



Mathematics Instructional Cycle Guide

Number and Operations – Fractions Develop understanding of fractions as numbers.

3.NF.A.2
3.NF.A.2.A
3.NF.A.2.B

Created by Kari Baransky, 2014 Connecticut Dream Team Teacher

CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

Number and Operations – Fractions

Develop understanding of fractions as numbers.

3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

3.NF.A.2.A Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

3.NF.A.2.B Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

MP.6 Attend to precision.

- Students specify the whole amount when referring to a unit fraction and explain what is meant by equal parts in their own words.

MP.7 Look for and make use of structure.

- Students understand and use the unit fraction as the basic building block or structure of all fractions on the number line.

WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings (**page 12**)
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint (**pages 3 - 7**)
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed (**pages 8 - 11**)
- Supporting lesson materials (**13-18**)
- Precursory research and review of standards **3.NF.A.2**, **3.NF.A.2.A**, **3.NF.A.2.B**, and assessment items that illustrate the standards (**19 - 22**)

HOW TO USE THIS DOCUMENT

- 1) Before the lesson, administer the **(Fractional Parts) [Mathematical Checkpoint](#)** individually to students to elicit evidence of student understanding.
- 2) Analyze and interpret the student work using the [Student Response Guide](#)
- 3) Use the next steps or **follow-up lesson plan** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
- 4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

MATERIALS REQUIRED

- Cuisenaire rods, fraction strips, Equivalency cubes, fractions towers, or other manipulative that can be used to show different sizes
- Worksheets
- Dice
- Response boards, markers
- Math response journal

TIME NEEDED

Number Line Fractions administration: **15 minutes**

Follow-Up Lesson Plan: **1 – 2 Instructional blocks**

Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

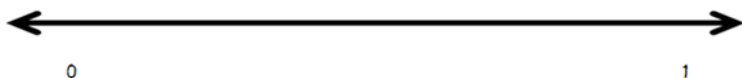
Connecticut State Department of Education

**Step 1: Elicit evidence of student understanding
Mathematical Checkpoint**

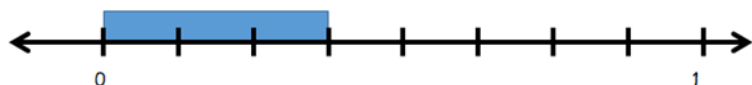
Question(s)

Purpose

Divide this number line into 3 equal parts



How many parts are shaded?
Write that number as a fraction in the box.



Label 0 and 1 on the number line below. Locate and label $\frac{1}{4}$ on the number line.



CT Core Standards:

3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

3.NF.A.2.A Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

3.NF.A.2.B Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

Target question addressed by this checkpoint:

- Do students understand a fraction as a number on the number line?
- Do students understand the ways to represent the values of numbers between 0 and 1?
- Can students equally partition a number line? (Draw one fewer lines than the number of parts desired.)
- Can students correctly place 0 and 1 on a number line?
- Can students define the interval from 0 to 1 as a whole?

Step 2: Analyze and Interpret Student Work
Student Response Guide

Got It

Divide this number line into 3 equal parts



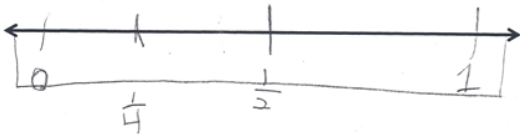
How many parts are shaded?

Write that number as a fraction in the box.

$$\frac{3}{8}$$



Label 0 and 1 on the number line below. Locate and label $\frac{1}{4}$ on the number line.



Developing

Divide this number line into 3 equal parts



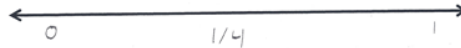
How many parts are shaded?

Write that number as a fraction in the box.

$$\frac{4}{7}$$



Label 0 and 1 on the number line below. Locate and label $\frac{1}{4}$ on the number line.



Getting Started

Divide this number line into 3 equal parts



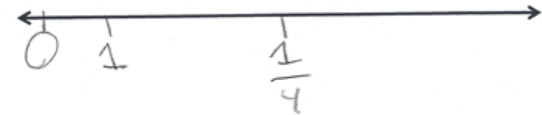
How many parts are shaded?

Write that number as a fraction in the box.



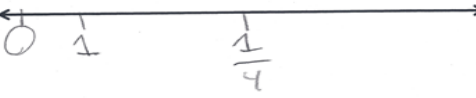
$$\frac{3}{0}$$



Label 0 and 1 on the number line below. Locate and label $\frac{1}{4}$ on the number line.



| | | |
|--|--|--|
| | | |
|--|--|--|

| Getting Started | |
|--|--|
| Student Response Example | Indicators |
| <p>Divide this number line into 3 equal parts</p>  <p>How many parts are shaded? Write that number as a fraction in the box.</p> <div style="display: flex; align-items: center; gap: 20px;">  <div style="border: 1px solid black; padding: 5px; display: inline-block;"> $\frac{3}{0}$ </div> </div> <p>Label 0 and 1 on the number line below. Locate and label $\frac{1}{4}$ on the number line.</p>  | <ul style="list-style-type: none"> • Student misunderstandings of dividing the line into 3 parts; represented 4 parts using 3 lines. • Equal parts not represented; the 4 parts represented in the illustration have different sizes, the student has a misconception that the number of lines equals the number of parts. • This student put 3 in the numerator of the fraction representing the 3 shaded parts of the number line, showing an understanding for what the numerator represents. • Student misunderstanding of the denominator of the fraction, difficulties with identifying that all of the available pieces are the whole. This student represented the denominator of the fraction as the starting point on the number line. • This student read the question and followed the numbers within the question to place on the number line. The first number in the question was 0; the next was 1, and finally $\frac{1}{4}$. Student placed the numbers in the order given from the question. • Student misunderstandings of $\frac{1}{4}$ and 1 in relationship to 0; difficulties with transferring understandings of a number line to the fractional whole. This student struggles with the understanding that on a number line the space between 0 and 1 is a whole and the fraction would be between these numbers. |
| In the Moment Questions/Prompts | Closing the Loop (Interventions/Extensions) |
| <p>Q – Show me a number line with 2 equal parts. How do you know that the parts are equal?</p> <p>Q – What would happen if I shaded 1 part? What part would you have?</p> <p>Q – What does the top number of the fraction represent?</p> <p>Q – What does the bottom number of the fraction represent?</p> | <p>Provide the student with paper and help him/her fold it into fractional parts. Walk the students through a variety of experiences with creating fractional parts such as $\frac{1}{2}$ or $\frac{1}{4}$. Then draw the same parts on a number line to show the student how the number line is partitioned into equal parts and how we count those parts.</p> <p>Students may need to use manipulatives to show the meanings of the parts of the fraction. A graham cracker can be used to show the whole, pieces can be broken off to show the 4 equal parts.</p> |


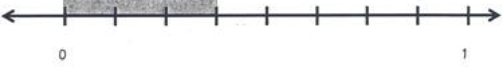

Q – Show me 0 and 1 on a number line? Why did you place them there?

<http://learnzillion.com/lessons/1727-plot-a-unit-fraction-on-a-number-line>

Q – Is $\frac{1}{4}$ greater than 1? Explain your thinking.

<http://learnzillion.com/lessons/1728-identify-a-fraction-as-a-point-on-a-number-line-by-dividing-the-number-line-into-equal-parts>

<http://learnzillion.com/lessons/1738-express-whole-numbers-as-fraction>

| Developing | |
|--|--|
| Student Response Example | Indicators |
| <p>Divide this number line into 3 equal parts</p>  <p>How many parts are shaded? Write that number as a fraction in the box.</p> <div style="border: 1px solid black; display: inline-block; padding: 2px; margin: 5px;">4</div>  <p>Label 0 and 1 on the number line below. Locate and label $\frac{1}{4}$ on the number line.</p>  | <ul style="list-style-type: none"> • Student misunderstands how to divide the line into 3 parts and represented 4 parts using 3 lines. • Student appears to understand fractions as equal parts because equal parts are represented. The 4 parts represented in the illustration have more or less the same sizes. • The student displays a misunderstanding of how parts are counted. On the lines representing the parts of the number line, student wrote a 4 for the numerator. This student is counting the lines as the fractional parts. • This is also evident in the first question where the student placed 3 lines to represent the 3 equal parts. • Student shows a misunderstanding that the denominator is one whole, not the equal number of parts represented on the number line. • Student understands where 0 and 1 are on a number line. • Student misunderstands where $\frac{1}{4}$ is in relationship to 0 and 1. • Student misunderstands the size of $\frac{1}{4}$ in relationship to one whole. |
| In the Moment Questions/Prompts | Closing the Loop (Interventions/Extensions) |
| <p>Q – Tell me how you solved each problem.</p> <p>Q – What does the top number of the fraction represent?</p> | <p>Students may need to be prompted with questions breaking down the value of each part of writing fractions. You might ask:</p> <p>Q: How many pieces are selected? How many pieces would you have if you had all of the pieces, the whole thing? How do we show that as a fraction?</p> |

Q – What does the bottom number of the fraction represent?

Q – Explain your answer on #3. How many parts are there between 0 and 1? How many parts do you need to show one half? How many parts to show one fourth?

Q – How could you change your answer? Explain your changes.

Using a physical model showing equal parts, you might ask:

Q: How many equal parts are there? How do you know? How would you show the equal parts on the number line?

<http://learnzillion.com/lessons/1729-identify-a-fraction-as-a-point-on-a-number-line-using-area-models>

| Got it | |
|--------------------------|------------|
| Student Response Example | Indicators |

What will a response include from a student who has demonstrated conceptual understanding and mastery?

Divide this number line into 3 equal parts



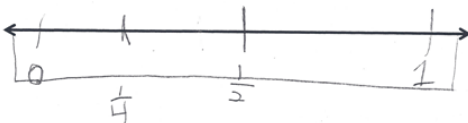
How many parts are shaded?

Write that number as a fraction in the box.

$\frac{3}{8}$



Label 0 and 1 on the number line below. Locate and label $\frac{1}{4}$ on the number line.



- Student understands how to break the number line into specific pieces.
- Student understands equal sized parts for each of the 3 pieces, which demonstrates the understanding that fractions show equal parts.
- Student knows how to show fractional parts as represented with the numerator showing the number of parts and the denominator showing the total number possible.
- Student understands the concept that the parts are between the lines, not the lines themselves.
- Student knew the position of 0 and 1 on the number line.
- Student knew the location of $\frac{1}{2}$ on the number line, demonstrating an understanding of the relationship of fraction sizes.
- Student understands the location of $\frac{1}{4}$ on the number line, greater than 0 but less than $\frac{1}{2}$.

| In the Moment Questions/Prompts | Closing the Loop (Interventions/Extensions) |
|---|---|
| <p>Q – Can you show me $\frac{1}{4}$ on a number line? Follow up fractions for this question would be: $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$, $\frac{5}{4}$, $\frac{6}{4}$.</p> | <p>Using several number lines, with a variety of denominators, students will show a variety of $\frac{1}{2}$ fractions. Students will also use various manipulatives to show $\frac{1}{2}$ of a larger set.</p> <p>Q: What would $\frac{1}{2}$ of 3 be? 4? 6?</p> <p>http://learnzillion.com/lessons/1730-plot-improper-fractions-on-a-number-line</p> |

Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction




| | |
|-------------------------------------|---|
| Lesson Objective: | Recognize fractions on a number line. Generate number lines with specific fraction points. |
| Content Standard(s): | <p><u>3.NF.A.2</u> Understand a fraction as a number on the number line; represent fractions on a number line diagram.</p> <p><u>3.NF.A.2.A</u> Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.</p> <p><u>3.NF.A.2.B</u> Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.</p> |
| Targeted Practice Standard : | <p>MP.6 Attend to precision.</p> <ul style="list-style-type: none"> Can the student specify the whole amount when referring to a unit fraction and explain what is meant by equal parts in their own words? <p>MP.7 Look for and make use of structure.</p> <ul style="list-style-type: none"> Can the student understand and use the unit fraction as the basic building block or structure of all fractions on the number line? |

| Mathematical Goals | Success Criteria |
|--|---|
| <ul style="list-style-type: none"> Students will gain an understanding that a fraction can be represented on a number on the number line and how to draw various common fractions between 0 and 1. Students will gain an understanding of the ways to represent the values of numbers between 0 and 1. | <ul style="list-style-type: none"> Students will create number lines with specific fractional representations. Students will explain their rationale for placing specific fractions on the number line. |

Launch (Probe and Build Background Knowledge)

Purpose: *Students will share prior understandings with the group and then build a deeper understanding of fractions on a number line between 0 and 1.*

Students will be told the following story: You are going to be sharing party food with two other friends. Think about how you would share the food if your friends get to pick their pieces before you get to pick your piece. You will be using pizza, chocolate, and cookies.

Pizza  Chocolate  Cookies 

The teacher will allow the students time to show how they would share the pizza, chocolate, and cookies. Students' choices will be on the response boards to ensure that all students give a response. The teacher will monitor for understanding by asking students to explain how they divided up their food.

Students will be shown pictures of foods that are sectioned off and asked to identify if it is really thirds. Students' choices will be on the response boards to ensure that all students give a response. Students will explain their thoughts

for their choices for which pictures represent thirds. Through this conversation with students the teacher will monitor for understanding of the following items:

- Do students understand the meaning of the numerator? Do they understand it represents part of the whole?
- Do students understand the meaning of the denominator? Do they understand that it shows the total number of parts in the whole?
- Do students understand the concept of equal (even) parts?

Questions and prompts to guide the discussion could include:

- What is the whole thing?
- How many pieces make the whole thing?
- How many pieces do you have?

Instructional Task

Purpose: Students will explore a task about sharing licorice in order to specify the whole amount when referring to a unit fraction and explain what is meant by equal parts. Students will understand and use the unit fraction as the basic building block or structure of all fractions on the number line. Students will use what they know about mathematical tools in order to create the structure of fractions between 0 and 1 on a number line.

Engage (Setting Up the Task)

Students will work on this task individually at first and then will work with a partner. Students will be given this question:

Sally would like to share licorice with her friends. She has one foot of licorice to share. If she has 2 friends (and she gets a piece), how could she cut the licorice? How much would they each get?

What about if she wanted to share with 3 friends? How much would they each get?

Show where the licorice would need to be cut in order to share it with friends.

She has decided that each of her friends would get $\frac{1}{3}$ feet of licorice. Show where the licorice would need to be cut in order to share it with friends.

Students will show their work on a number line, folding paper strips, or by using another strategy that they have chosen. Then students will explain how they decided where to draw the lines for Sally's licorice. The teacher will use the licorice page for this activity.

Explore (Solving the Task)

Circulate to observe, support, and gather information on student thinking using some of the possible questions and prompts below. Note student misconceptions or strategies which would be valuable to share with the whole group.

Clarifying questions/prompts:

- Can you explain how you divided your number line?
- Why did you divide it into ___ number of pieces?
- How many lines did you have to draw to get ___ number of pieces?

Advancing Questions/Prompts

- Could you divide the number line in any other ways?
- Can you explain why each of those ways would give equal parts for ___ friends?

Elaborate (Discuss Task and Related Mathematical Concepts)

Select students to share their work. Ask students to explain their representations.

Some questions which may lead to discussion include:

- How did you make equal parts on your number line?

- How did you know how many parts to section the number line into?
- How did you show the numerator on your number line?
- How do you show that this number line is less than 1?
- Did you show the same fraction in different ways?
- How did you know that they were the same fraction?

Checking for Understanding

Purpose: A clear understanding of 0 and 1 on the number line should be established. Students will be demonstrating an understanding of equal parts on the number line representing fractional parts of the whole.

Each student will be given the fractions on number lines page. Each student group will be expected to explain why each of their number line models is correct for the given fraction. Each group will answer three self-check questions:

1. Is my numerator correct? (Do I have the correct number of equal pieces selected?)
2. Is my denominator correct? (Do I have the correct total amount of equal pieces?)
3. Are each of the parts of my number line equal? (Did the parts of my fraction get placed on the number line in the correct spot?)

The teacher will need to monitor the progress of the students. The students will be asked to show $\frac{4}{6}$ on a number line. The teacher will check for understanding by asking questions of the students:

- Explain how you made equal parts on your number line?
- How did you know how many parts to section the number line into?
- How did you show the numerator on your number line?
- How do you know the value of the parts?

Common Misunderstanding

Purpose: The teacher will ask students to explain the sections of the number line.

If students are showing a misunderstanding of how to break the number line into the correct sections, the teacher will ask:

- How did you section your number line?

While listening to the student response it may be necessary to make the students aware of the misunderstanding and clarify the correct understanding.

For example: A student's work shows that the $\frac{2}{2}$ number line is broken into 3 pieces; it may be necessary, after listening to the student, to clarify that the sections between the lines would be the equal parts of the number line, not the lines. The teacher can demonstrate folding paper into 2 parts, or use an example number line representing 2 equal parts.

Checking for Understanding

Purpose: Students will show and explain their understanding of number lines using their previous mathematical checkpoint work.

Students will receive their Mathematical Checkpoint work and discuss their previous answers with a partner. Specifically, they will answer the following questions:

1. Is my numerator correct? (Do I have the correct number of equal pieces selected?)
2. Is my denominator correct? (Do I have the correct total amount of equal pieces?)
3. Are each of the parts of my number line equal? (Did the parts of my fraction get placed on the number line in the correct spot?)

If the students have a disagreement then the teacher will ask the students to explain their thinking for each of the questions. Students may use manipulatives to explain how they sectioned out their number line or can explain it in words. Specific language about parts being equal, the number representing the numerator, the number representing the denominator and where, specifically each number is represented on the number line.

Closure

Purpose: Students will reflect upon their understanding of number lines. Students will share a way that changed or corrected their understanding.

Students will complete the Exit Slip as a way to demonstrate their understanding.

Extension Task

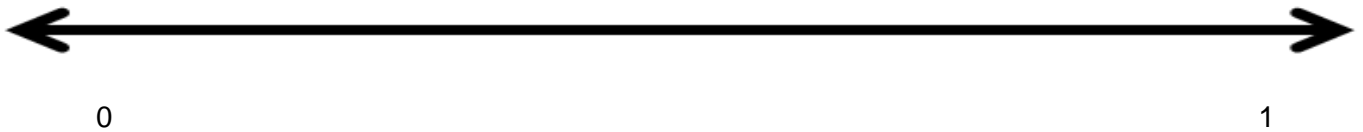
Purpose: Students with a solid conceptual understanding of number lines will apply their knowledge to a variety of fractions and place them correctly onto a number line.

Students who are ready to extend their understanding of the number line will be given an extension activity that will have them problem solving where on the number line a specific fraction would be, illustrate the number line, and explain if the fraction is closer to 0, $\frac{1}{2}$, or 1.

Mathematical Checkpoint

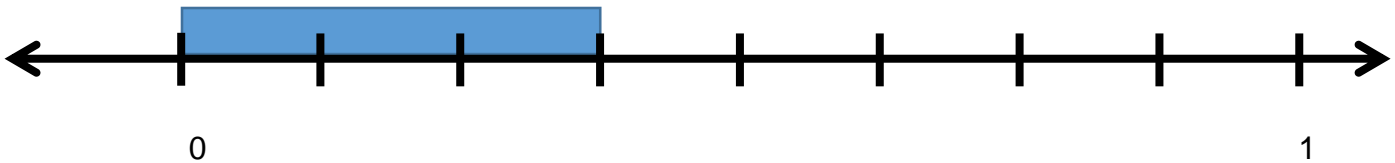
Name _____

Divide this number line into 3 equal parts



How many parts are shaded?

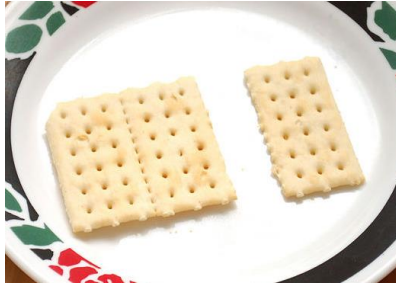
Write that number as a fraction in the box.



Label 0 and 1 on the number line below. Locate and label $\frac{1}{4}$ on the number line.



Are they thirds?



Licorice Activity Page

Name _____

Sally would like to share licorice with her friends. She has one foot of licorice to share. If she has 2 friends (and she gets a piece), how could she cut the licorice? How much would they each get?

What about if she wanted to share with 3 friends? How much would they each get?

Show where the licorice would need to be cut in order to share it with friends.

Sally's shares with 2 friends







Sally's shares with 3 friends



Explain how you solved the problem.

Name: _____ Date: _____

Fractions on Number Lines

| Fraction | Number Line |
|---------------|--|
| $\frac{3}{8}$ |  |
| $\frac{2}{2}$ |  |
| $\frac{4}{6}$ |  |
| $\frac{2}{3}$ |  |

Student Self-Check

1. Is my numerator correct? (Do I have the correct number of equal pieces selected?)
2. Is my denominator correct? (Do I have the correct total amount of equal pieces?)
3. Are each of the parts of my number line equal? (Did the parts of my fraction get placed on the number line in the correct spot?)

Exit Slip

Name: _____ Date: _____

Put an X on the spot where $\frac{4}{8}$ would be on the number line.



Explain your thinking.

Exit Slip

Name: _____ Date: _____

Put an X on the spot where $\frac{4}{8}$ would be on the number line.

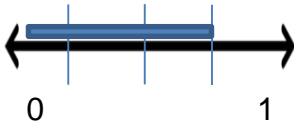


Explain your thinking.

Name _____ Date _____

Closer to 0, $\frac{1}{2}$, or 1 Recording Sheet

Directions: The first player will roll 2 dice. The smaller number will be the numerator, and the larger number will be the denominator. Record the fraction below, circle what you think the fraction is closer to, draw the fraction on a number line, and write your evidence supporting your thinking. Once the game is over, list the fractions rolled in the game in order from least to greatest in your math journal.

| Fraction | Closer to 0, $\frac{1}{2}$, or 1 | Number Line | How Do You Know? |
|--|---|---|---|
| <p>Example:</p> <p>$\frac{3}{4}$</p> | <p>0 $\frac{1}{2}$ 1</p> |  | <p>The number line is sectioned into 4 equal pieces and 3 pieces are shaded in. This shows 3 of the 4 equal pieces.</p> |
| | <p>0 $\frac{1}{2}$ 1</p> | | |
| | <p>0 $\frac{1}{2}$ 1</p> | | |
| | <p>0 $\frac{1}{2}$ 1</p> | | |


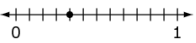

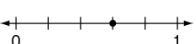

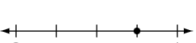
| | | | |
|--|-------------------|--|--|
| | 0 $\frac{1}{2}$ 1 | | |
| | 0 $\frac{1}{2}$ 1 | | |
| | 0 $\frac{1}{2}$ 1 | | |
| | 0 $\frac{1}{2}$ 1 | | |
| | 0 $\frac{1}{2}$ 1 | | |

Student Self-Check

1. Is my numerator correct? (Do I have the correct number of equal pieces selected?)
2. Is my denominator correct? (Do I have the correct total amount of equal pieces?)
3. Are each of the parts of my number line equal? (Did the parts of my fraction get placed on the number line in the correct spot?)

Research and review of standard

| Content Standard(s): | Standard(s) for Mathematical Practice: |
|--|---|
| <p>Number and Operations – Fractions</p> <p>Develop understanding of fractions as numbers.</p> <p>3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.</p> <p>3.NF.A.2.A Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.</p> <p>3.NF.A.2.B Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.</p> | <p>MP.6 Attend to precision. Students specify the whole amount when referring to a unit fraction and explain what is meant by equal parts in their own words.</p> <p>MP.7 Look for and make use of structure. Students understand and use the unit fraction as the basic building block or structure of all fractions on the number line.</p> |

| Smarter Balanced Claim | Smarter Balanced Item |
|--|---|
| <p>Overall Claim for Grades 3–8 “Students can demonstrate progress toward college and career readiness in mathematics.”</p> <p>Claim #1 – Concepts & Procedures “Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.”</p> | <div style="border: 1px solid gray; padding: 10px;"> <p>Look at point P on the number line.</p>  <p>Look at number lines A – E. Is the point on each number line equal to the number shown by P? Choose Yes or No.</p> <div style="display: flex; flex-direction: column; gap: 10px;"> <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">A.</div>  <div style="margin-left: 10px;"> <input type="radio"/> Yes <input type="radio"/> No </div> </div> <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">B.</div>  <div style="margin-left: 10px;"> <input type="radio"/> Yes <input type="radio"/> No </div> </div> <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">C.</div>  <div style="margin-left: 10px;"> <input type="radio"/> Yes <input type="radio"/> No </div> </div> <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">D.</div>  <div style="margin-left: 10px;"> <input type="radio"/> Yes <input type="radio"/> No </div> </div> <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">E.</div>  <div style="margin-left: 10px;"> <input type="radio"/> Yes <input type="radio"/> No </div> </div> </div> </div> |

CPR Pre-Requisites

(Conceptual Understanding, Procedural Skills, and Representations)

Look at the Progressions documents, Learning Trajectories, LZ lesson library, unpacked standards documents from states, NCTM Essential Understandings Series, NCTM articles, and other professional resources. You'll find links to great resources on your PLC Platform.

Conceptual Understanding and Knowledge

- Understand a fraction as a number on the number line.
- Understand the ways to represent the values of numbers between 0 and 1.

Procedural Skills

- Equally partition a number line. (Draw one fewer lines than the number of parts desired.)
- Correctly place 0 and 1 on a number line.
- Defining the interval from 0 to 1 as the whole.

Representational

- Represent fractions on a number line diagram.
- Represent equal parts on a number line.
- Represent a fraction by marking off equal lengths from 0 to 1.

Social knowledge

- Fractional unit (half, third, fourth, etc.)
- Equal parts (parts with equal measurements)
- Number line
- Halves, thirds, fourths, sixths, eighths ($1/2$, $1/3$, $1/4$, $1/6$, $1/8$)
- Half of, one third of, one fourth of, etc. ($1/2$, $1/3$, $1/4$, $1/6$, $1/8$)
- Equal shares (pieces of a whole that are the same size)
- Whole (e.g., 2 halves, 3 thirds, etc.)
- Fraction (e.g., $1/3$, $2/3$, $3/3$, $4/3$)
- Partition (divide a whole into equal parts)

Standards Progression

| Grade(s) below | Target grade | Grade(s) above |
|---|--|--|
| <p>2.G.A.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.</p> <p>2.G.A.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.</p> | <p>3.NF.A.1 Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.</p> <p>3.NF.A.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.</p> <p>3.NF.A.3.A Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.</p> | <p>Extend understanding of fraction equivalence and ordering.</p> <p>4.NF.A.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</p> |

| | | |
|--|--|---|
| <p>2.MD.B.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, and represent whole-number sums and differences within 100 on a number line diagram.</p> | <p>3.NF.A.3.B Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.</p> <p>3.NF.A.3.C Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3 = 3/1$; recognize that $6/1 = 6$; locate $4/4$ and 1 at the same point of a number line diagram.</p> <p>3.NF.A.3.D Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.</p> | <p>4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.</p> |
|--|--|---|

Common Misconceptions/Roadblocks

What characteristics of this problem may confuse students?

- Students may not be familiar with dividing a number line into equal parts.
- Students may not be familiar with recognizing that each segmented part of a number line represents the same length.
- Students may not be familiar with the numerator of the fraction is the number of selected parts.
- Students may not be familiar with the denominator of the fraction is the total number of parts of the whole.
- Students may not be familiar with the concept that there are fractional numbers between 0 and 1.
- Students may not be familiar with how to represent numbers correctly in a fraction. Unfamiliar with definitions of numerator and denominator.

What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?

- Students may think that half means - one whole cut into two pieces, where size does not matter.
- Students may incorrectly identify the fraction symbol.
- Students may think that the size of a fraction depends solely on the number at the bottom (denominator) and you can ignore the number on the top (numerator).
- Students may think that fractions of the whole are whole numbers in themselves.
- Students may count the lines on the number line instead of the spaces between the lines and the parts.
- Students may represent the shaded parts as $3/3$ or 3; instead of $3/8$.
- Students may not label Point A; but rather label $1/4$ on the number line.

What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?

- The students think the numerator will always show one piece of the whole.