Mathematics Instructional Cycle Guide

A.CED.2: Create equations in 2 or more variables and use them to solve problems

Created by William McKinney, 2014 Connecticut Dream Team teacher

CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

- A.CED.2: Create equations in two or more variables and use them to solve problems.

This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

- MP.1: Make sense of problems and persevere in solving them.
- MP.3: Construct viable arguments and critique the reasoning of others
- MP.4: Model with mathematics

WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings **(p2)**
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint **(pp3-8)**
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed **(pp9-13)**
- Supporting lesson materials **(pp14-22)**
- Precursory research and review of standard **A.CED.2** and assessment items that illustrate the standard **(pp23-26)**

HOW TO USE THIS DOCUMENT

1) Before the lesson, administer the **Calorie Task** ***Mathematical Checkpoint*** individually to students to elicit evidence of student understanding.
2) Analyze and interpret the student work using the ***Student Response Guide***
3) Use the next steps or ***follow-up lesson plan*** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

MATERIALS REQUIRED

- Copies of Check-Point
- Copies of Extension Task Handout
- Chart paper with Stations Activities
- Post-It Notes
- Markers
- Graph Paper (in case students choose to solve using graph paper)

TIME NEEDED

Calorie Task administration: **10 minutes**
Follow-Up Lesson Plan: **75 minutes**

Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

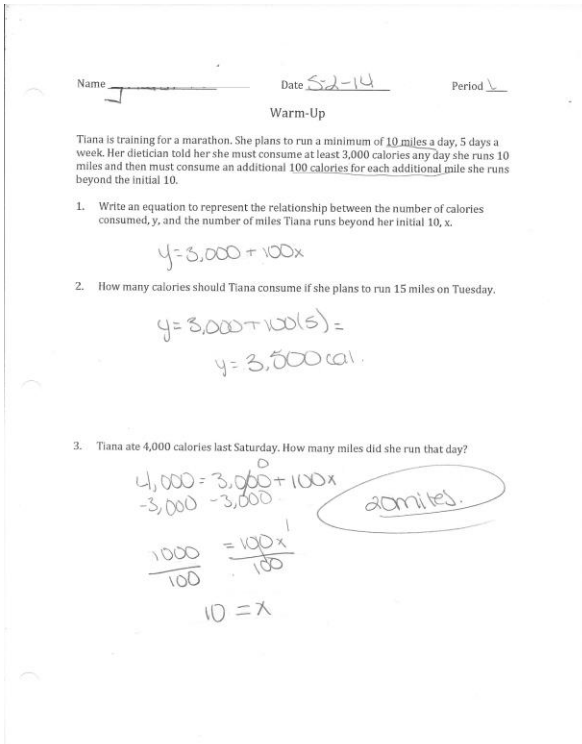
<table>
<tr><th colspan="3">Step 1: Elicit evidence of student understanding</th></tr>
<tr><th colspan="3">Mathematical Checkpoint</th></tr>
<tr><th>Question(s)</th><th colspan="2">Purpose</th></tr>
<tr><td rowspan="2">Tiana is training for a marathon. She plans to run a minimum of 10 miles a day, 5 days a week. Her dietician told her she must consume at least 3,000 calories any day she runs 10 miles and then must consume an additional 100 calories for each additional mile she runs beyond the initial 10.

1) Write an equation to represent the relationship between the number of calories consumed, y, and the number of miles Tiana runs beyond her initial 10, x.
2) How many calories should Tiana consume if she plans to run 15 miles on Tuesday?
3) Tiana ate 4,000 calories last Saturday. How many miles did she run that day?</td><td>CT Core Standard:</td><td>A.CED.2: Create equations in 2 or more variables and use them to solve problems.</td></tr>
<tr><td>Target question addressed by this checkpoint:</td><td>• How do students approach a linear modeling context when the value of x has been adjusted to already include a given amount of y?
• Can students model a linear relationship to determine specific values within the context of the problem?
• Can students interpret the results and relate it back to the original question to verify that it makes sense in the context of the problem?</td></tr>
</table>

LearnZillion

Step 2: Analyze and Interpret Student Work		
Student Response Guide		
Got It	**Developing**	**Getting Started**

Got It


Name J Date 5-2-14 Period 1

Warm-Up

Tiana is training for a marathon. She plans to run a minimum of 10 miles a day, 5 days a week. Her dietician told her she must consume at least 3,000 calories any day she runs 10 miles and then must consume an additional 100 calories for each additional mile she runs beyond the initial 10.

1. Write an equation to represent the relationship between the number of calories consumed, y, and the number of miles Tiana runs beyond her initial 10, x.

   y = 3,000 + 100x

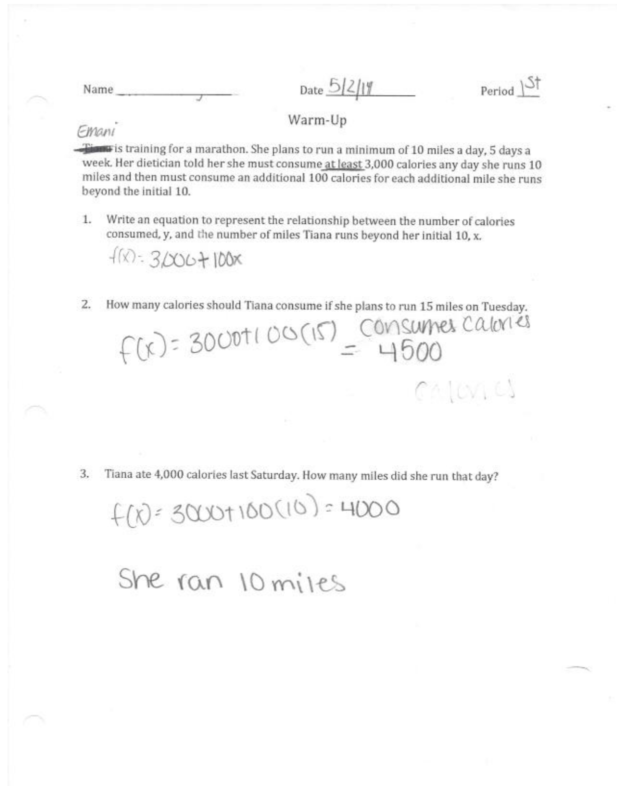
2. How many calories should Tiana consume if she plans to run 15 miles on Tuesday.

   y = 3,000 + 100(5) =
   y = 3,500 cal.

3. Tiana ate 4,000 calories last Saturday. How many miles did she run that day?

   4,000 = 3,000 + 100x
   -3,000 -3,000
   1000/100 = 100x/100
   10 = x
   20 miles.

Developing


Name Date 5/2/14 Period 1st

Warm-Up

Emani
[redacted] is training for a marathon. She plans to run a minimum of 10 miles a day, 5 days a week. Her dietician told her she must consume at least 3,000 calories any day she runs 10 miles and then must consume an additional 100 calories for each additional mile she runs beyond the initial 10.

1. Write an equation to represent the relationship between the number of calories consumed, y, and the number of miles Tiana runs beyond her initial 10, x.

   f(x) = 3000 + 100x

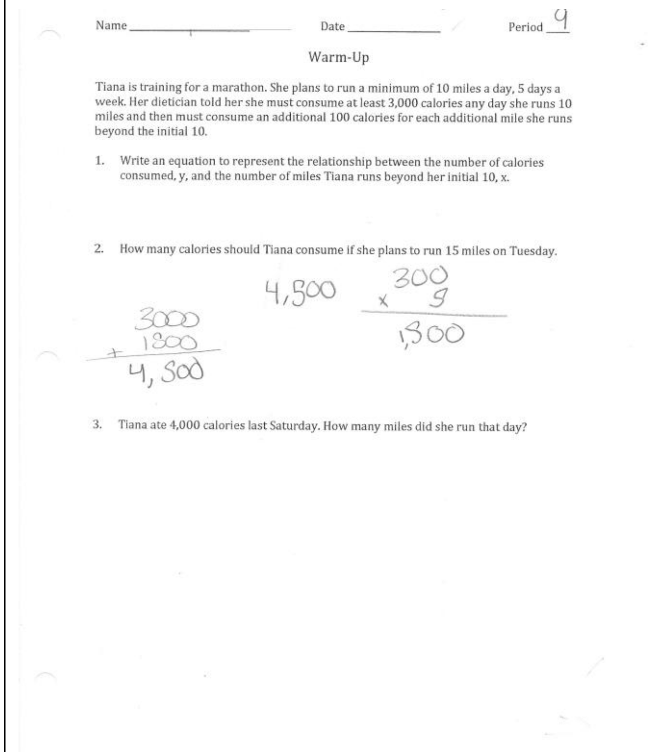
2. How many calories should Tiana consume if she plans to run 15 miles on Tuesday.

   f(x) = 3000 + 100(15) = 4500 consumes calories
   calories

3. Tiana ate 4,000 calories last Saturday. How many miles did she run that day?

   f(x) = 3000 + 100(10) = 4000
   She ran 10 miles

Getting Started


Name Date Period 9

Warm-Up

Tiana is training for a marathon. She plans to run a minimum of 10 miles a day, 5 days a week. Her dietician told her she must consume at least 3,000 calories any day she runs 10 miles and then must consume an additional 100 calories for each additional mile she runs beyond the initial 10.

1. Write an equation to represent the relationship between the number of calories consumed, y, and the number of miles Tiana runs beyond her initial 10, x.

2. How many calories should Tiana consume if she plans to run 15 miles on Tuesday.

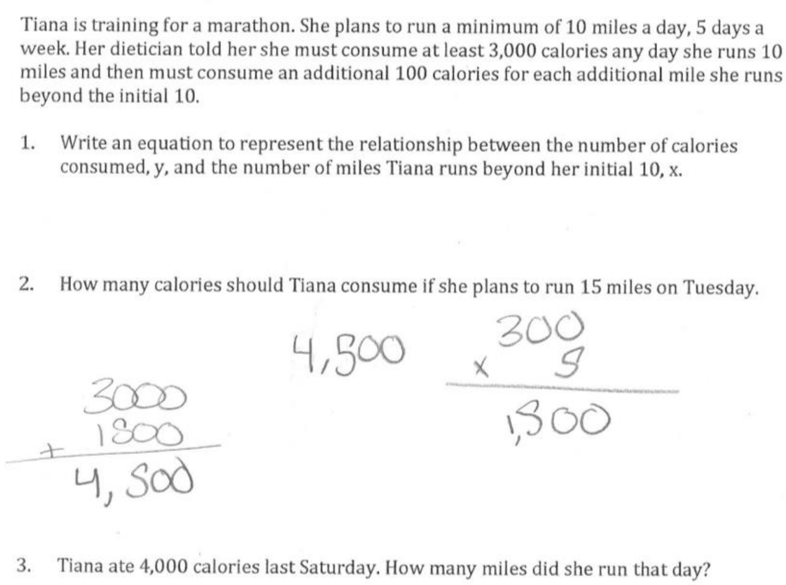
   4,500
   300 × 9 = 1,500
   3000 + 1500 = 4,500

3. Tiana ate 4,000 calories last Saturday. How many miles did she run that day?

LearnZillion

Getting Started

Student Response Example


Tiana is training for a marathon. She plans to run a minimum of 10 miles a day, 5 days a week. Her dietician told her she must consume at least 3,000 calories any day she runs 10 miles and then must consume an additional 100 calories for each additional mile she runs beyond the initial 10.

1. Write an equation to represent the relationship between the number of calories consumed, y, and the number of miles Tiana runs beyond her initial 10, x.

2. How many calories should Tiana consume if she plans to run 15 miles on Tuesday.

4,500

300 × 9 = 1,300

3000 + 1800 = 4,500

3. Tiana ate 4,000 calories last Saturday. How many miles did she run that day?

Indicators

- The student is unable to write a linear equation or unable to use the equation to evaluate or solve the problem.
- The student will not attempt to use the equation, create a table of values, or a graph to try and solve the problem. This student response shows procedural fluency of how to write an equation, but lacks an understanding of how to apply the equation to solve real world problems.
- This student doesn't understand how an equation relates the two variables; that is, the student doesn't understand that one can determine the value of y given x, or x given y. Student understanding of the equation is superficial, and doesn't show depth of understanding that equations are conduits for expressing the relationship between variables.

In the Moment Questions/Prompts

Q: What is the purpose of writing an equation?

Q: What does the x-variable represent in this problem? The y-variable?

Q: What does the slope represent in this problem? The y-intercept?

Q: Do you know any strategies besides using the equation to examine the pattern?

Q: What if you were to create a table of values? What would that table reveal?

Closing the Loop (Interventions/Extensions)

The following video could help students learn to write an equation in slope-intercept form from a real-world scenario. The video defines a linear relationship and walks students through the process of writing a linear equation and interpreting word problems. Students will learn how to set up and solve a linear equation. Students will also learn to graph linear relationships, which provide students an alternative approach and way to visualize the function.

https://ctdreamteam.learnzillion.com/lessons/2526-write-and-graph-a-linear-function-by-examining-a-reallife-scenario

Developing

Student Response Example

[redacted] is training for a marathon. She plans to run a minimum of 10 miles a day, 5 days a week. Her dietician told her she must consume at least 3,000 calories any day she runs 10 miles and then must consume an additional 100 calories for each additional mile she runs beyond the initial 10.

1. Write an equation to represent the relationship between the number of calories consumed, y, and the number of miles Tiana runs beyond her initial 10, x.

   $f(x) = 3{,}000 + 100x$

2. How many calories should Tiana consume if she plans to run 15 miles on Tuesday.

   $f(x) = 3000 + 100(15)$ consumes calories $= 4500$ calories

3. Tiana ate 4,000 calories last Saturday. How many miles did she run that day?

   $f(x) = 3000 + 100(10) = 4000$

   She ran 10 miles

Indicators

- The student wrote an accurate linear equation. The student used the equation to evaluate for given values of x in order to find a value of y, or solved the equation for x given a value of y.
- A student might use the equation directly to evaluate and solve. Alternatively, the student might use a table of values to answer the question. The work shows an understanding of the relationship between the x- and y-values of the given pattern.
- This student interpreted his/her results incorrectly. The student assumes that the given value of x represents the total number of miles run, instead of the additional miles run beyond the initial 10 miles.
- This mistake is most obvious in the third question, when the student claims that the runner ran 10 miles on the day he consumed 4,000. Yet, in the question itself, the dietician states that if you run 10 miles, you should only consume 3,000 calories. There is a direct contradiction of information here that the student fails to recognize.
- This mistake might also appear in the student's table of values, if students label the x-column as number of miles run instead of number of miles run beyond the initial 10.

In the Moment Questions/Prompts

Begin redirecting student thinking by first analyzing the results to question 3.

Q: Should Tiana consume 3,000 or 4,000 calories if she runs 10 miles?

Q: What was the dietician's recommendation? Is she supposed to consume 100 calories for every mile she runs?

Closing the Loop (Interventions/Extensions)

It might be necessary to reemphasize the definition of variables. The problem clearly defines x as the number of miles Tiana runs beyond the initial 10. These students would benefit from a lesson that emphasizes what each variable represents. The student work shows a fluency in the required skills, but a lack of understanding of the variables.

The piecewise approach used to extend the "Got It" students (described on the next page) may be beneficial to the "Developing" students as it further

Q: If Tiana consumes just 100 calories for each mile she runs, how many calories should she consume if she runs 10 miles? Is this consistent with what the dietician recommended?	emphasizes that Tiana only consumes 100 calories per mile for each mile she runs BEYOND her initial 10. It may also be helpful to discuss with students that the more you exercise, the more calories you must consume since you're burning calories as you run. This might be social knowledge that students are unaware of.

Got it

Student Response Example

Tiana is training for a marathon. She plans to run a minimum of 10 miles a day, 5 days a week. Her dietician told her she must consume at least 3,000 calories any day she runs 10 miles and then must consume an additional 100 calories for each additional mile she runs beyond the initial 10.

1. Write an equation to represent the relationship between the number of calories consumed, y, and the number of miles Tiana runs beyond her initial 10, x.

$y = 3{,}000 + 100x$

2. How many calories should Tiana consume if she plans to run 15 miles on Tuesday.

$y = 3{,}000 + 100(5) =$

$y = 3{,}500$ cal.

3. Tiana ate 4,000 calories last Saturday. How many miles did she run that day?

$4{,}000 = 3{,}000 + 100x$

$-3{,}000 \quad -3{,}000$

$\frac{1000}{100} = \frac{100x}{100}$

$10 = x$

20 miles.

Indicators

- The student correctly writes a linear equation in slope-intercept form that describes the relationship between the x- and y-variables. The student is also able to use the equation or a table of values to accurately answer the questions. The student understands that x represents the number of miles run beyond the initial 10 and accurately interprets his/her results to reflect this fact.
- The student might choose to evaluate the equation to determine how many calories to consume. Additionally, the student would solve the equation for x to determine how many additional miles Tiana ran. Alternatively, the student might choose to create a table of values. Both representations show an understanding of how the two variables are related.
- The student should show a full understanding of the problem and not display any of the common misconceptions.

In the Moment Questions/Prompts

Q: Can you think of other ways to approach this problem?

Closing the Loop (Interventions/Extensions)

Students that display a full understanding of this problem should be given the Extension Task Handout. This task will ask students to consider the same problem, but defined as a piece-wise function over multiple intervals. The video

Q: In question 2, how would you explain to someone else why you substituted in the value you did for x? Where did that value come from? **Q:** Your final solution to question 3 is different from the value you found for x. Why is that so?	below demonstrates how to write and graph a piece-wise function, which will ultimately help students approach the extension task. Students might choose to graph the piece-wise function as a way to visualize which interval the solution will lie in. https://ctdreamteam.learnzillion.com/lessons/2957-graph-piecewise-functions

Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction	
Lesson Objective:	Write, evaluate, and solve a linear equation to model a real-world situation
Content Standard(s):	**A.CED.2**: Create equations in 2 or more variables and use them to solve problems
Targeted Practice Standard :	**MP.1: Make sense of problems and persevere in solving them** *How do students explain the meaning of the problem in the Miles Run v. Calories Consumed Check-Point and look for entry points to its solution? Are students able to explain the correspondence between the variables and how the equation models the given situation?* **MP.3: Construct viable arguments and critique the reasoning of others** *Do students attend to the meaning of the variables in the Miles Run v. Calories Consumed Check-Point? Are students able to understand that their solutions indicate how many ADDITIONAL miles Tiana ran, NOT her TOTAL number of miles run? Can they rationalize that a solution is inaccurate because it directly contradicts given information in the problem?* **MP.4: Model with mathematics** *Can students write a linear equation that shows the relationship between two variables and use this equation to determine specified values according to a designated context?*

Mathematical Goals	**Success Criteria**
Students will demonstrate an understanding of how to… • *write a linear equation to model the relationship between two variables.* • *evaluate an equation for a given value of x.* • *solve an equation to find a value of x. and solve the equation.*	*Students must be able to…* • *define what different variables represent within the context of a problem.* • *substitute a given value into the correct variable in a linear equation.* • *Accurately interpret the values of the variables.*

Launch (Probe and Build Background Knowledge)

Purpose: ***Initiate student understanding that marginal change is not the same at total change***

1.) **Think**-Pair-Share

The following warm-up problem will be written on the board (or on a piece of chart paper) prior to students arriving to class. The problem is designed to help guide the students' thinking about the largest misconception from the check-point: misinterpreting the meaning of the variables within the context of the problem. The warm-up also models what a linear equation looks like while framing it like a two-step problem they've worked on in earlier grades. As students arrive, direct them to begin thinking individually about the problem and to write an initial reflection.

Johnny wants to buy a new gaming system that includes 2 pre-loaded video games and 1 controller for $300. He wants to purchase 3 more games for $60 each, for a total of 5 games. Johnny tells his mom that he needs

$300 + ($60 per game)(5 games) = $600

to buy the system and the 5 games. Is he asking his mom for the right amount of money or not? Explain your reasoning.

2.) Think-**Pair**-Share

After about 3 minutes of individual student reflection, ask students to share their reflections with their neighbors. Pairs must come to a consensus and write a single reflection in support of or against Johnny's math. When pairs have finished, ask students to hang their reflections on the white board according to whether they think Johnny did his math correctly or incorrectly. This will give you a good gauge as to whether or not the students understand that Johnny only spends $60 on three of the games. Students that agree should hang their reflections on the left side of the problem. Students that disagree should hang their reflections on the right side of the problem. Once all the student reflections are on the board, begin a class discussion of the answer.

3.) Think-Pair-**Share**
Students should be able to explain that Johnny only needs to pay for three of the games; two games are included with the console. Each **additional** game costs $60. So if Johnny gets $600, he is actually getting the system, the two included games, plus five additional games, for a total of seven games.

Prompts/questions during the discussion might include:
- What is included in Johnny's initial $300 purchase?
- Can you create a table that shows the total number of games Johnny has and the total price spent?
- Does the total price spent change with the addition of *every* game?
- Does Johnny spend $60 on ALL the games?
- What is the difference between the phrases "cost per game" and "cost per 'additional' game?"

4.) Extension
To push the students further, ask students, in pairs with their neighbor, to write an equation to model the scenario. Be sure to have students define the variables. Circulate around the room to listen in on conversation and prompt students' thinking.

Prompts/questions during the discussion might include:
- How would you define the variables?
- Did you define x as the total number of games he has or the number of additional games he purchased? How does the way in which you define the variable change the way in which you write your equation?
  - If x = total number of games: y = 60(x – 2) + 300
  - If x = number of additional games: y = 60x + 300
  - What are the benefits of each representation (i.e. point-slope v. slope-intercept forms)?
- What does the slope represent in the context of this problem?
- What does the y-intercept represent in the context of this problem?

5.) Share Out
Ask students to share out their equations. Write their responses on the board, along with how they chose to define their variables. You may see multiple representations depending on how students chose to define their variables. Below are a few possibilities.

- X represents the total number of games purchased
  - y = 60(x – 2) + 300
  - y = 300 + 60(x – 2)
  - y – 300 = 60(x – 2)
- X represents the additional games purchased
  - y = 60x + 300
  - y = 300 + 60x

Instructional Task

Engage and Explore

6.) Provide the Rationale Behind the Lesson

Explain that many misconceptions arise when defining variables and writing equations that coincide with the variables chosen. Today students will learn to write linear equations that model real-world situations by analyzing student work for accuracy and verifying that it makes sense within the provided context. This is a lesson on close reading and supporting your arguments.

7.) Give Directions / Introduce Primary Task
Instruct students that 5 questions are stationed around the room. Each problem has already been solved. Several are solved incorrectly, but at least one problem has been solved correctly. Split the students into small heterogeneous groups of 3-5. In their small groups, students will rotate through each station and determine whether the problem was solved correctly or incorrectly. If the problem was solved correctly, students are to write down at least one other method for solving the problem. If the problem was solved incorrectly, students are to explain what the student did incorrectly and why his or her mistake is incorrect. Students will have 8-10 minutes at each station. If you have a shorter class, you might choose to do only 3-4 stations or move onto the discussion section after students complete 2-3 stations.

Since the teacher cannot be at each station the entire cycle, each poster has been designed to spark discussion among the students. Posters will have a set of sticky notes that will cover written prompts designed to help them think more deeply about a specific misconception or element of the problem. Set an alarm for 3 minutes. When students hear the alarm sound, they may lift the sticky note to reveal the clue below. When students have finished their analysis, they will rotate clockwise until they have been to all stations.

8.) Provide students with time to work on the Error Analysis Stations activity in groups
Students will record the misconception on the provided handout with an explanation of why what the student did is incorrect. Circulate to observe, question, and note students who are strategic candidates to share out responses. Possible questions/prompts to ask students to help them engage in the task include:

9.) Extra Time
If any groups finishes before it is time for groups to rotate, have the group consider alternative representations and methods for solving the problem. This is similar to the task students must complete if the problem was solved correctly.

Elaborate

10.) Facilitate a class discussion of the task after students have rotated through each of the stations. Discussion prompts will include prompts/questions like:

- How many of the stations were solved correctly?
- How many students think Station X was solved correctly? If not, what was the big mistake the student made? Can anyone summarize the mistake and why the student's thinking is incorrect?
- What would you have to do to fix the student's mistake?
- What is something that each of these problems had in common with one another? What did you have to pay close attention to when solving these problems?
- Based on how the prompts asked you to define the variables, would it have been appropriate to write the equations in point-slope form?
- Could point-slope form of a linear equation have facilitated the analysis portion of the problems in any way if we were able to redefine what x represented?
- Are there any misconceptions that would have arisen from using point-slope form instead of slope-intercept form?
- Looking at Station B, assume we redefined x to represent how many students went on the camping trip in total and wrote the equation as $y = 5(x – 10) + 100$. Would it be correct to say that the cost for 6 students to go camping is $y = 5(6 – 10) + 100 = 5(-4) + 100 = \$80$? Explain why or why not.

It is important to explain that both slope-intercept and point-slope forms present a different set of misconceptions, but both require you to properly assess what the variable represents within the context of the problem and to justify whether or not your solution makes sense within the given context. It's always important to ask yourself whether or not you've answer the given question and whether your answer makes sense with the given information.

Checking for Understanding

Purpose: Give the following question as an Exit Slip. Have students answer the questions in their math journals, answer the prompt from the Closure, and then turn in their response as they leave class.

Nathan does his laundry at the local laundromat, which doesn't take coins, but instead uses a special card. The card costs $20 and gives you 10 wash and dry cycles. Each additional wash and dry cycle costs another $1.50. Customers are free to add value to their card whenever they like at the counter. If a customer loses his card, he must purchase another card at the counter.

(a) Write an equation that represents the total cost Nathan spends at the laundromat, y, with respect to how many additional wash and dry cycles he must purchase, x.
(b) If Nathan purchases a wash card for $50, how many wash and dry cycles can he run?
(c) Nathan does one load of laundry each week. How much money will he spend at the laundromat in one year?

Common Misunderstanding

Purpose: ***Verify that students have a thorough understanding of the common misconceptions that arise from word problems that involve writing and solving linear equations.***

While students are working on the Check for Understanding, ask the students probing questions to check for the common misunderstandings that were addressed in the lesson: incorrectly writing the equation, misinterpreting the variables, misinterpreting units, and incorrectly substituting values into the wrong variable. Circulate around the room as students work. Utilize the same types of questions and probes that were used in the main lesson to check for misunderstandings.

Closure

Purpose: ***Provide students an opportunity to self-assess their own learning related to the success criteria by having students respond to the following prompt in their math journals below where they wrote their response to their Exit Slip:***

1.) I can write a linear equation to model a real-world scenario.

Strong Disagree	**Disagree**	**Neither Agree nor Disagree**	**Agree**	**Strongly Agree**

2.) I can evaluate and solve a linear equation to solve real-world problems.

Strong Disagree	**Disagree**	**Neither Agree nor Disagree**	**Agree**	**Strongly Agree**

3.) I understand that the way in which we define a set of variables impacts how I analyze the results.

Strong Disagree	**Disagree**	**Neither Agree nor Disagree**	**Agree**	**Strongly Agree**

4.) After this lesson, I feel like I need more time learning ________________________________.

Extension Task

Students that are prepared for an extended learning opportunity will be introduced to the concept of piece-wise functions. In this case, students will be given a modified check-point problem with a set of conditions over specific intervals. The questions will remain virtually unchanged, but the strategy for how students approach the problem might vary some. Refer to the **Extension Task Handout** to see the modified check-point activity.

The biggest change for how students approach this problem comes in question 3, where students have to determine how many miles Tiana ran if she consumed 4,000 calories. Students can substitute 4,000 for y in each equation and see if the resulting value of x fits in the given interval. If it does not, students must reason that the solution is not feasible and check the next equation from the piece-wise function.

While students are working on the task, prompt them with questions to help direct their thinking. Questions might include

- How does defining Tiana's caloric intake over specific intervals affect your approach to this problem? What new considerations must you make?
- What other ways can you represent the information that might allow you to better visualize how many calories Tiana has consumed?
- Is there a way to determine the fewest and greatest number of calories Tiana can consume over each interval?
- When you originally did this problem, you found that Tiana had run 10 additional miles. Does this answer impact how you might approach this problem?

When students have finished this task, ask them to compare their solution when using the piece-wise function verses when they solved it originally. Attempt to show as many different student approaches as possible. Display the different approaches on the board and ask students to compare and critique the reasoning of others. Prompt students with questions like

- What approaches did people use to solve this problem? Explain your rationale.
- How did changing Tiana's diet requirements affect how you approached the problem?

Mathematical Check-Point – Calorie Task

Tiana is training for a marathon. She plans to run a minimum of 10 miles a day, 5 days a week. Her dietician told her she must consume at least 3,000 calories any day she runs 10 miles and then must consume an additional 100 calories for each additional mile she runs beyond the initial 10.

1) Write an equation to represent the relationship between the number of calories consumed, y, and the number of miles Tiana runs beyond her initial 10, x.

2) How many calories should Tiana consume if she plans to run 15 miles on Tuesday?

3) Tiana ate 4,000 calories last Saturday. How many miles did she run that day?

Explore

The posters on the next 5 pages depict the problems that will be hung around the room for the Error Analysis portion of this lesson. Each poster has a word problem, set of questions, student work, and post-it note prompts. Below is a summary of the mistakes seen on each poster. Note that Station E was solved correctly, and students will therefore need to present an alternative representation or method for solving this problem.

Answer Key

The four misconceptions demonstrated in the first four stations include:

1. Station A: Misinterpreting what value should be substituted for a given variable (e.g. y = .25(4.75) + 1.5 = $2.69)
2. Station B: Misinterpreting the solution (e.g. x = 10 means 10 students went on the camping trip)
3. Station C: Incorrectly substituting a given value in for the wrong variable (e.g. y = 250 + 75(471) = 35,575 baskets)
4. Station D: Incorrectly writing the equation (reversing the slope and y-intercept; e.g. y = 1.50x + 0.25)
5. Station E: Solved correctly using equations. Students may choose to solve the problem using tables or a graph as an alternative approach. An alternative representation of the correct answer might be:

Hours of Overtime, x	0	1	2	3	4	5
Pay ($), y	400	415	430	445	460	475

LearnZillion

Station A

Arianna is going to a business conference in Washington, D.C. She decided to take a taxi from the airport to her hotel. The taxi costs \$1.50 for the first mile and then an additional \$0.25 for each additional quarter mile.

(a) Write an equation to represent the relationship between the price of the cab ride, y, and the number of miles Arianna rides beyond the initial mile, x.

(b) How much is Arianna's bill if she rides in the taxi for 4.75 miles? Show your work in the space provided.

(c) Arianna's bill is actually \$6.25. How many miles did she ride the taxicab? Show your work in the space provided.

---

a) $y = 0.25x + 1.50$

b) $y = 0.25(4.75) + 1.50$

$= 1.1875 + 1.50$

$\approx \$2.69$

c) $6.25 = .25x + 1.50$

$-1.50 \qquad -1.50$

$4.75 = .25x$

$\frac{4.75}{.25} = \frac{.25x}{.25}$

$19 = x$

Arianna rode the taxi for 19 miles.

Post-It Prompts:

1. When writing a linear equation, is the x multiplied with the slope or the y-intercept? Why?
2. What does the slope represent in this problem?
3. What does the y-intercept represent in this problem?

LearnZillion

Station B

Mr. Brady is chaperoning the senior camping trip. He found a great campsite that costs $100 for the first 10 people, and then an additional $5 per person beyond the initial 10.

(a) Write an equation to represent the relationship between the price of the campsite, y, and the number of people who go camping beyond the initial 10, x.

(b) How much is the total bill if Mr. Brady chaperones a camping trip with 18 students? Show your work in the space provided.

(c) Mr. Brady receives a bill for the campsite. If the bill is for $150, how many students went on the camping trip? Show your work in the space provided.

---

a) $y = 100 + 5x$

b) 18 students

$\Rightarrow x = 10$

$y = 100 + 5(10)$

$= 100 + 50$

$= \$150$

c) $150 = 100 + 5x$

$-100 \quad -100$

$50 = 5x$

$\frac{50}{5} = \frac{5x}{5}$

$10 = x$

10 students went on the camping trip.

Post-It Prompts:

1. According to the problem, what is the cost for 10 students to go camping? Do any of your answers contradict the cost as stated in the problem?
2. What does x represent in this problem?

Station C

Chris is a participant on a game show with his family. He is chosen to shoot blindfolded baskets. He earns his family 250 points if he makes 4 blindfolded baskets. Each additional basket will earn his family 75 points.

(a) Write an equation to represent the relationship between how many points Chris's family has earned, y, and the number of blindfolded shots Chris makes beyond the initial 4, x.

(b) How many points will Chris earn his family if he makes 8 blindfolded baskets? Show your work in the space provided.

(c) Chris earned his family 475 points. How many baskets did he make? Show your work in the space provided.

---

a) $y = 75x + 250$

b) 8 baskets $\Rightarrow x = 4$

$y = 75(4) + 250$

$= 300 + 250$

$=$ 550 points

c) 475 points $\Rightarrow$

$x = 471$

$y = 75(471) + 250$

$= 35{,}325 + 250$

$=$ 35,575 points

Post-It Prompts: (on question c)

1. What does x represent in this problem? What does y represent in this problem?
2. What does it mean if you substitute 475 in for x?
3. Do questions (b) and (c) ask the same thing? How are they different?

Station D

Derrick bought an aquarium with 5 tropical fish for $200. A month later he decided to purchase more fish. Each additional fish costs $15.

(a) Write an equation to represent the relationship between how much money Derrick spent in total, y, and how many additional fish he purchased, x.

(b) If Derrick has a total of 9 fish, how much money did he spend in total? Show your work in the space provided.

(c) Derrick spent a total of $335. How many fish does Derrick have in total? Show your work in the space provided.

---

a) $y = 200x + 15$

b) $9 - 5 = 4$ additional fish

$y = 200(4) + 15$

$= 800 + 15$

$= \$815$

c) $335 = 200x + 15$

$-15 \qquad\qquad -15$

$320 = 200x$

$\frac{320}{200} = \frac{200x}{200}$

$1.6 = x$

Derrick has $5 + 1.6 = 6.6$ fish in total.

Post-It Prompts: (on question b)

1. Was the purchase of the aquarium and the 5 fish a one-time purchase, or did he purchase it multiple times?
2. What does Derrick spend the $15 to purchase?

LearnZillion

Station E

Eric gets paid $400 a week for the first 40 hours he works and then receives $15 for each additional hour of overtime he works beyond the initial 40 hours.

(a) Write an equation to represent the relationship between how much Eric gets paid, y, and the number of hours of overtime he works, x.

(b) How much will Eric get paid if he works 60 hours this week? Show your work in the space provided.

(c) Eric's paycheck this week is for $490. How many hours did he work this week in total? Show your work in the space provided.

---

a) $y = 15x + 400$

b) 60 hrs − 40 hrs = 20 hrs overtime

$y = 15(20) + 400$

$= 300 + 400$

$= \$700$

c) $490 = 15x + 400$

$-400 \qquad -400$

$\frac{90}{15} = \frac{15x}{15}$

$6 = x$

Eric worked 6 hours of overtime for a total of 46 hours this week.

Post-It Prompts:

1. What other ways can you represent the relationship between two variables?

Extension Task

Tiana is training for a marathon. She plans to run a minimum of 10 miles a day, 5 days a week. Her dietician told her she must consume at least 3,000 calories any day she runs 10 miles. If she runs more than 10 miles, she must consume additional calories as indicated below.

Additional Miles Run, x	Required Additional Calories per Mile
$0 < x \leq 5$	100
$5 < x \leq 10$	150
$10 < x \leq 15$	200
$15 < x$	300

1) Create a piece-wise function to represent the relationship between the number of calories consumed, y, and the number of miles Tiana runs beyond her initial 10, x.

2) How many calories should Tiana consume if she plans to run 15 miles on Tuesday?

3) Tiana ate 4,000 calories last Saturday. How many miles did she run that day?

LearnZillion

Extension Task Answer Key

Tiana is training for a marathon. She plans to run a minimum of 10 miles a day, 5 days a week. Her dietician told her she must consume at least 3,000 calories any day she runs 10 miles. If she runs more than 10 miles, she must consume additional calories as indicated below.

Additional Miles Run, x	Required Additional Calories per Mile
$0 < x \leq 5$	100
$5 < x \leq 10$	150
$10 < x \leq 15$	200
$15 < x$	300

1) Create a piece-wise function to represent the relationship between the number of calories consumed, y, and the number of miles Tiana runs beyond her initial 10, x.

2) How many calories should Tiana consume if she plans to run 15 miles on Tuesday?

3) Tiana ate 4,000 calories last Saturday. How many miles did she run that day?

---

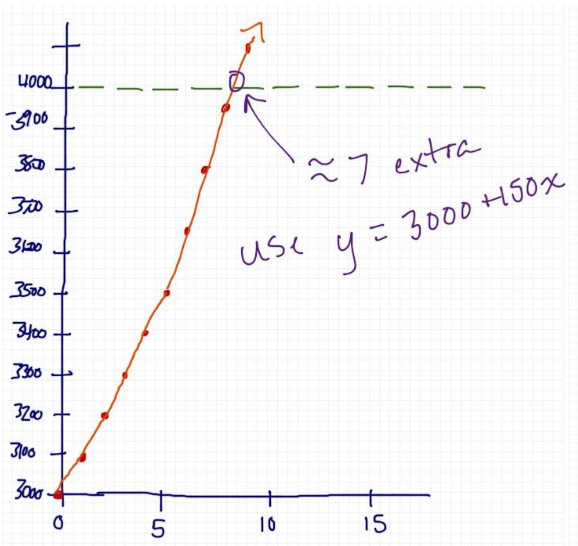
1)
$$y = \begin{cases} 3000 + 100x, & 0 < x \leq 5 \\ 3000 + 150x, & 5 < x \leq 10 \\ 3000 + 200x & 10 < x \leq 15 \\ 3000 + 300x, & 15 < x \end{cases}$$

2) 15 miles ⇒ 5 extra miles

$y = 3000 + 100(5)$

$= 3000 + 500$

$= 3500$ calories



3)
$4000 = 3000 + 100x$
$-3000 \quad -3000$
$\frac{1000}{100} = \frac{100x}{100}$
$10 = x$

Not the solution because $x > 5$

$4000 = 3000 + 150x$
$-3000 \quad -3000$
$\frac{1000}{150} = \frac{150x}{150}$
$6.\overline{6} = x$

Therefore, Tiana ran a total of $10 + 6.\overline{6} \approx 17$ miles that day.

Research and review of standard

Content Standard(s):

A.CED.2: Create equations in two or more variables and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

Standard(s) for Mathematical Practice:

MP.1: Make sense of problems and persevere in solving them.
MP.4: Model with mathematics.
MP.7: Look for and make use of structure.

Smarter Balanced Claim

Primary Claim: Problem Solving
Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

Smarter Balanced Item

Book Size	Base Price	Cost for Each Additional Page
7-in. by 9-in.	$20	$1.00
8-in. by 11-in.	$25	$1.00
12-in. by 12-in.	$45	$1.50

The base price reflects the cost for the first 20 pages of the photo book.

1. Write an equation to represent the relationship between the cost, *y*, in dollars, and the number of pages, *x*, for each book size. Be sure to place each equation next to the appropriate book size. Assume that *x* is at least 20 pages.

7-in. by 9-in.

8-in. by 11-in.

12-in. by 12-in.

2. What is the cost of a 12-in. by 12-in. book with 28 pages?

3. How many pages are in an 8-in. by 11-in. book that costs $49?

CPR Pre-Requisites

(Conceptual Understanding, Procedural Skills, and Representations)

Look at the Progressions documents, Learning Trajectories, LZ lesson library, unpacked standards documents from states, NCTM Essential Understandings Series, NCTM articles, and other professional resources. You'll find links to great resources on your PLC Platform.

Conceptual Understanding and Knowledge

- Understand the relationship between the independent and dependent variable.
- Understand the y-intercept as an initial value
- Understand slope as a rate of change; that is, as a ratio of the change in y to the change in x.
- Understand that the value of the dependent variable can be determined by evaluating the equation for a given value of x
- Understand that the value of the independent variable can be determined by solving the equation for x given a specific value for y

Procedural Skills

- Write linear equations in slope-intercept form
- Evaluate an algebraic expression by substituting a given value for the variable
- Solve a one-variable equation

	Representational • Flexibly complete the problem by solving equations algebraically, using a table, or using a graph **Social knowledge** • Know that slope is the coefficient of the independent variable and represents the cost per page • Know that the y-intercept is a constant term that represents the initial value or base price • Know that the base price includes the first 20 pages of the book • Know linear equations written in slope-intercept form as y = ax + b

Standards Progression **Look at LearnZillion lessons and expert tutorials, the Progressions documents, learning trajectories, and the "Wiring Document" to help you with this section*		
Grade(s) below	**Target grade**	**Grade(s) above**
6.EE.9: Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. 7.EE.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. 8.F.4: Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x,y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	A.CED.1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* A.CED.3: **Represent constraints by equations** or inequalities, and by systems of equations and/or inequalities, **and interpret solutions as viable or non-viable options in a modeling context.** F-IF.2:	A.CED.2: Create equations in two or more variables and use them to solve problems. *Include equations arising from linear and* ***quadratic functions, and simple rational and exponential functions****.* F-BF.1: Write a function that describes a relationship between two quantities.

Common Misconceptions/Roadblocks
What characteristics of this problem may confuse students? • Students might get confused by the base price including the first 20 pages. • Students may struggle with the fact that the base price represents the cost for the first 20 pages, only defining the function over the interval $x \geq 20$. Students may forget to subtract 20 from the total page count; the cost per additional page is only for pages beyond 20. **What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?** • Students may not understand whether to substitute given values in for x or y, especially if they don't understand how the variables were defined. • Students may struggle to write the given information as a linear function or may reverse the slope and y-intercept. • Students may substitute values of x or y into their equations for the slope or y-intercept. **What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?** • Students may assume that the base price is for 0 pages, not the first 20. • Students may write the equation in point-slope form, eg. $y = 1(x - 20) + 20$, which may give the misconception that the price is less than $20 if you print fewer than 20 pages.