Mathematics Instructional Cycle Guide

Measurement and Data (3.MD.7.c)

Created by Andrew Hutchinson, 2014
Connecticut Dream Team teacher

CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

Measurement and Data - Relate area to the operations of multiplication and addition. (3.MD.7.c)

This Instructional Cycle Guide also relates to the following Mathematical Standard of Practice in the *CT Core Standards for Mathematics*:
MP 2 Reason abstractly and quantitatively.

WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings **(p. 3)**
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint **(pp. 4-7)**
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed **(pp. 8-11)**
- Supporting lesson materials **(pp. 12-16)**
- Precursory research and review of standard 3.MD.7.c and assessment items that illustrate the standard **(pp. 21-22)**

HOW TO USE THIS DOCUMENT

1) Before the lesson, administer the ***Mr. Jackson's Wall*** ***Mathematical Checkpoint*** individually to students to elicit evidence of student understanding.
2) Analyze and interpret the student work using the ***Student Response Guide***.
3) Use the next steps or ***follow-up lesson plan*** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint.
4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

MATERIALS REQUIRED (for 30 students in pairs)

- Teacher Demonstration Materials include:
  - ✓ Demonstration set of color tiles, and you will need 40 red and 10 blue to complete the task modeling
  - ✓ Demonstration Task
- Student Task Materials include -
  - ✓ Work mat of "Office Floor Design" - **Special Note -** The student work mat shall measure 10 in. long by 4 in. wide. This will allow the design to match the size of the standard 1 square inch color tile.
  - ✓ Recording Sheets
  - ✓ Containers of color tiles for student access
  - ✓ Colored pencils or crayons for each group of students to match the colors of the color tiles**.**

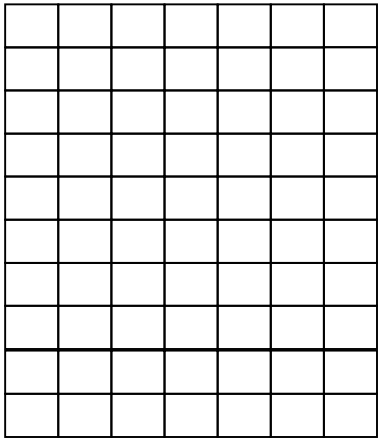
TIME NEEDED

Mr. Jackson's Wall administration: **20 minutes**
Follow-Up Lesson Plan: **60-75 minutes**

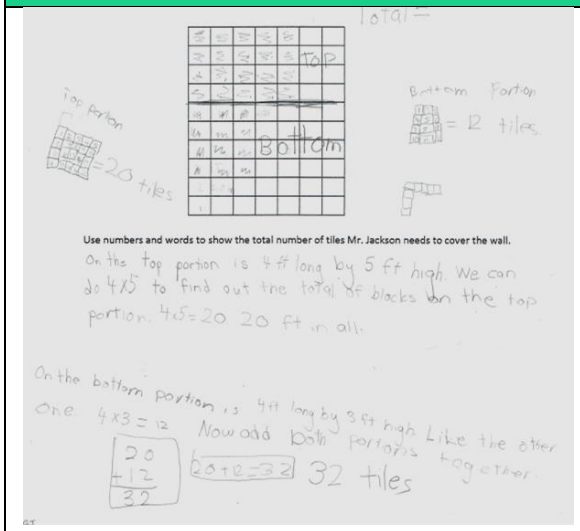
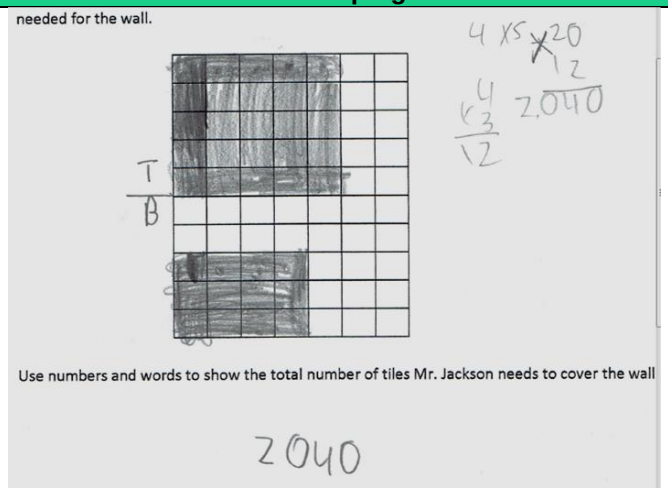
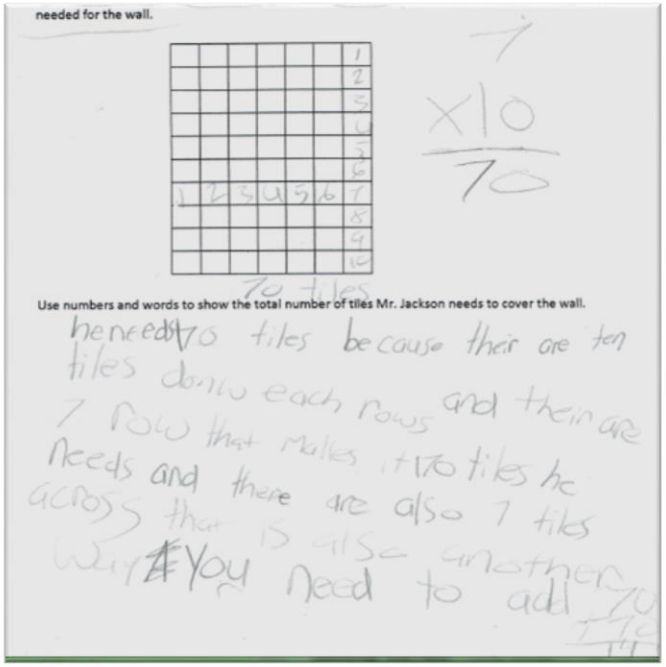
Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

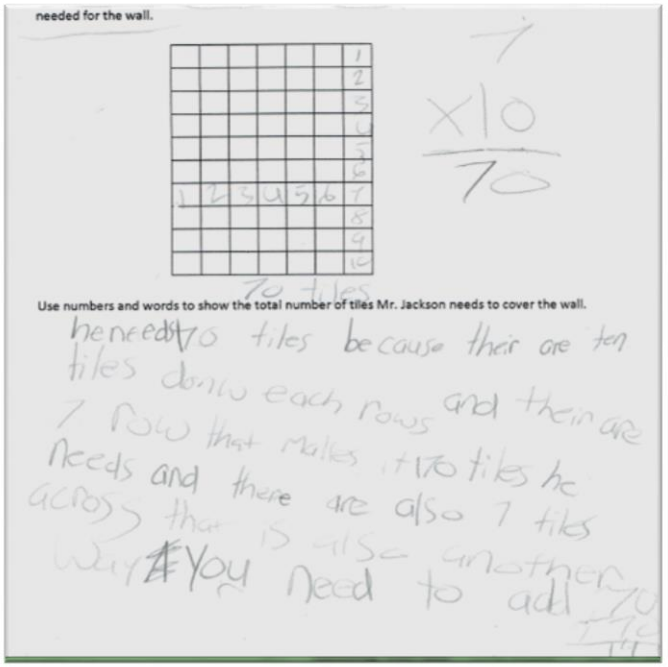
<table>
<tr><th colspan="3">Step 1: Elicit evidence of student understanding</th></tr>
<tr><th colspan="3">Mathematical Checkpoint</th></tr>
<tr><th>Question(s)</th><th colspan="2">Purpose</th></tr>
<tr><td rowspan="2">Mr. Jackson's Wall

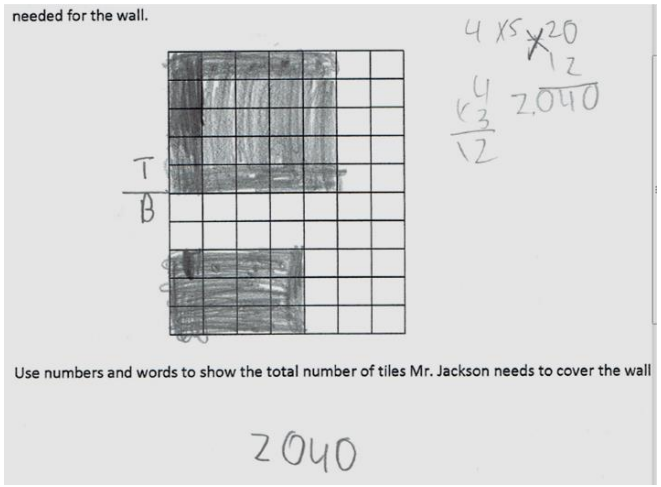
Mr. Jackson wants to cover his bathroom wall using 2 different styles of 1 square foot tiles. He wants to use one style of tile to cover the top portion of the wall and another style to cover the bottom portion.
The top portion of the wall measures 4 feet long by 5 feet high.
The bottom portion of the wall measures 4 feet long by 3 feet high.

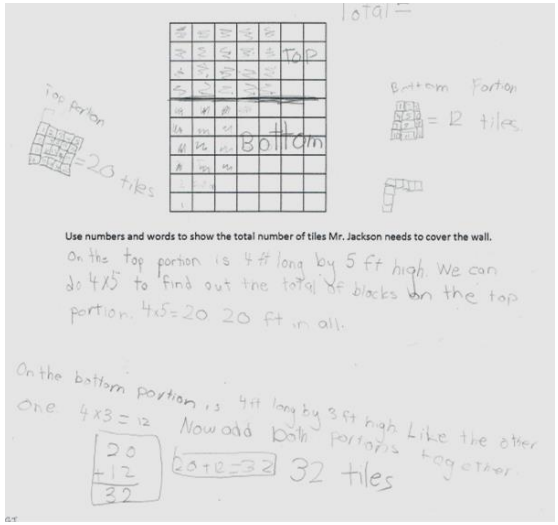
Mr. Jackson needs to figure out how many tiles total he will need to cover the wall. Below is a grid to help plan for the new tiles. You may use this tool to help you decide how many tiles are needed for the wall.
 

Use numbers and words to show the total number of tiles Mr. Jackson needs to cover the wall.</td><td>CT Core Standard:</td><td>3.MD.C.7
Relate area to the operations of multiplication and addition.
c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b and b + c is the sum of a x b and a x c. Use area models to represent the distributive property in mathematical reasoning.</td></tr>
<tr><td>Target question addressed by this checkpoint:</td><td>Do students understand that the area of a rectangle can be determined using the distributive property?</td></tr>
</table>

Step 2: Analyze and Interpret Student Work		
Student Response Guide		
Got It	**Developing**	**Getting Started**
		

Getting Started	
Student Response Example	**Indicators**
	• Student explanation provides no rational evidence • Student does not show understanding of dimensions provided and may use entire grid. • Student may not understand that area is the number of square units needed to tile a given figure. • Student provides no evidence of understanding the distributive property.
In the Moment Questions/Prompts	**Closing the Loop (Interventions/Extensions)**
Q: Tell me about your equation? Q. What do you think Mr. Jackson was trying to do? Q. Can you explain your picture? P: Provide the student with a rectangle cut from construction paper that is 4" x 8" and ask the student to tile the rectangle. Discuss and investigate what the length and width (height) of the rectangle is and how these terms relate to describing an array.	Find the area of a rectangle using an array: https://ctdreamteam.learnzillion.com/lessons/2509-find-the-area-of-a-rectangle-using-an-array Cover several rectangles by tiling them with color tiles. Discuss and connect the length and width of the rectangles to array expressions that will help to determine the area in square units.

Developing	
Student Response Example	**Indicators**
	• Student work reveals misunderstanding of area being additive and students multiply the products of both portions • Student may understand that area is found using length X height for the area of each portion • Model used correctly for one or both portions • Gaps may exist showing the wall in two separate portions, revealing that students may not see this as one rectangle
In the Moment Questions/Prompts	**Closing the Loop (Interventions/Extensions)**
Q. Tell me about your numbers. Q Can explain your picture to me? Q. How are the two rectangles related? A; Provide students with grid paper to cut out 2 rectangles. Then have students assemble them together to determine the area of the whole rectangle through adding the areas of the 2 cut-out rectangles.	Use area models to represent the distributive property https://ctdreamteam.learnzillion.com/lessons/3721-use-area-models-to-represent-the-distributive-property

<table>
<tr><th colspan="2">Got it</th></tr>
<tr><th>Student Response Example</th><th>Indicators</th></tr>
<tr><td> 
</td><td>

Student correctly names the number of total tiles
Explanation is clear and precise
All equations are written and solved correctly
Portions on the model are correctly represented with no errors
The wall may not be represented as one rectangle

</td></tr>
<tr><th>In the Moment Questions/Prompts</th><th>Closing the Loop (Interventions/Extensions)</th></tr>
<tr><td>Q: How are the two rectangles related?
Q: Is there another way to draw Mr. Jackson's wall?
Q. Can you explain your equations?</td><td>Find area using distributive property
https://ctdreamteam.learnzillion.com/lessons/3930-find-area-using-distributive-property

See Appendix A, pp. 18-19 for Premium LearnZillion Performance Task, 3.MD.7 - for 'Relate area to multiplication and addition using arrays '</td></tr>
</table>

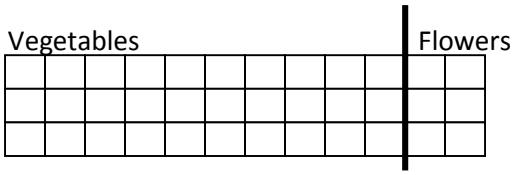
Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction	
Lesson Objective:	Apply the distributive property to find the area of a rectangle
Content Standard(s):	3.MD.C.7.c Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* and *b* + *c* is the sum of *a* x *b* and *a* x *c*. Use area models to represent the distributive property in mathematical reasoning.
Targeted Practice Standard :	MP2 -Reason abstractly and quantitatively ○ Do students reason abstractly and quantitatively when applying the distributive property to find the area of a rectangle?

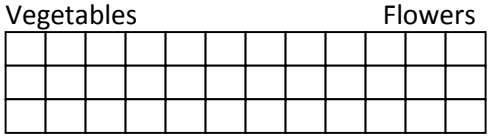
Mathematical Goals	Success Criteria
○ Understand how to find the area of a rectangle ○ Understand how to partition a rectangle and find the area of the two smaller rectangles ○ Understand that applying the distributive property will find the area of a rectangle	○ Application of the distributive property to determine the area of a rectangle

Launch (Probe and Build Background Knowledge)

Purpose: Determine if students can find the area of a rectangle by partitioning it and applying the distributive property to do so

I am considering putting in a garden in my backyard. I'd like the garden to have some flowers, but mostly vegetables. Once I decide how to arrange my garden, I will then need to buy enough fertilizer to cover the entire area. Below is a model of the area I will use for my garden.





Vegetables area is 30 square feet.
Flowers area is 6 square feet.
Total garden area is 30 + 6 = 36 square feet

Instruct students to use the second model and show a different way to divide the vegetables from the flowers by partitioning the area into 2 smaller rectangles. Then ask students to use numbers and words to show their idea.

- Can the number of square units in either section be represented with an equation?
- Can you think of a model used in math that this garden represents?
- How is the area of a rectangle determined?
- How can we write an equation that shows the combination of the 2 rectangles (arrays)?

Instructional Task

Purpose:

Students are given a container of color tiles to construct two adjoining rectangles. The rectangles will represent a new classroom floor the principal is interested in looking at. Along with the new design, the principal will need know how many square tiles it takes to measure the area of the classroom.

Engage:

Introduce the task of designing a new classroom for the principal.

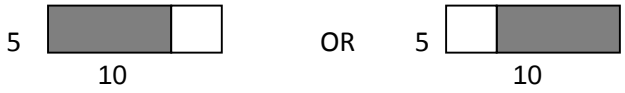
Task: The principal is replacing the floor in his office and needs help with the new design. Students are being asked to create new designs, but there are requirements in the design. Those requirements are as follows:

1. The floor is made up of 2 colors.
2. The floor is made up of square feet.
3. The floor is 10 feet long and 5 feet wide.
4. There is no mixing of colors and the two colors make rectangles.

Introduce the container of color tiles as the tools that will help provide the information the principal is requesting. Elicit student response as to the connection of the 1 inch square tile to the 1 square foot tile in the design. Choose which two colors you will use to demonstrate with. Present the demonstration area model. Ask the following questions:

- What do you notice about this model?
- Where do you see numbers on the model?
- What do you think these numbers represent?

Model think aloud and begin to complete the model like one presented here:



Model as 8 to represent the shaded part and 2 the unshaded part. The shaded part shall be represented by 5 x 8 and the unshaded part as 5 x 2.

Ask the following questions:

- What expression can I use to represent the shaded area?
- What expression can I use to represent the unshaded area?
- What can I do with these equations to determine how many square tiles the principal needs? Elicit student response for addition.

Discuss the expressions that match each color. Demonstrate writing the equation and inserting the addition sign between the two multiplication expressions. Explain that this equation represents using the distributive property to find the area of the classroom in square units.

Discuss the use parentheses is used to help see and identify the two expressions used in the equation.

Demonstrate writing the equation: (5x2) + (5x8) = area

Prompt students to solve the expressions and solve and label the equation as: 10 + 40 = 50 square feet

Introduce the task directions

Teacher will:

1. Distribute the work packet for each pair of students. Inform the students that they may submit four different designs.
2. Distribute containers of color tiles to students.

Students will:

3. Build the rectangles using your tiles.
4. Color the model to match your color tiles.

5. Write the multiplication expression for each color.
6. Determine the area for each design by combining the multiplication equations.

Extending the task:

Successful students should recognize the common factor in both multiplication equations and investigate how that may be represented differently in an equation.

Present the following equation: (a x b) + (a x c) = a x (b + c) and have students identify what numbers the letters stand for from the demonstration model. (5 x 8) + (5 x 2) = 5 x (8 + 2)

Students shall go back into their packets of rectangles and write the distributive property using both equations.

Explore:

Students shall work in heterogeneous groups of 2-3 students.

What questions will you ask as students work on the task to elicit evidence of their understanding and support mathematical connections?

- How do your equations represent the colors in your designs?
- Why do you add the two multiplication equations?

What are some anticipated student responses or solution paths?

- Students will count the number of total tiles.
- Students may write the whole equation with addition signs.
- Students will use the multiplication symbol when combining instead of the addition symbol.
- Students will successfully combine both equations.

Elaborate:

Students will present their work and allow groups to compare their equations with others. Facilitate a whole class discussion to elicit evidence of students understanding and develop mathematical connections using some of the possible questions below:

- Why can more than one equation represent the same area?
- How does the distributive property measure the area of a rectangle?
- Why would you use the distributive property to find area?

Checking for Understanding

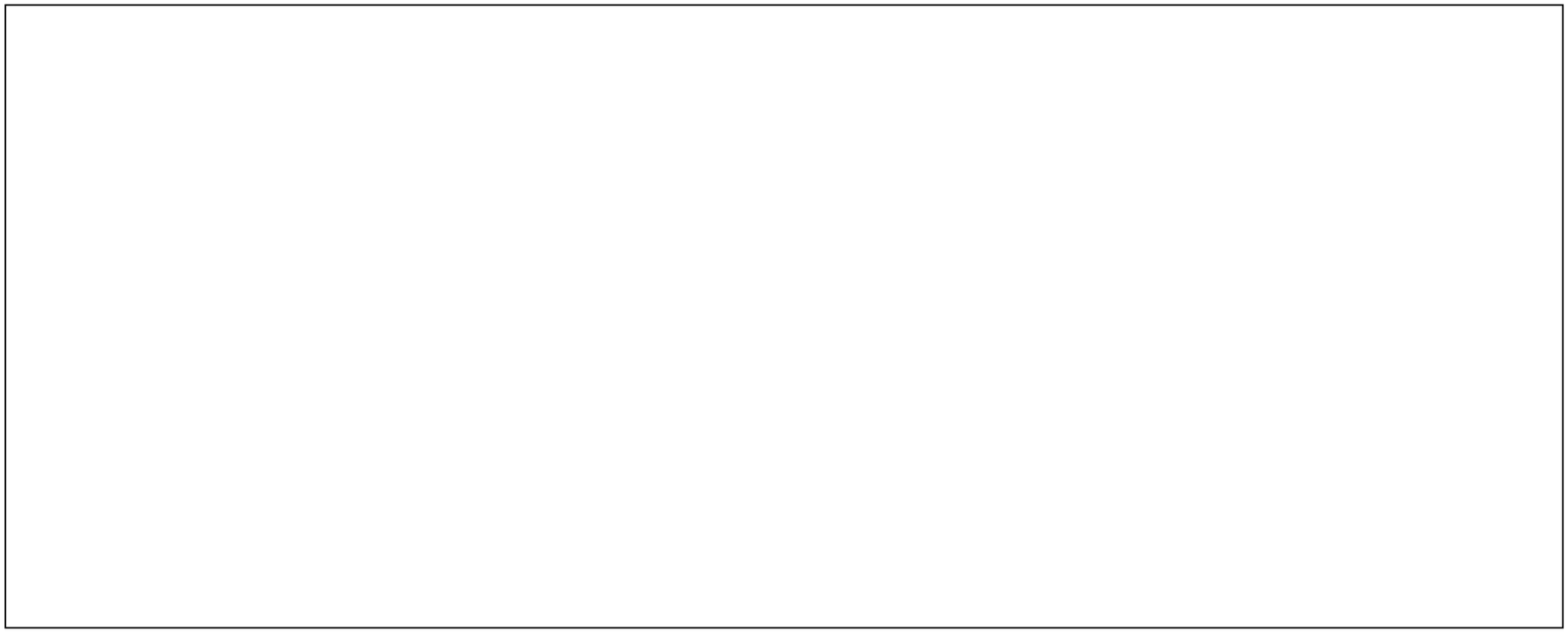
Purpose:

Teacher will monitor student progress by circulating to the groups and asking the following questions. These questions can be used for any of the "classroom" designs.

- Tell me how your equations represent your picture.
- How do your equations prove the area of this classroom? (Teacher witnesses equations that do or do not equal 40 square units
- How can you be sure your answer is correct?
- Possible observations: total area is/is not equal to 40 square units; rectangles include or are missing the factor of 4; gaps or overlays exist; mixing of colors, equation and expressions are accurately written

Common Misunderstanding
Purpose: Present students with "Common Misunderstanding" illustration and ask students to identify the error. (separate expressions are multiplied and not added) See Appendix B, pg. 20.
Closure
Purpose: Students will complete the exit slip to determine level of understanding when applying the distributive property to find area. Students will complete self-assessment of their own understanding. See Appendix C, pg. 21.
Extension Task
Purpose: To increase students' understanding of using the distributive property to find the area, use the link to the following Illustrative mathematics task. This task challenges students to use the distributive property with more than two expressions. https://www.illustrativemathematics.org/illustrations/1836

DEMONSTRATION - CLASSROOM FLOOR LAYOUT

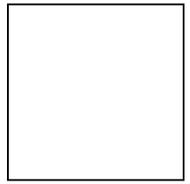


Task Instructions:

1. Use color tiles to fill this rectangle.
2. Choose two colors for your design.
3. Make 2 rectangles inside the figure.
4. Colors may not be mixed.
5. Write an equation using the distributive property to represent the area of the whole rectangle.

Use numbers and words to show the total number of square units needed to cover the area of this rectangle.

Names _____

 = 1 square foot

PRINCIPAL'S OFFICE FLOOR WORK MAT

Task Instructions:

1. Use color tiles to fill this rectangle.
2. Choose two colors for your design.
3. Make 2 rectangles inside the figure.
4. Colors may not be mixed.
5. Use the recording grids to show your designs by coloring the grids to match your designs. Then write an equation using the distributive property to represent the area of the whole rectangle.

DESIGN GRIDS to REPRESENT THE PRINCIPAL'S OFFICE FLOOR

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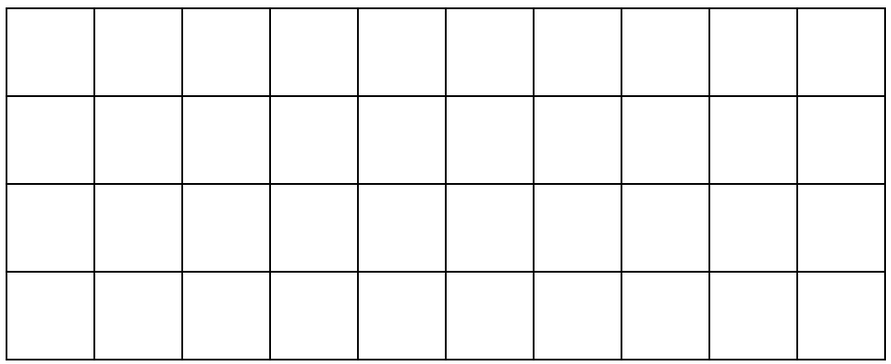
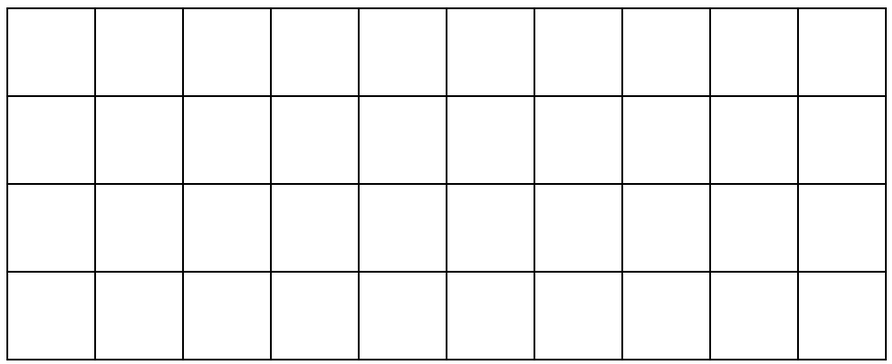


WORKSPACE



WORKSPACE

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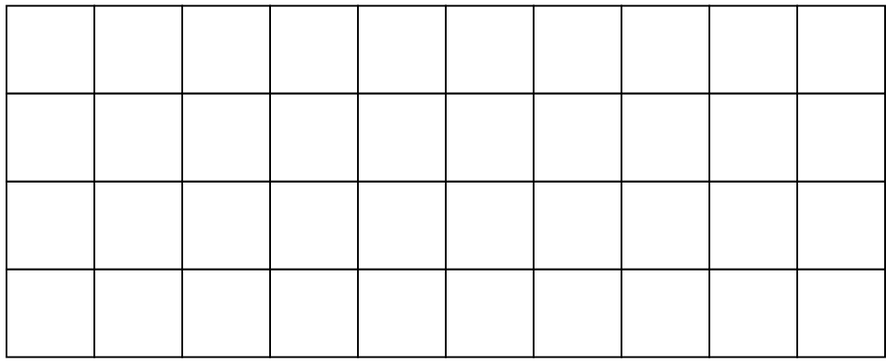
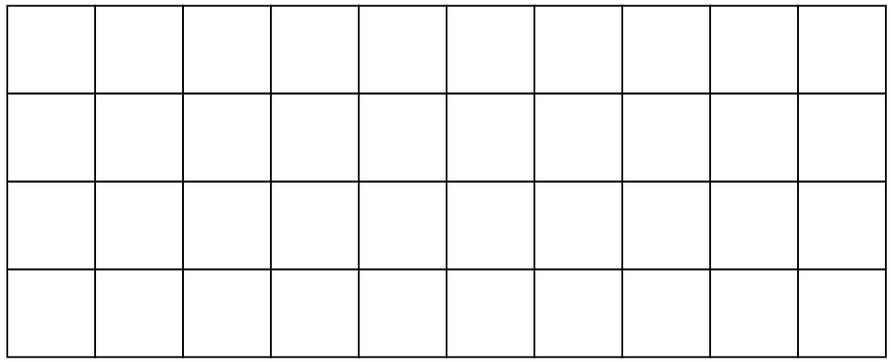


WORKSPACE



WORKSPACE

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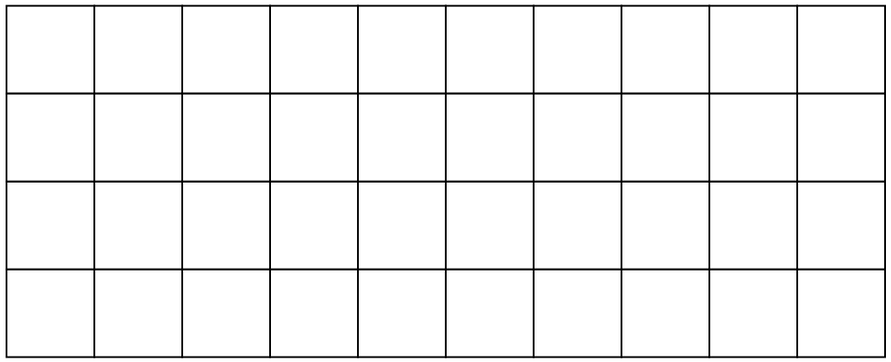

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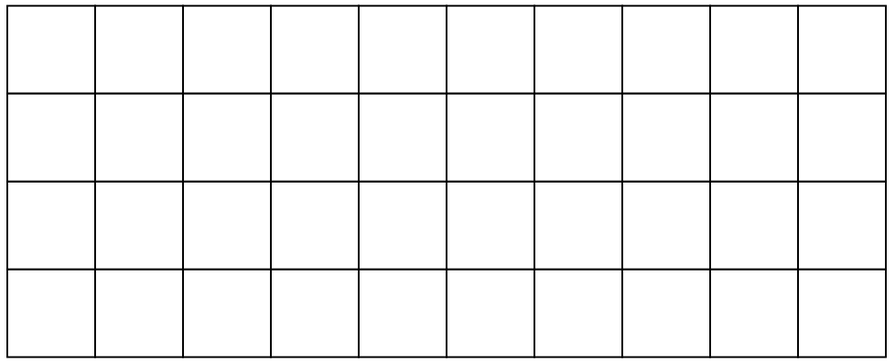


WORKSPACE

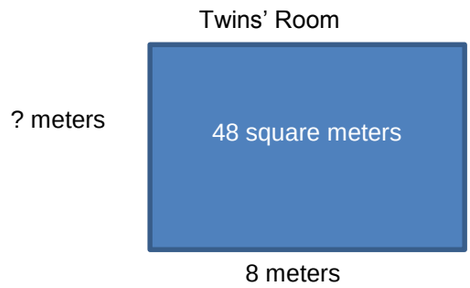
Bedroom Area

Content Standard: 3.MD.7. Relate area to the operations of multiplication and addition.
a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
Standards for Mathematical Practice:

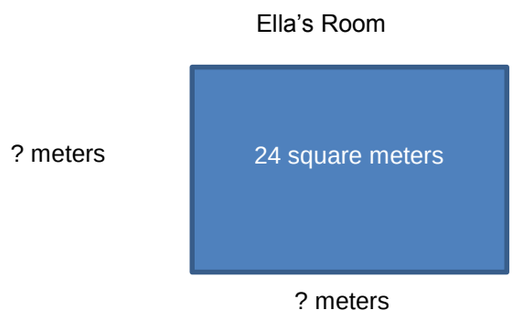
MP.2 Reason abstractly and quantitatively
MP.3 Construct viable arguments and critique the reasoning of others.

Twin brothers Adam and Robbie share a bedroom that is shaped like a rectangle. They know that the room has an area of 48 square meters.

Part 1: Adam measures and finds that the room is 8 meters long. He says it must be 16 meters wide, because 8 + 8 + 16 + 16 = 48. Robbie says the bedroom is 6 meters wide, because 8 x 6 = 48. Who is correct? Use pictures, numbers, and/or words to explain your thinking.



Part 2: Their sister Ella has a rectangular bedroom with an area of 24 square meters. What could be the length and width of Ella's bedroom? Give two different possibilities.

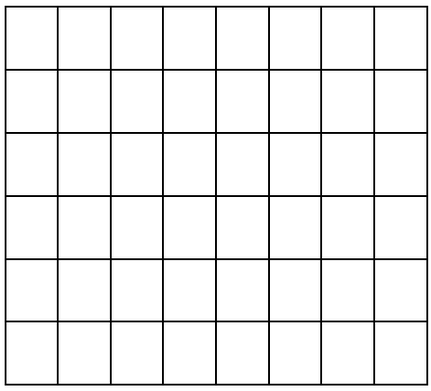


Bedroom Area (Solution)

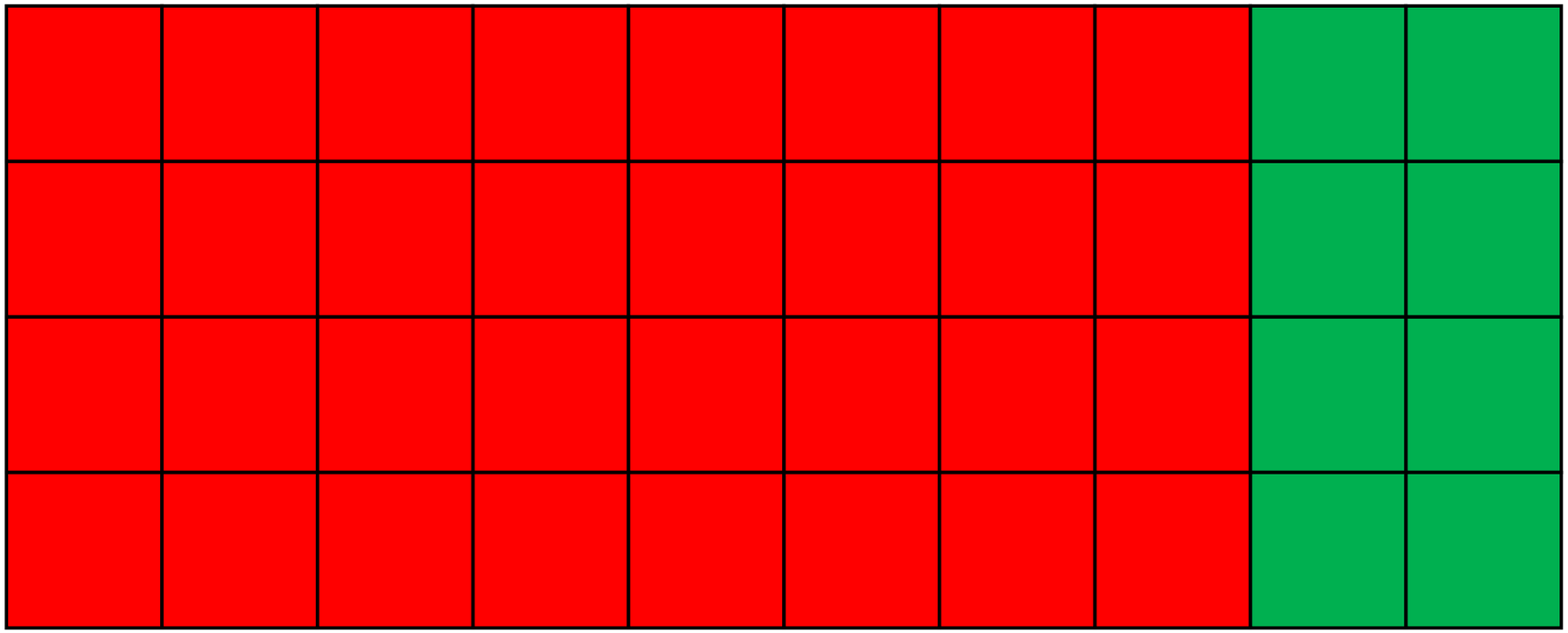
Content Standard: 3.MD.7. Relate area to the operations of multiplication and addition.
a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
Standards for Mathematical Practice:

MP.2 Reason abstractly and quantitatively
MP.3 Construct viable arguments and critique the reasoning of others.

Part 1 – Robbie is correct. Area = length x width. The area of the room is 48 square meters. 48 = 8 x 6, so the width has to be 6 meters. Students may draw a diagram that shows a rectangle with an array, such as this:



Part 2 – Students should list two of the following choices: Ella's bedroom could be 1 by 24, 2 by 12, 3 by 8, or 4 by 6 meters.

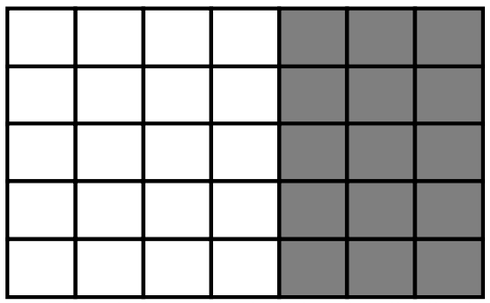


Cory says the area of the whole rectangle is 256 square feet because 32 x 8 = 256. Do you agree with Cory? Why or why not?

Name _____

APPENDIX C - LESSON EXIT SLIP

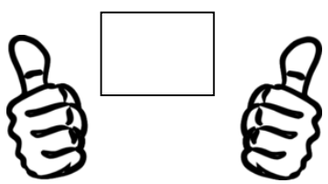
The picture below is the flooring design of a new office. Use the model to measure the area of the entire rectangle.

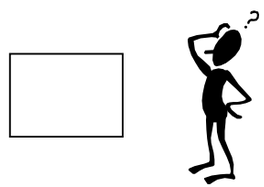


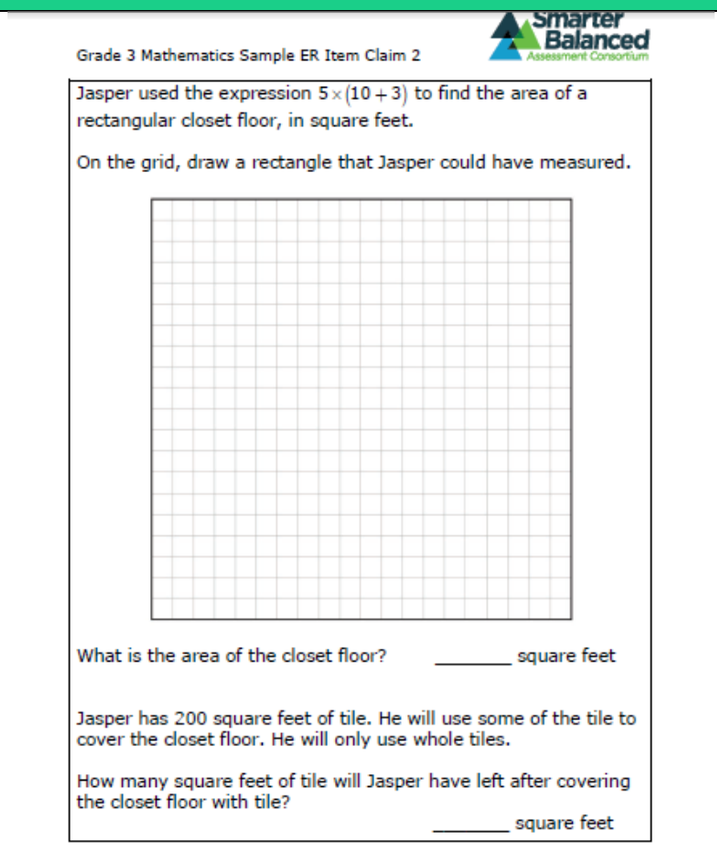
Express the area of the whole rectangle using the distributive property.

Self-Assessment - Place a check in the box that describes how you think you did on finding the answer.

My answer shows that I can represent the area of a rectangle using the distributive property.	I am not confident that my answer uses the distributive property to find the area of the rectangle.





<table>
<tr><th colspan="2">Research and review of standard</th></tr>
<tr><th>Content Standard(s):</th><th>Standard(s) for Mathematical Practice:</th></tr>
<tr><td>3.MD.C.7
Geometric Measurement: understand concepts of area and relate area to multiplication and addition.
7. Relate area to operations of multiplication and addition.
c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a x b and b x c. Use area models to represent the distributive property in mathematical reasoning.</td><td>2 - Reason abstractly and quantitatively</td></tr>
<tr><th>Smarter Balanced Claim</th><th>Smarter Balanced Item</th></tr>
<tr><td>Primary Claim - Claim 2 Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.</td><td> 
</td></tr>
<tr><td>CPR Pre-Requisites
(Conceptual Understanding, Procedural Skills, and Representations)</td><td>Conceptual Understanding and Knowledge
understand a plane figure's area is measured by tiling it with square units leaving no gaps or overlays
understand a rectangle can be partitioned into 2 smaller rectangles
understand that area is always additive as represented by the distributive property
understand properties of rectangles
Procedural Skills
counting unit squares
partitioning rectangles
tiling to leave no gaps or overlays
addition and multiplication skills
Representational
understand arrays are representational of rectangles
understand rectangles can be decomposed as arrays
understand that the distributive property can be used to measure the area of a rectangle
represent arrays with an equation
Social knowledge
definition of area
understand how floors and walls are tiled with square units and are measured to determine area</td></tr>
</table>

Standards Progression		
Grade(s) below	**Target grade**	**Grade(s) above**
2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. 2.G.A.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	3.OA.A.1 - Interpret products of whole numbers, e.g. interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. 3.OA.C.7 - Multiply and divide within 100 3.MD.5.b - A plane figure that can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. 3.MD.5.a - A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. 3.MD.6 - Measure areas by counting unit squares (square cm, square m, square in, square ft., and improvised units). 3.MD.7.a - Find the area of a rectangle with whole number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. 3. MD.7.b - Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems and represent whole-number products as rectangular areas in mathematical reasoning.	4.MD.3 Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. 5.MD.5 Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.

Common Misconceptions/Roadblocks

What characteristics of this problem may confuse students?

- the context of "tiling" a wall
- the work area may be used as an exact model
- application of length and width

What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?

- inability to apply length and width as representational of an area model and connect it conceptually to an array
- area is measured with square units and it's not a linear measurement
- not understanding that the distributive property is an effective strategy when finding the area of rectangles

What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?

- using multiplication to combine instead of addition
- overuse of the work area provided and not using the information provided