student whiteboards, dominoes, number cards**

TIME NEEDED

The Bakery administration: **15 minutes**
Follow-Up Lesson Plan: **60 minutes**

Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

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Step 1: Elicit evidence of student understanding		
Mathematical Checkpoint		
Question(s)	**Purpose**	
A bakery started its day with $7\frac{3}{4}$ pounds of flour. After baking some cakes, there are now $3\frac{1}{3}$ pounds of flour left. How many pounds of flour were used to bake the cakes? A. Write an equation that could be used to solve the word problem above: _____ B. Show how you solved the equation you created in Part A: _____ pounds	**CT Core Standard:**	**5.NF.A.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2*.
	Target question addressed by this checkpoint:	Can students write an equation to represent a word problem? Do students know how to correctly solve an addition/subtraction word problem with mixed numbers that have unlike denominators?

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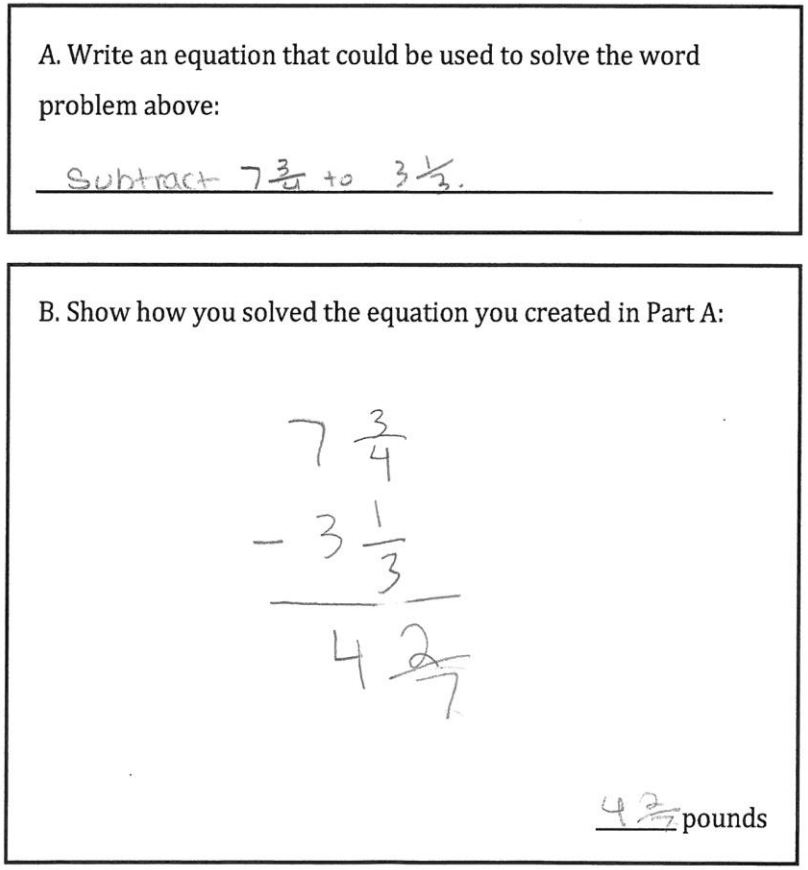
Step 2: Analyze and Interpret Student Work

Student Response Guide

Got It	Developing	Getting Started
A. Write an equation that could be used to solve the word problem above: $7\frac{3}{4} - 3\frac{1}{3} = f$	A. Write an equation that could be used to solve the word problem above: $7\frac{3}{4} - 3\frac{1}{3} = \square$	A. Write an equation that could be used to solve the word problem above: Subtract $7\frac{3}{4}$ to $3\frac{1}{3}$.
B. Show how you solved the equation you created in Part A: $7\frac{3}{4} = 7\frac{9}{12}$ $3\frac{1}{3} = 3\frac{4}{12}$ $4\frac{5}{12}$ $3\frac{3}{4} - 3\frac{1}{3} = \frac{9}{12}$ $\frac{1}{3} \times \frac{1}{4} = \frac{4}{12}$ $4\frac{5}{12}$ pounds	B. Show how you solved the equation you created in Part A: 4, 4, 8, (12) 3, 3, 6, 9, (12) $7\frac{3}{12} - 3\frac{1}{12} = 4\frac{2}{12}$ $4\frac{2}{12}$ pounds	B. Show how you solved the equation you created in Part A: $7\frac{3}{4} - 3\frac{1}{3} = 4\frac{2}{7}$ $4\frac{2}{7}$ pounds

<table>
<tr><th colspan="2">Getting Started</th></tr>
<tr><th>Student Response Example</th><th>Indicators</th></tr>
<tr><td>


</td><td>

Students might add instead of subtracting (not recognizing the context as involving subtraction).
Students may subtract and end up with an answer that is not correct because they have overgeneralized their work with like denominators and whole numbers.
Students will not use knowledge of equivalent fractions to create fractions with like denominators in order to perform the subtraction.
Students will not have the correct equation that represents the problem.

</td></tr>
<tr><th>In the Moment Questions/Prompts</th><th>Closing the Loop (Interventions/Extensions)</th></tr>
<tr><td>Q: Think about what is happening in the problem. How could you write an equation that represents this word problem?

Q: Can you tell me about the fractions you wrote? How do they connect to the problem?</td><td>http://learnzillion.com/lessons/2541-subtract-mixed-numbers-with-unlike-denominators-using-fraction-bars

http://learnzillion.com/lessons/121-add-and-subtract-fractions-with-unlike-denominators</td></tr>
</table>

Q: How do you add or subtract fractions with like denominators? How do you add or subtract fractions with unlike denominators? Is the process different? Q: If the student writes 4 2/1) What does 2/1 cup of flour mean? What would that look like? Can you draw a picture to show how you took away some of the flour to see what remained?	Activity: Using fraction strips or circles, students will practice solving simple subtraction problems, first with like denominators, and then with unlike, related denominators (example: 3/4 – 1/8)

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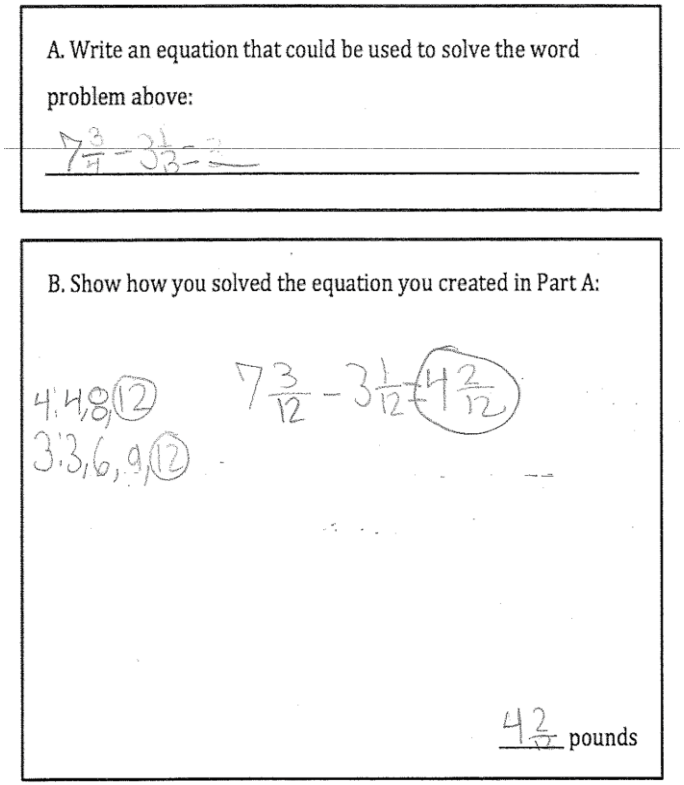
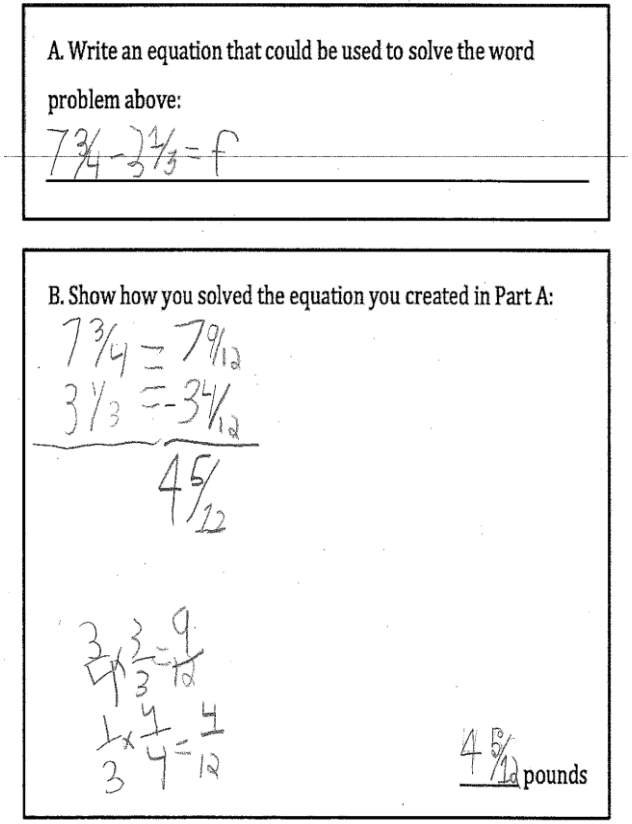
Developing	
Student Response Example	**Indicators**
	• Student recognized the context as a subtraction situation and attempts to subtract the mixed numbers involved. • Student includes an equation that represents the problem situation (with or without a variable for the unknown). • Students attempt to create an equivalent fraction to subtract, but make a procedural error.
In the Moment Questions/Prompts	**Closing the Loop (Interventions/Extensions)**
Q: How do we subtract fractions with unlike denominators? Q: How can equivalent fractions help us solve this problem? Q: What is a common multiple for 3 and 4? Q: How can we represent what the question is asking us to do using an equation? Q: Tell me about the equation you wrote.	http://learnzillion.com/lessons/2443-subtract-fractions-and-mixed-numbers-by-decomposing http://learnzillion.com/lessons/2624-subtract-fractions-with-unlike-denominators-by-creating-area-models http://learnzillion.com/lessons/989-solve-word-problems-using-number-lines-to-subtract-unlike-mixed-number-fractions

P: Provide students with fraction strips or circles and ask them to generate an equivalent fraction for 2/3 and 5/6 (unlike, related fractions). What do you notice about the equivalent fractions you created? How can this help us solve this problem?	

Got it	
Student Response Example	**Indicators**
	• Students create an equation (with a variable for the unknown) that accurately represents the problem. (examples: $7\frac{3}{4} - 3\frac{1}{3} = f$ or $3\frac{1}{3} + f = 7\frac{3}{4}$) • Students generate equivalent fractions for ¾ and 1/3 in order to solve the problem. • If students show how the equivalent fractions were generated, the mathematics is correct (both the numerator and denominator are multiplied by n/n)
In the Moment Questions/Prompts	**Closing the Loop (Interventions/Extensions)**
Q: How do you know your equation represents the problem? Is there another equation you could use that would also be correct? P: What if the amount of flour we started with was 7 1/4 pounds and we were left with 3 2/3 pounds at the end of the day. How could we figure out the amount of flour we used? How is this problem different from our original problem? Q: Are 9/12 and 4/12 the only equivalent fractions we could use to solve this problem? Is there a reason why they would be the best?	http://learnzillion.com/lessons/345-subtract-mixed-fractions-regrouping Provided with other mixed numbers (or even actual recipes), students could generate and solve their own word problems. Another challenge could be to ask students to create a problem that results in the same solution as this problem but uses different mixed numbers than those used here.

Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction	
Lesson Objective:	Students will create solve word problems involving addition and subtraction of mixed numbers.
Content Standard(s):	**5.NF.A.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem.
Targeted Practice Standard :	**MP2** Reason abstractly and quantitatively. **MP7** Look for and make use of structure.

Mathematical Goals	Success Criteria
• Understand a mixed number is comprised of a whole number and a fraction smaller than one • Fractions (and mixed numbers) can be added or subtracted (through joining or separating parts of the same whole).	• Students will choose the correct operation for solving a word problem with mixed numbers. • Students will use equivalent fractions as a strategy to add and subtract fractions. • Students will correctly add and subtract mixed numbers.

Launch (Probe and Build Background Knowledge)

Purpose: *Teacher will write the following two problems on the board and ask students to solve on scrap paper or whiteboards:*

1. 4 2/5 - 1 1/5= ___

2. 7 5/8 – 4 1/4 = ___

Teacher will ask students: How did you solve these problems? What was different between the first problem and second problem? What prior knowledge or strategies did you use to help you make sense of the problems?

Teacher will review adding or subtracting mixed numbers with unlike denominators by finding equivalent fractions as needed (if possible, let a student instruct the class in doing this). Teacher will introduce that today students will take their understanding of adding and subtracting mixed numbers and apply it to solving real-world problems.

Instructional Task

Purpose: *Students will create and correctly solve word problems with mixed numbers.*

Engage (Setting Up the Task)

- *Teacher will instruct students that today they will play a game that will help them practice solving addition and subtraction word problems that include mixed numbers.*
- *Teacher will allow each student to pick a domino out of a bag and a number card from a stack (index cards with single digit numbers 1-5 written on them, enough created by the teacher so that each student can pull a card).*
  ***Printable dominos can be found at this website: http://www.first-school.ws/theme/printables/dominoes-math.htm*

- *Teacher will demonstrate how to use the card and domino to create a mixed number. Example: If I pulled a card with a 2 and a domino with 4 dots on one side, and 3 dots on the other side, then my mixed number would be 2 and 3/4 (the index card represents the whole number and the domino is turned so that it displays a proper fraction with the smaller amount of dots on top and larger number of dots on the bottom).*
  ***Teacher can either remove doubles from the bag or leave them in and use this opportunity to discuss that the same numerator and denominator (same number of dots on each side of the domino) equals one whole that can be added to the whole number they already selected.*
- *Once students have their mixed numbers created they can write that down on a piece of paper, post-it note or mini-whiteboard. They will carry that around along with scrap paper, and the word problem stems during the game.*

Explore (Solving the Task)

- *Teacher will explain that the game is like Musical Chairs. He or she will play music for the class and students will walk around with their mixed number and scrap paper. When the music stops, they must partner with the student closest to them. Teacher will call out either "add" or "subtract." Together students will choose a word problem stem from their paper (see attachment) that fits the criteria (addition or subtraction) and plug their mixed numbers into the proper boxes to create a unique word problem. After identifying the operation needed to solve the problem, students will work together to solve the problem and use whatever knowledge they have to write out and solve the equation. When a pair thinks they have correctly solved their problem, they will raise their hand to have their answer checked by the teacher. Since the teacher will not be able to check each pair before the next round, students will continue recording their equations for each round on lined paper. This can later be turned in and checked to determine reasonableness of answers.*
  ***Note: In some instances, the pair might create an addition or subtraction problem that requires regrouping (such as 4 3/4 + 2 5/6). If regrouping was not taught prior to this lesson, students may require additional assistance in solving such problems.*
- *Rounds will continue with students creating and solving problems with other classmates (teacher repeats the process of playing the music, stopping, and calling out "addition" or "subtraction"), using as many word problem stems as they can. Teacher will monitor the pairs as they work to help clarify misconceptions or support mathematical thinking and conversation, ensuring accuracy.*
  ***Differentiation: Students who may need more assistance in order to take part in this lesson can select dominoes from a bag which includes smaller numbers (dominoes with numbers on both sides less than 4) resulting in more easily manipulated fractions. Additionally, those students can utilize a multiplication chart when creating equivalent fractions.*

Elaborate (Discuss Task and Related Mathematical Concepts)

After playing several rounds of the game, teacher will prompt students to reflect. He or she will ask "Think of a round where you and your partner created an especially challenging expression that needed to be solved. How did you solve that problem?"– Students can reflect on this in writing and/or orally.

Checking for Understanding

Purpose: *Teacher will monitor students as they work in pairs to complete the task. He or she will prompt students with such questions as:*

- *How did you determine if this was an addition or subtraction problem?*
- *Which fraction did you put into the first blank? Into the second blank? Does it matter?*

- *What are the rules for adding or subtracting mixed numbers? Do those rules change if the denominators are different?*

Common Misunderstanding

Purpose: *Students will solve the following problem (written on the board or handed out): "Sarah cut a piece of ribbon for a project that she was working on. It was 7 6/8 inches long. The ribbon was too long so she cut 2 1/4 inches off. Sarah says her ribbon is now 5 5/4 long. Is she correct? Why or why not?"*

Checking for Understanding

Purpose: *Teacher will share an expression that he or she noticed was not solved correctly by a pair and write that on the board (including the wrong answer). Teacher will ask students to comment on why and how the pair arrived at the answer.*
***When choosing a problem to use as the model for this selection, teacher should look for students who incorrectly interpreting a problem (example: they used addition to solve but it was a subtraction problem) and/or students who incorrectly performed the operation due to common misconceptions (such as adding both the numerators and denominators).*

If teacher is having difficulty selecting a problem that was solved incorrectly by a pair of students in his or her class, then use the following question:

Maria made 3 4/6 cups of lemonade for her lemonade stand last weekend. This weekend she made 2 1/3 cups of lemonade. How many more cups did she make last weekend than this weekend?

Answer: 5 5/6

Prompt students by asking: "Do you agree with this solution?" "What did the person solving this do wrong?" (misinterpreted this to an addition problem, didn't correctly generate an equivalent fraction for 1/3) "Why do you think he/she added?" "How can we fix the solution?"

Closure

Purpose: Students will complete the Post Lesson Reflection (see attachment). Teacher will use the information collected from this self-reflection to plan future instruction.

Extension Task

Students will complete the "Which Punch Is Best?" worksheet (attached). While the first problem asks students to use their knowledge of addition and subtraction of mixed numbers (the focus of the previous lesson), the second question asks students to extend their thinking and now apply this to pricing (money). Finally students must justify their decision in the third question.

A bakery started its day with $7\frac{3}{4}$ pounds of flour. After baking some cakes, there are now $3\frac{1}{3}$ pounds of flour left. How many pounds of flour were used to bake the cakes?

A. Write an equation that could be used to solve the word problem above:

_____

B. Show how you solved the equation you created in Part A:

_____ pounds

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Word Problem Stems

Directions: With your partner, plug your mixed numbers into the boxes below to create a word problem that makes sense. Write the equation to represent the problem, and then find the solution.

1. Sarah walked ☐ miles this week. Nancy walked ☐ this week. How many more miles did Nancy walk than Sarah?

2. Whiskers ate ☐ pounds of cat food last week. This week she ate ☐ pounds of cat food. How much cat food has Whiskers eaten so far this month?

3. To make punch for a party, Mom mixed ☐ cups of seltzer with ☐ cups of cranberry juice. How many cups of punch did she make?

4. Kareem collected ☐ pounds of candy when he went trick-or-treating on Halloween. His younger brother collected ☐ pounds of candy. If the boys combined their candy, how many pounds of candy would they have?

5. Derek was helping his dad build a deck on their house. His dad asked him to cut a piece of wood that was ☐ feet long to ☐ feet long. How long was the piece of wood that Derek cut off?

6. Antonio ran ☐ laps around the track at school. His goal was to run ☐ laps altogether. How many laps was he short of making his goal?

7. Maria made ☐ cups of lemonade for her lemonade stand last weekend. This weekend she made ☐ cups of lemonade. How many more cups did she make last weekend than this weekend?

8. A restaurant is serving fresh fruit salad. The recipe calls for ☐ pounds of strawberries and ☐ pounds of blueberries. How many pounds of berries are used in the recipe?

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Which Punch is the Best?

Taylor is in charge of making punch for the school picnic. She has two recipes to choose from.

Lemon-Raspberry Fizz

- $2\frac{1}{4}$ quarts ginger ale
- $1\frac{2}{3}$ quarts lemon sherbet
- $1\frac{1}{2}$ quarts lemonade
- $\frac{1}{2}$ quart raspberry juice

Orange-Lime Fizz

- $1\frac{2}{3}$ quarts club soda
- $2\frac{3}{4}$ quarts lime sherbet
- $1\frac{1}{3}$ quart orange juice

1. Which recipe makes more quarts of punch? _____
   How much more? _____ Show your thinking.

2. The price per package of the different ingredients is listed below.
   Which recipe would be cheapest to make? _____
   What is the difference in cost between the two recipes? _____

Ginger ale (1 quart) $1.50

Lemon sherbet ($1\frac{3}{4}$ quarts) $2.75

Lemonade (2 quarts) $1.58

Raspberry juice (2 quarts) $3.49

Club soda (1 quart) $1.79

Lime sherbet ($1\frac{3}{4}$ quarts) $3.25

Orange juice (2 quarts) $3.99

3. Lemon-Raspberry Fizz serves 24 people. Orange-Lime Fizz serves 20 people. If Taylor needs to serve 70 people at the party, which recipe would you recommend for her to use based on serving size and cost? Explain your reasoning.

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Post Lesson Reflection

Direction: Circle the face that best describes how you feel.

- ✓ I feel comfortable reading a word problem and deciding whether to use addition or subtraction to solve it.

Yes	Maybe	No
☺	😐	☹

- ✓ I can write an equation that shows how I solved a word problem.

Yes	Maybe	No
☺	😐	☹

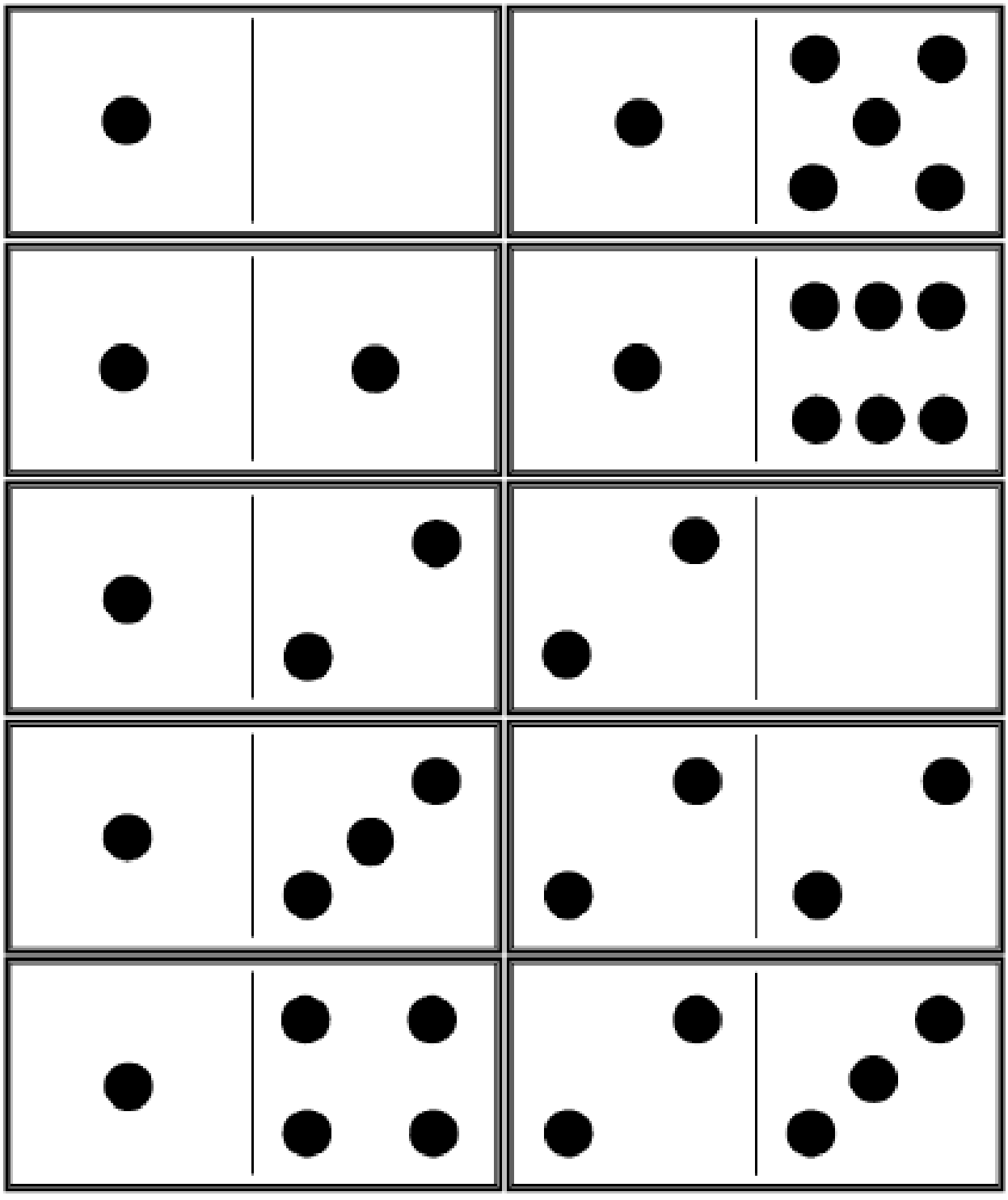
- ✓ I can use equivalent fractions as a strategy for adding and subtracting fractions.

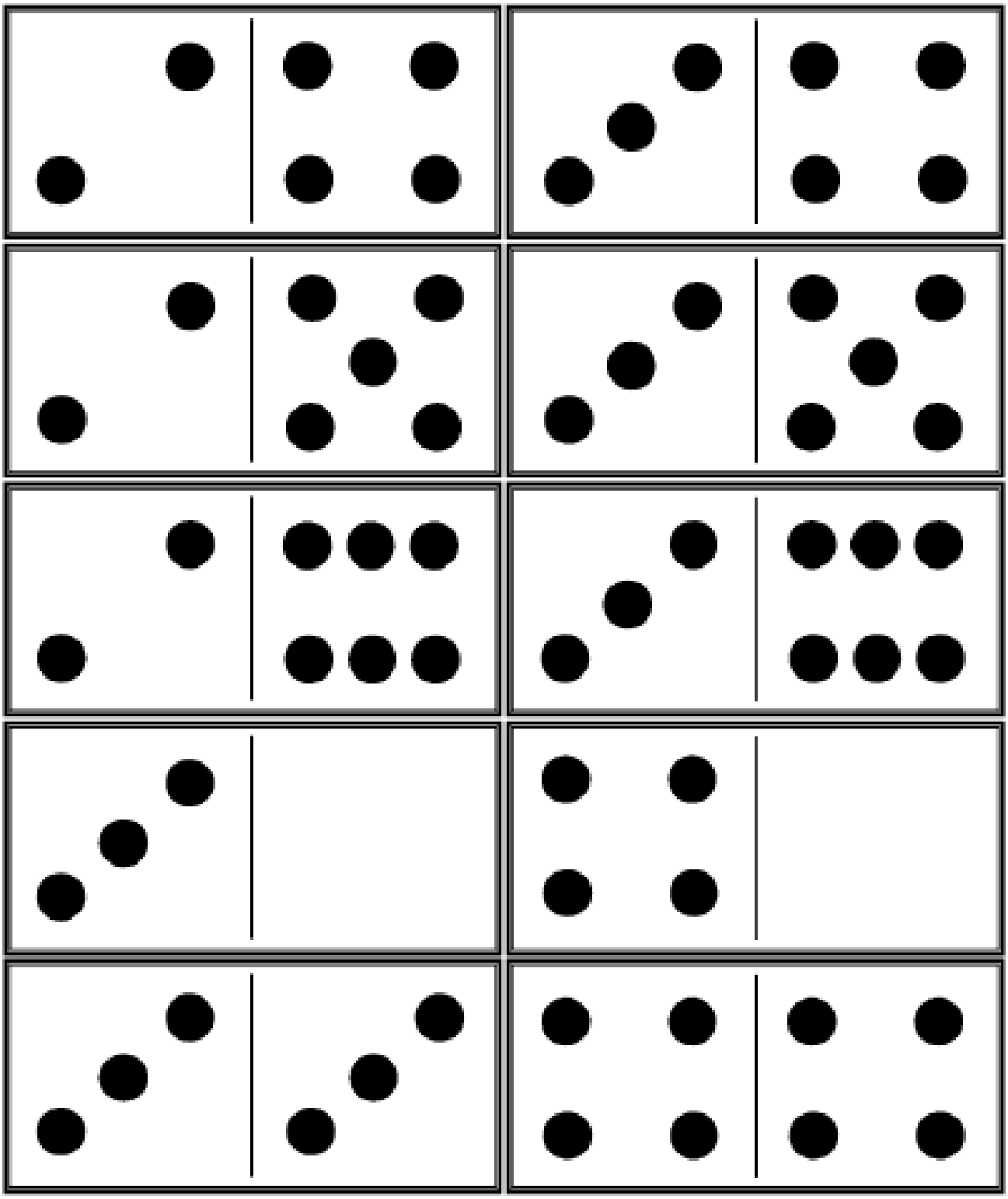
Yes	Maybe	No
☺	😐	☹

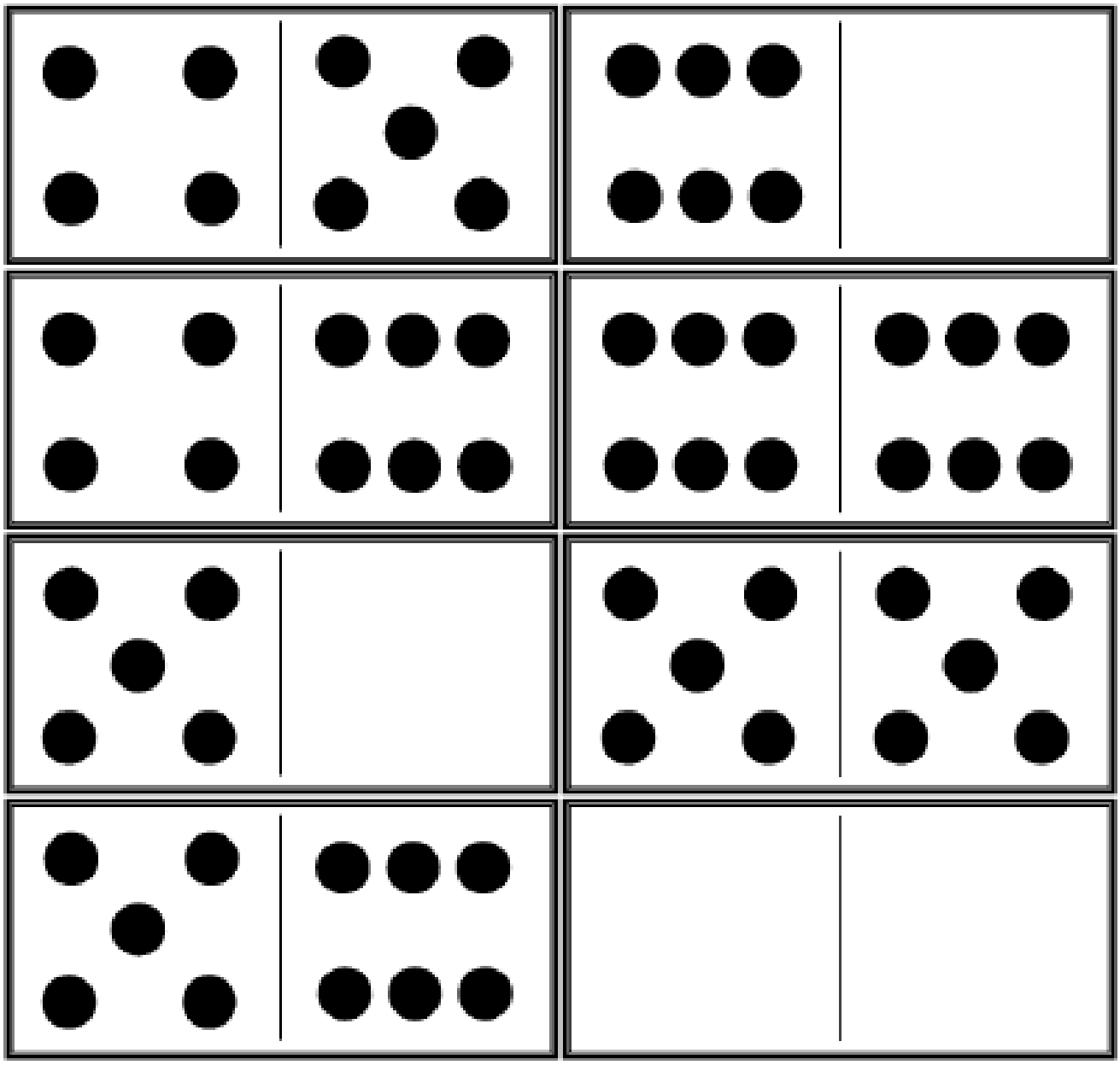
- ✓ I know how to add and subtract mixed numbers with unlike denominators.

Yes	Maybe	No
☺	😐	☹

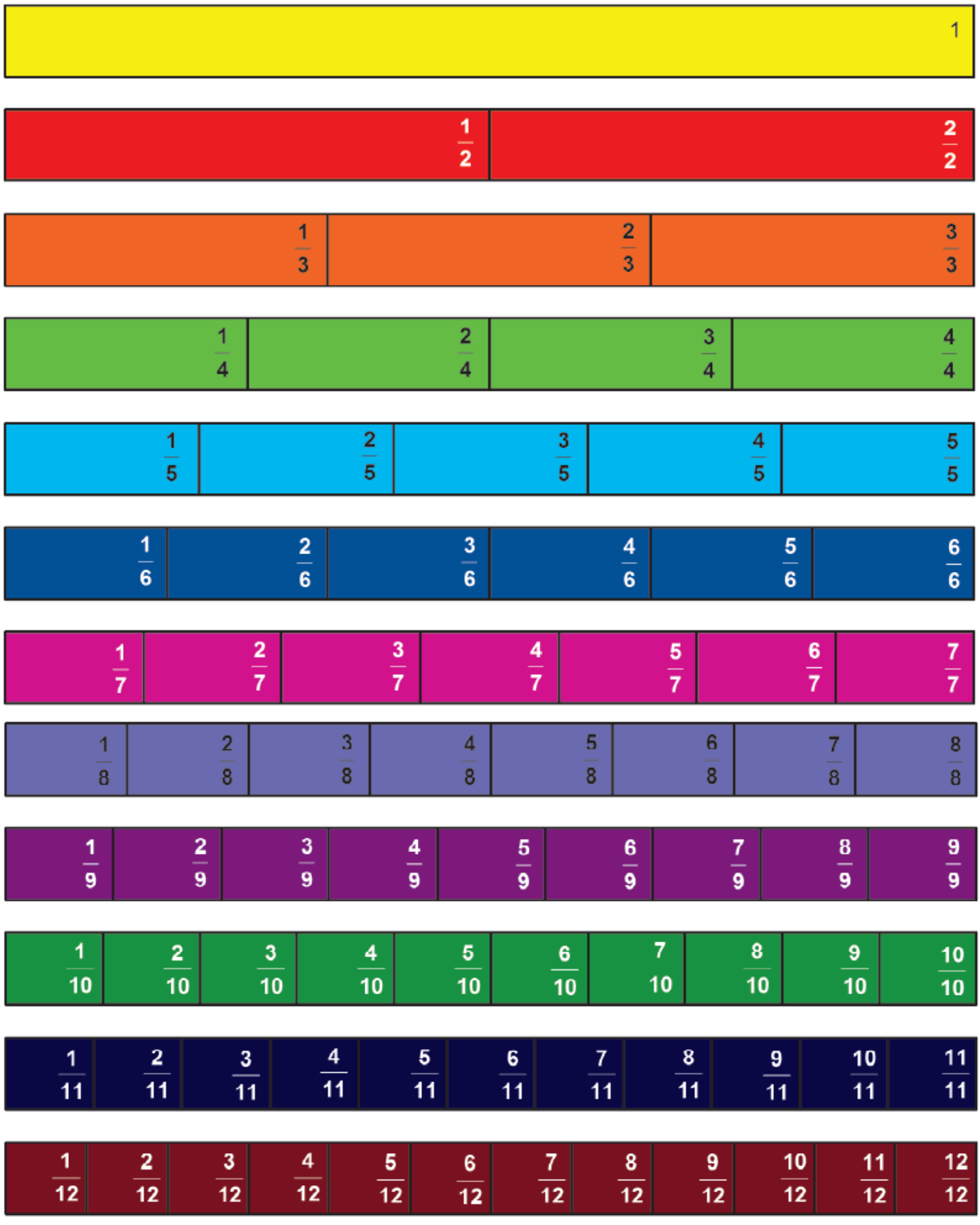
Domino Set http://www.first-school.ws/theme/printables/dominoes-math.htm







Fraction Strips - http://www.math-drills.com/fractions/fraction_strips_color_labeled.pdf



Research and review of standard	
Content Standard(s):	**Standard(s) for Mathematical Practice:**
Grade 5 – Numbers and Operations: Fractions **Cluster:** Use equivalent fractions as a strategy to add and subtract fractions. **5.NF.A.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2*.	**MP2** Reason abstractly and quantitatively. **MP7** Look for and make use of structure.
Smarter Balanced Claim	***Smarter Balanced Item***
Claim 1: Concepts and Procedures-Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.	Grade 5 Mathematics Sample CR Item C1 T1 Smarter Balanced Assessment Consortium The table below shows the length of ribbon, in yards, needed to make different art projects. **Project** / **Length of Ribbon (in yards)** Flower / $1\frac{3}{4}$ Bulletin board / $3\frac{1}{3}$ Costume / 2 Mask / $\frac{1}{3}$ Puppet / $2\frac{1}{2}$ Picture frame / $\frac{1}{4}$ Lance has $3\frac{2}{3}$ yards of ribbon. He is making a puppet. How much ribbon, in yards, will Lance have left? ☐ Yards *Key:* $3\frac{4}{6} - 2\frac{3}{6} = 1\frac{1}{6}$
CPR Pre-Requisites *(Conceptual Understanding, Procedural Skills, and Representations)* *Look at the Progressions documents, Learning Trajectories, LZ lesson library, unpacked standards documents from states, NCTM Essential Understandings Series, NCTM articles, and other professional resources. You'll find links to great resources on your PLC Platform.*	**Conceptual Understanding and Knowledge** • Understand that a fraction is a number • Understand a mixed number is comprised of a whole number and a fraction smaller than one • Equivalence of fractions (two fractions are equal if they are the same size or represent the same point on a number line) • Fractions can be added or subtracted (through joining or separating parts of the same whole) and the same

	properties that apply for whole number addition and subtraction apply to fractions (associative, distributive, commutative) **Procedural Skills** • Creating equivalent fractions for fractions and mixed numbers with like and unlike denominators • Adding and subtracting fractions and mixed numbers with like denominators **Representational** • Visual representations of fractions • Equations • Number lines **Social knowledge** • Terms: Numerator; denominator; equivalent fraction (equivalence); mixed number • Conventions: How to write a mixed number

Standards Progression **Look at LearnZillion lessons and expert tutorials, the Progressions documents, learning trajectories, and the "Wiring Document" to help you with this section*		
Grade(s) below	**Target grade**	**Grade(s) above**
3.NF.3.a Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line. **4.NF.3.a** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. **4.NF.3.c** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. **4.NF.3.d** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	**5.NF.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.	

Common Misconceptions/Roadblocks

What characteristics of this problem may confuse students?

- Students may not understand that this a subtraction problem.
- Students may get confused by the extra information provided in the table and choose the incorrect ribbon length to subtract from Lance's total
- Student may use other ribbon lengths in the table and use them to calculate something other than what the question is asking them to do

What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?

- Students may not recognize fractions as numbers and treat the numerator and denominator as isolated numerals when computing (example: 1/10 + 1/10 = 2/20 or 5/6 – 1/6 = 4/0)
- Students may not understand meanings of whole number subtraction can extend to fraction subtraction.
- Students may not recognize the importance of the whole unit and the need for the whole to be the same when computing with fractions (you can't add ½ of a large pizza to ¼ of a small pizza)
- Students may not generate equivalent fractions in order to subtract mixed numbers

What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?

- Students may overgeneralize their work with whole numbers and apply this understanding when subtracting fractions.