Mathematics Instructional Cycle Guide

Concept A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

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CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

A-CED Create equations that describe numbers or relationships

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
7. Look for and make use of structure.

WHAT IS INCLUDED IN THIS DOCUMENT?
  - A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings **p.2**
  - A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint **p.3-7**
  - A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed **p.8-11**
  - Supporting lesson materials **p.14-23**
  - Precursory research and review of standard **insert standard code** and assessment items that illustrate the standard **p.24-27**

HOW TO USE THIS DOCUMENT
  1) Before the lesson, administer the **Pizza Party *Mathematical Checkpoint*** individually to students to elicit evidence of student understanding.
  2) Analyze and interpret the student work using the ***Student Response Guide***
  3) Use the next steps or ***follow-up lesson plan*** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
  4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

MATERIALS REQUIRED
  - **20 Quarters (play money) per group**
  - **5 One Dollar Bills (play money) per group**
  - **Pizza Topping cutouts (from handout) – 1 set per group**
  - **Handouts:**
  - **Pop quiz**
  - **Exit Ticket**
  - **Extension Handout**

TIME NEEDED
Pizza Party administration: **10 minutes**
Follow-Up Lesson Plan: **20 minutes**

Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

Step 1: Elicit evidence of student understanding		
Mathematical Checkpoint		
Question(s)		**Purpose**

Question(s)		Purpose
PIZZA	**CT Core Standard:**	A-CED.2 Create equations that describe numbers or relationships 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
	Target question addressed by this checkpoint:	In question form, note the conceptual understandings and procedural skills the checkpoint targets. (What is it you want to find out about students' understanding? Why are you targeting this concept?) For example: "Do students understand the value of a digit as it moves from a single to a double-digit number?" How do students approach writing equations to represent a relationship between quantities? To what extent do they… • Identify and interpret the relationship? • Identify unknown input and output quantities and use variables to represent these quantities? • Identify parameters? • Combine the parameters and variables as expressions (e.g. fixed and variable quantities)? • Discerning between an expression and an equation • Write the equation using the expressions?

PIZZA

Small – 9 inches – 6 slices		Large – 16 inches – 12 slices	
Plain	$5.10	Plain	$9.80
Additional Items	.50	Additional Items	1.40

(I) You are ordering pizza for a party and you want to order toppings that everyone likes but you have a limit on what you can spend on a single pizza. Identify the input and output quantities and label with a variable.

(II) Write an expression to represent the fixed and variable costs for a small pizza.

(III) Write an equation to represent the cost of a small pizza as a function of the number of toppings.

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Step 2: Analyze and Interpret Student Work		
Student Response Guide		
Got It	**Developing**	**Getting Started**
What will a response include from a student who has demonstrated conceptual understanding and mastery? A student who demonstrates mastery will do the following: • Indicate the functional relationship of total cost as a function of the number of toppings by choosing total cost as output (y) and # of toppings as input (x) • Choose the correction information from the table, i.e. values for small pizza • Write an expression with 5.10 as a constant and .50 or .5 as a coefficient for a written variable of the student's choosing • Use the expression in an equation with a second variable as the equivalent expression (Note: this student incorrectly identified the input and output values which, using his variables, would look like: y = total cost, x = # toppings)	What will a response include from a student who demonstrated some understanding and possibly some misunderstandings or undeveloped understanding? A student who demonstrates some understanding omits1-3 of the bullet points from "Got It." He would likely commit one of the following mistakes: • Misidentify the input and output • Use information from the large column • Either confuse the fixed with the variable costs or use the fixed as a variable cost as well • Would add the fixed and variable costs.	What will a response include from a student who demonstrated minimal understanding and possibly misconceptions? A student who demonstrates minimal understanding fails to do the following: • Identify either the toppings or cost as an element of an unknown value. • Discerns fixed versus variable costs by writing the fixed as a constant and the variable as a coefficient • Lists input and output with respective variables • Writes fixed and variable costs as an expression in an equation

Got It (student work)

PIZZA

Small – 9 inches - 6 slices		Large – 16 inches – 12 slices	
Plain	$5.10	Plain	$9.80
Additional Items	.50	Additional Items	1.40

(I) You are ordering pizza for a party and you want to order [...] can spend on a single pizza. Identify the input and output qu[...]

y is type of pizza cost
x = additional stuff

(II) Write an expression to represent the fixed and variable c[...]

$y = 5.10 + .5x$

(III) Write an equation to represent the cost of a small pizza

$S = 5.10 + .5x$

Developing (student work)

PIZZA

Small – 9 inches - 6 slices		Large – 16 inches – 12 slices	
Plain	$5.10	Plain	$9.80
Additional Items	.50	Additional Items	1.40

(I) You are ordering pizza for a party and you want to order toppings tha[...] can spend on a single pizza. Identify the input and output quantities and[...]

input is money ouput PPSSg

(II) Write an expression to represent the fixed and variable costs for a s[...]

$.50P + 5.80 = y$

(III) Write an equation to represent the cost of a small pizza as a functic[...]

$5.10 + .50 = y$

Getting Started (student work)

PIZZA

Small – 9 inches - 6 slices		Large – 16 inches – 12 slices	
Plain	$5.10	Plain	$9.80
Additional Items	.50	Additional Items	1.40

(I) You are ordering pizza for a party and you want to order toppings that everyone likes but you have [...] can spend on a single pizza. Identify the input and output quantities and label with a variable.

The input is the pizza cost and the otput is the toppings

(II) Write an expression to represent the fixed and variable costs for a small pizza.

$5.10

(III) Write an equation to represent the cost of a small pizza as a function of the number of toppings.

$5.10 x + .50 y = something$

Getting Started	
Student Response Example	**Indicators**

What will a response include from a student who demonstrated minimal understanding and possibly misconceptions?
 	
*Input a student work sample that shows a "Getting Started" response if possible.
 	
(Same as above)
 	
	• What possible indicators may be included in a student response who has demonstrated minimal understanding of the standard?
	   ○ Not using a value as a coefficient of a variable
	   ○ Adding unlike terms
	   ○ Does not attempt to identify some relationship between an amount associated with pizza and some type of cost.
	
	• What strategies, and representations will or will not be used? What understandings or procedural fluency does the student response reveal?
	   ○ Symbolic representation in the form of slope-intercept form would likely be overlooked
	   ○ Labeling unknowns with a variable would likely be overlooked
	   ○ Including irrelevant information in a response is likely
	   ○ Responses may not be appropriate for the prompt
	
	• What undeveloped understandings, misconceptions, and common mistakes may be revealed in the student response to this item?
	   ○ He did not identify the specific unknown quantities
	   ○ He did not identify the input and output variables
	   ○ He appears to not understand the functional relationship of cost as a function of # of toppings
	   ○ He did not identity fixed and variable costs
	   ○ He does not appear to understand the concepts of fixed and variable costs or amounts

In the Moment Questions/Prompts	**Closing the Loop (Interventions/Extensions)**
What questions could you ask, or feedback could you provide in the moment to develop student understanding, create disequilibrium, or advance student thinking? • What are the unknown values (what do we not know in this problem, what changes)? • What do you pay for? • What does "additional items…50" mean? • What type of toppings do you like? How much would you pay for a small pizza with those toppings?	LZ video lesson links that may help develop conceptual understanding and procedural skill needed If no LZ video lessons address the error or misunderstanding, provide strategies or notes that could be useful in planning follow up action • Give the students the menu from which this problem was developed and have students order a pizza and computer total. Have them change the # number of toppings and recomputed. Continue and record data and identify a pattern. • Create cutouts of toppings and dollars for CRA interaction with this topic. • Create a scaffolded handout with spaces for the total cost, fixed cost and variable cost.

Developing	
Student Response Example	**Indicators**

Student Response Example	Indicators

What will a response include from a student who demonstrated some understanding and possibly some misunderstandings or undeveloped understanding?
 	
 PIZZA
 Small – 9 inches - 6 slices | Large – 16 inches – 12 slices
 Plain | $5.10 | Plain | $9.80
 Additional Items | .50 | Additional Items | 1.40
 	
 (I) You are ordering pizza for a party and you want to order toppings tha can spend on a single pizza. Identify the input and output quantities and
 	
 input is money ouput pissg
 	
 (II) Write an expression to represent the fixed and variable costs for a s
 	
 $.50P + \$.80 = y$
 	
 (III) Write an equation to represent the cost of a small pizza as a functic
 	
 $5.10 + .50 \leq y$
	• What possible indicators may be included in a student response who has demonstrated some understanding of the standard?
	    o Using a value as a coefficient of a variable
	    o Adding terms
	    o Referring to the general topic associated with either an input or output quantity
	
	 • What strategies, and representations will or will not be used? What understandings or procedural fluency does the student response reveal?
	    o Symbolic representation in the form of slope-intercept form may be used.
	    o Labeling unknowns with a variable may be used
	    o Including relevant information only in a response is likely
	    o Responses are likely appropriate for the prompt
	
	    o He correctly identified the existence of a relationship between cost and pizza
	    o He correctly identified fixed and variable amounts (although he omitted the variable in part III
	
	 • What undeveloped understandings, misconceptions, and common mistakes may be revealed in the student response to this item?
	    o He did not identify the specific unknown quantities
	    o He did not identify the input and output variables
	    o He appears to not understand the specific values associated with "money" and "pissa" [pizza] in terms of input and output

In the Moment Questions/Prompts	**Closing the Loop (Interventions/Extensions)**
What questions could you ask, or feedback could you provide in the moment to develop student understanding, create disequilibrium, or advance student thinking? • When you pay for the pizza what exactly are you paying for? • How do you know how much money to pay? • What comes first, you pay money then order your toppings or you order your toppings then pay money. • In your equation, what does the P represent? The y? • What do you pay for to get a total? (pizza and toppings)	If no LZ video lessons address the error or misunderstanding, provide strategies or notes that could be useful in planning follow up action • Show a video or photos of a dollar bill change machine and use the terms "input" and "output" for the # of dollar bill and # of quarters respectively. • Have the student complete a t-table for values associated with input and output for change machine. • Use the function Q = 4d to represent the change machine (Q = # quarters, d = # dollar bills) and have the student enter a value for d then compute Q.

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Got it	
Student Response Example	**Indicators**

What will a response include from a student who has demonstrated conceptual understanding and mastery?
 	
*Initially, you will write this response. If you are able to get student work that is reflective of an exemplar response, insert it here.
 	
PIZZA
Small – 9 inches - 6 slices | Large – 16 inches – 12 slices
Plain | $5.10 | Plain | $9.80
Additional Items | .50 | Additional Items | 1.40
 	
(I) You are ordering pizza for a party and you want to order ' can spend on a single pizza. Identify the input and output qu
 	
y is type of pizza cost
$x =$ additional stuff
 	
(II) Write an expression to represent the fixed and variable c
 	
$y = 5.10 + .5x$
 	
(III) Write an equation to represent the cost of a small pizza
 	
$S = 5.10 + .5x$
	• What indicators must be included in an exemplar student response
	  o Indicate the functional relationship of total cost as a function of the number of toppings by choosing total cost as output (y) and # of toppings as input (x)
	  o Choose the correction information from the table, i.e. values for small pizza
	  o Write an expression with 5.10 as a constant and .50 or .5 as a coefficient for a written variable of the student's choosing
	  o Use the expression in an equation with a second variable as the equivalent expression
	
	• What strategies, and representations will or will not be used? What understandings or procedural fluency does the student response reveal?
	  o
	
	• What undeveloped understandings, misconceptions, and common mistakes may be revealed in the student response to this item?
	  o The student does not appear to differentiate between an expression and an equation.
	  o The student did not provide a strict definition of his output – used "type" but did refer to cost
	  o The student may not be clear as to what "additional items" means

In the Moment Questions/Prompts	**Closing the Loop (Interventions/Extensions)**
What questions could you ask, or feedback could you provide in the moment to extend or push student understanding, create disequilibrium, or advance student thinking? • Ask him to explain what "additional stuff" or "additional items" represents. Also ask him to explain what he would order for his choice of a pizza to gauge his understanding of pizza (this is an ELL student). • Have him compute the cost of a pizza and have him explain his computations to gauge his understanding of the parts of the equation. • Have him use his equation (from part II or III) and compute. Have him explain his computations – clinical interview.	If no LZ video lessons are appropriate, provide strategies or notes that could be useful in planning follow up action • Have him complete a t-table for this function for various # of toppings • Have him write a function for the large pizza cost and complete a t-table. • If identifying input and output amounts continues to be problematic I would give him various applications and have him identify input and output amounts.

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Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction

Lesson Objective:	The student will, given a real life problem that can be modeled by a linear function, identify two unknown quantities as input and output and will identify fixed and variable amounts.
Content Standard(s):	A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
Targeted Practice Standard :	1. Make sense of problems and persevere in solving them. 4. Model with mathematics. 7. Look for and make use of structure.

Mathematical Goals	Success Criteria
• Understand that there is a functional relationship between two quantities. • Understand that a situation can be modeled as opposed to looking for key words that lead to operation. • Describe in the student's own words the functional relationship between two quantities.	• Identify the input and output quantities. • Write equations to represent word problems using a variable to represent an unknown quantity.

Launch (Probe and Build Background Knowledge)

Purpose:
A brief description of how you will probe and build student background knowledge
I will begin with contexts that are prior knowledge for the students: coin change machine, pizza. I will ask students to explain, in their own words, related mathematical topics associated with this objective with the terms presented in language they understand, e.g. what do you put into the machine and what do you get out?

Instructional Task

Purpose: *A brief description of the mathematics and/or the mathematical practices the task is intended to engage students in and what students will be doing.*

- Use a variable to represent an unknown quantity.
- Identify input and output for a given real life situation or problem.
- Identify the fixed and variable amounts for a given or real life situation or problem.

Engage (Setting Up the Task)
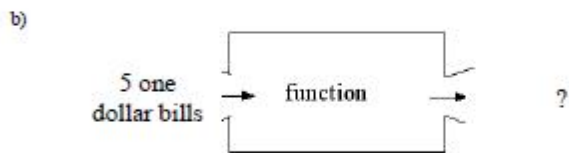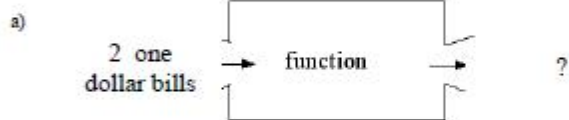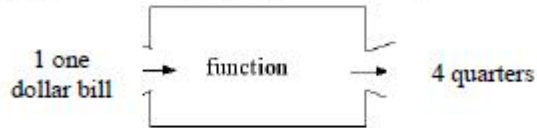*How will you introduce and set up the task?*
- Show a video or photos of a dollar bill change machine and use the terms "input" and "output" for the # of dollar bill and # of quarters respectively (http://youtu.be/OhYu1HRK30M)
- Have the student complete a t-table for values associated with input and output for change machine and formally identify input and output quantities (see handout below).

Intro to Input and Output Using Change Machine

1. If you put a dollar bill into the change machine, what comes out?

2. If you put in 1 dollar bill you get 4 quarters. Complete parts a and b.

1 one dollar bill → function → 4 quarters

a)

2 one dollar bills → function → ?

b)

5 one dollar bills → function → ?

4. Complete the table

X = # one dollar bills	Y = # quarters
1	
2	
5	

5. Which of these quantities, # dollar bills or # quarters, would be the input? Explain.

Explore (Solving the Task)
How will students work on the task? What questions will you ask as students work on the task to elicit evidence of their understanding and support mathematical connections? What are some anticipated student responses or solution paths?

- When you pay for the pizza what exactly are you paying for?
- How do you know how much money to pay?
- What comes first, you pay money then order your toppings or you order your toppings then pay money.
- In your equation, what does the P represent? The y?
- What do you pay for to get a total? (pizza and toppings)
- (See handout below)

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Pizza costs $10 and toppings cost $2 each					
# toppings	1		2		3
Total cost					
	10	2	10	4	10	6

Elaborate (Discuss Task and Related Mathematical Concepts)

How will you facilitate the sharing of student work and discussion to support students in making mathematical connections?

I will present teams different problems for the same prompts. Students will generate responses individually and then as a team on poster paper and share out to the class. The class will in turn evaluate the presenting teams work. (See handout below.)

Group Work and Presentations on Poster Paper   Name_____

1. Lin is tracking the progress of her plant's growth. Today the plant is 5 cm high. The plant grows 1.5 cm per day.

a. What are the input and output? Explain.

b. What are the fixed and variable amounts? Explain.

2. Mr. E is on a diet. He currently weighs 260 pounds. He loses 4 pounds per month.

a. What are the input and output? Explain.

b. What are the fixed and variable amounts? Explain.

3. The population of town of Windsor is 35,000 today. Every year the population of Windsor increases by 750 people.

a. What are the input and output? Explain.

b. What are the fixed and variable amounts? Explain.

4. Paul opens a savings account with $350. He saves $150 per month. Assume that he does not withdraw money or make any additional deposits.

a. What are the input and output? Explain.

b. What are the fixed and variable amounts? Explain.

Checking for Understanding

Purpose: *A brief description of what questions or prompts you will use to elicit evidence of student understanding and the strategy you will use to elicit the evidence during the lesson*

I will provide a formative pop quiz asking students to independently identify input and output values and to explain in their own words the functional relationship between these two values. (See handout below.)

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Pop Quiz: Input, Output, Fixed Amount, Variable Amount   name _____

Jeff wants to buy the new iPhone and needs to know how long it will take him to afford it. Currently he has $60. He saves $20 a week. How many weeks will it take for Jeff to have enough money?

1. What is the input? Explain.

2. What is the output? Explain.

3. What is the fixed amount? Explain.

4. What is the variable amount? Explain.

Common Misunderstanding

Purpose: *A brief description of a probe or prompt students could engage in to make them aware of a common misunderstanding*
A common mistake in the student artifacts is not identifying input and output as measurable quantities, e.g. "pizza".

To address this I would provide students examples of properly worded inputs/outputs and improperly worded ones. The students would then be prompted to explain the problem with the improperly worded ones. (See handout below)

	Input	Possible value	Output	Possible value
1.	# dollar bills	2	# quarters	8
2.	# toppings		Total cost	
3.	Pizza		Money	
4.	Type of pizza		toppings	

Explain what is wrong with the input and output for #3 and 4.

Checking for Understanding

Purpose: *A brief description of what questions or prompts you will use to elicit evidence of student understanding and the strategy you will use to elicit the evidence at the end of the lesson*

Scaffolded handout structured similarly to the coin change machine handout with a box for identifying the input and one for the output. (See handout below)

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Identifying Input and Output for a word problem

input → function → output

Input	→	Function Machine	→	Output
# of dollar bills	→	4 quarters for a dollar	→	# of quarters
	→	Cost of toppings + cost of pizza	→	
	→		→	
	→		→	

Closure

Purpose: *A brief description of how students will engage in reflecting on their own learning and understanding*

Exit ticket prompting students to identify input, output, fixed and variable cost for a new real life situation. (See handout below)

Exit Slip   name _____ date _____

1. The cost of buying a pizza with an unknown number of toppings is modeled by the expression 10+2x.

    a. In this expression, what does the "x" represent?

    b. In this expression, what does the 10 represent?

    c. In this expression, what does the 2 represent?

Wireless Order Summary

Ship By:
11/02/2012
Line 1

Apple iPhone 5 Black & Slate - 16GB
$199.99 - with 2 year contract with SIM card

Price Plan
SHARE EVERYTHING UNL TALK & TEXT 300MB SJ0 0512   Share Plan

Features
Accessories   Not Selected
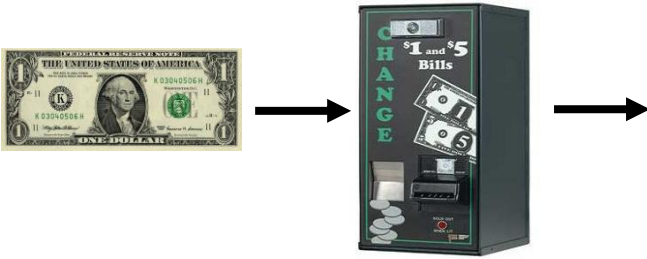
Share Plans Summary
Share Plans Total $40.00

Total Due : $199.99
Monthly Estimate : $40.00

2. What is the fixed cost for this order? Explain.

3. What is the variable cost for this order? Explain.

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Extension Task

Purpose: *A brief description of how you will extend the learning for students who are ready to go deeper*

If students complete the lesson, they will be provided additional problems that can be modeled by a functional relationship. The context would shift from those that directly relate to their prior knowledge to ones that are more abstract, e.g. the costs associated with a plumber. (See handout below)

Identify Input and Output   Name: _____

1. A season's pass to Six Flags is $60, and it costs $15 dollars to park each visit. Use C for cost and V for number of visits.

Input	output

Independent variable: (input) _____

Dependent variable: (output) _____

Fixed amount: _____

Variable amount: _____

2. Your phone company charges $30 each month plus $0.10 per minute over your plan. Use C for charges and M for number of minutes.

Input	output

Independent variable: (input) _____

Dependent variable: (output) _____

Fixed amount: _____

Variable amount: _____

3. An airline charges $525 for a flight to L.A. plus $25 for each piece of baggage. Use C for cost and B for number of pieces of baggage.

Input	output

Independent variable: (input) _____

Dependent variable: (output) _____

Fixed amount: _____

Variable amount: _____

4. At a tag sale, Martin saw an original Nintendo Game boy for only $20 and games for $3 each! Use C for cost and G for number of games.

Input	output

Independent variable: (input) _____

Dependent variable: (output) _____

Fixed amount: _____

Variable amount: _____

1. The cost of buying a pizza with an unknown number of toppings is modeled by the expression 10+2x.

    a. In this expression, what does the "x" represent?

    b. In this expression, what does the 10 represent?

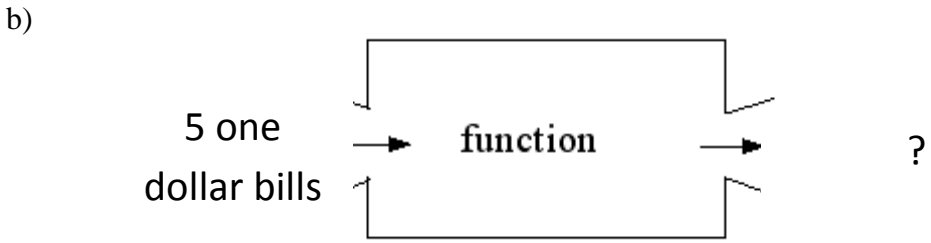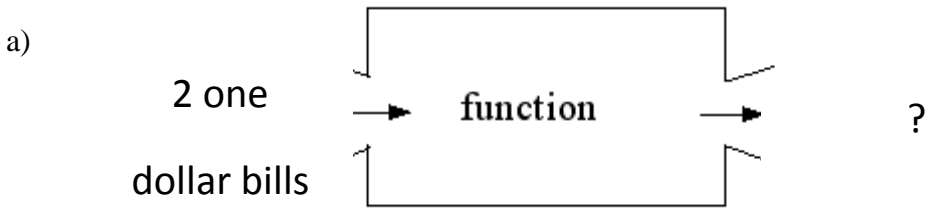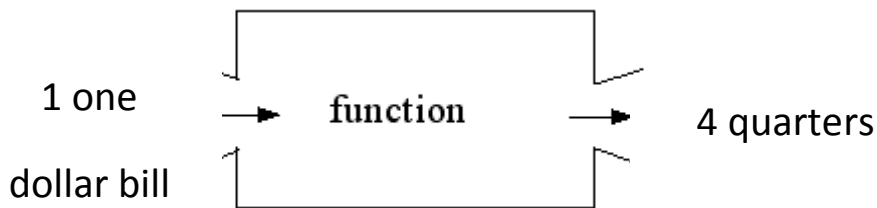   c. In this expression, what does the 2 represent?

2. What is the fixed cost for this order? Explain.

3. What is the variable cost for this order? Explain.

Intro to Input and Output Using Change Machine

1.   If you put a dollar bill into the change machine, what comes out?

2.   If you put in 1 dollar bill you get 4 quarters. Complete parts a and b.

1 one

dollar bill   →   function   →   4 quarters

a)

2 one

dollar bills   →   function   →   ?

b)

5 one
dollar bills   →   function   →   ?

4.   Complete the table

5.   Which of these quantifies, # dollar bills or # quarters,
     would be the input? Explain.

X = # one dollar bills	Y = # quarters
1	
2	
5	

Identify Input and Output   Name: _____

1. A season's pass to Six Flags is $60, and it costs $15 dollars to park each visit. Use

Input	output

C for cost and **V for number of visits**.

Independent variable: (input) _____

Dependent variable: (output) _____

Fixed amount: _____

Variable amount: _____

2. Your phone company charges $30 each month plus $0.10 per minute over your plan. Use

Input	output

C for charges and **M for number of minutes**.

Independent variable: (input) _____

Dependent variable: (output) _____

Fixed amount: _____

Variable amount: _____

3. An airline charges $525 for a flight to L.A. plus $25 for each piece of baggage. Use

Input	output

C for cost and **B for number of pieces of baggage**.

Independent variable: (input) _____

Dependent variable: (output) _____

Fixed amount: _____

Variable amount: _____

Input	output

4. At a tag sale, Martin saw an original Nintendo Game boy for only $20 and games for $3 each! Use

C for cost and **G for number of games**.

Independent variable: (input) _____

Dependent variable: (output) _____

Fixed amount: _____

Variable amount: _____

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Fixed and Variable Cost – Final product should look like this.

Pizza costs $10 and toppings cost $2 each					
# toppings	1		2		3
Total cost						
	10	2	10	4	10	6

Fixed and Variable Cost – Table on which cut outs are glued

# toppings	1		2		3	
Pizza costs $10 and toppings cost $2 each						
Total cost						

Fixed and Variable Cost – Cut Outs

Group Work and Presentations on Poster Paper   Name_____

1. Lin is tracking the progress of her plant's growth. Today the plant is 5 cm high. The plant grows 1.5 cm per day.

a. What are the input and output? Explain.

b. What are the fixed and variable amounts? Explain.

2. Mr. E is on a diet. He currently weighs 260 pounds. He loses 4 pounds per month.

a. What are the input and output? Explain.

b. What are the fixed and variable amounts? Explain.

3. The population of town of Windsor is 35,000 today. Every year the population of Windsor increases by 750 people.

a. What are the input and output? Explain.

b. What are the fixed and variable amounts? Explain.

4. Paul opens a savings account with $350. He saves $150 per month. Assume that he does not withdraw money or make any additional deposits.

a. What are the input and output? Explain.

b. What are the fixed and variable amounts? Explain.

Pop Quiz: Input, Output, Fixed Amount, Variable Amount   name _____

Jeff wants to buy the new iPhone and needs to know how long it will take him to afford it. Currently he has $60. He saves $20 a week. How many weeks will it take for Jeff to have enough money?

1. What is the input? Explain.

2. What is the output? Explain.

3. What is the fixed amount? Explain.

4. What is the variable amount? Explain.

Pop Quiz: Input, Output, Fixed Amount, Variable Amount　　　　name _____

Jeff wants to buy the new iPhone and needs to know how long it will take him to afford it. Currently he has $60. He saves $20 a week. How many weeks will it take for Jeff to have enough money?

1. What is the input? Explain.

2. What is the output? Explain.

3. What is the fixed amount? Explain.

4. What is the variable amount? Explain.

Identifying Input and Output for a word problem
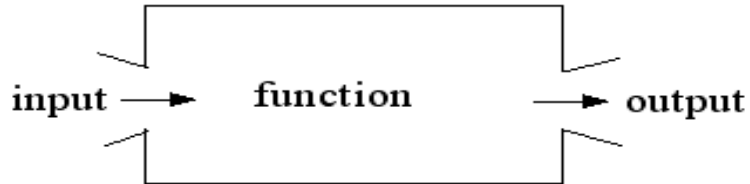
Input	→	Function Machine	→	Output
# of dollar bills	→	4 quarters for a dollar	→	# of quarters
	→	**Cost of toppings + cost of pizza**	→	
	→		→	
	→		→	

	Input	Possible value		Output	Possible value
1.	# dollar bills	2		# quarters	8
2.	# toppings			Total cost	
3.	Pizza			Money	
4.	Type of pizza			toppings	

Explain what is wrong with the input and output for #3 and #4.

Research and review of standard	
Content Standard(s):	**Standard(s) for Mathematical Practice:**
A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	1. Make sense of problems and persevere in solving them. 4. Model with mathematics. 7. Look for and make use of structure.
Smarter Balanced Claim	***Smarter Balanced Item***
Claim 2: Problem Solving Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.	

| **CPR Pre-Requisites**
(Conceptual Understanding, Procedural Skills, and Representations)

Look at the Progressions documents, Learning Trajectories, LZ lesson library, unpacked standards documents from states, NCTM Essential Understandings Series, NCTM articles, and other professional resources. You'll find links to great resources on your PLC Platform. | **Conceptual Understanding and Knowledge**

• Understand the use of a symbol for the unknown number.
• Understand that a variable can represent an unknown number.
• Understand that there is a functional relationship between two quantities.
• Understand that a graph can represent a functional relationship.
• Understand that a situation can be modeled as opposed to looking for key words that lead to operation.
• Discern between a parameter and a variable.
• Describe qualitatively the functional relationship between two quantities.

Procedural Skills

• Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
• Use a variable to represent an unknown quantity.
• Write a linear equation in one variable to model a problem.
• Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers.
• Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations |

	used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?
	• **Construct a function to model a linear relationship between two quantities.**
	• Determine the rate of change and initial value of the function from a description of a relationship.
	• Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
	Representational
	• Use visual models to represent a functional relationship.
	• Graph a given data set in two variables.
	• Write equations to represent word problems using a variable to represent an unknown quantity.
	Social knowledge
	• Know that the x typically represents the input and the y typically represents the output
	• Know that the coefficient of x is the rate (slope)
	• Know that the x variable is the independent variable and the y variable is the dependent variable
	• Know that the x variable is represented on the horizontal axis and the y variable is represented on the vertical axis of a coordinate plane

Standards Progression
Look at LearnZillion lessons and expert tutorials, the Progressions documents, learning trajectories, and the "Wiring Document" to help you with this section

Grade(s) below	Target grade	Grade(s) above
What previous grade level standards build up to the grade level standard this item assesses?	*What other grade level standards are connected to the standard this item assesses?*	*What subsequent grade level standards build off of the grade level standard this item assesses?*
F.IE.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$.	A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
8.EE.7 Solve linear equations in one variable. b. Solve linear equations with rational number coefficients, including equations whose solutions require	F.LE.1 Construct and compare linear, quadratic, and exponential models and solve problems	

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expanding expressions using the distributive property and collecting like terms. 7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? 8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two px; yq values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. 8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. 7.RP.1 Analyze proportional relationships and use them to solve real-world and mathematical problems.	1. Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. F.BF.1 Build a function that models a relationship between two quantities 1. Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	A.REI.12 Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

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1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*		

Common Misconceptions/Roadblocks

What characteristics of this problem may confuse students?
- Students may not discern necessary information from a table.
- Students may not decode the context of the problem (decontextualize)
- Students may not determine that the fixed cost (base cost) includes first twenty pages and write an equation accordingly

What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?
- Students may not use a variable (maybe parameters) to represent unknown quantities
- Students may not understand the concept of a relationship between input and output
- Students may not be able to identify fixed and variable amounts
- Students may not discern between fixed and variable amounts
- Students do not know how to start graphing a function
- Students may not use grouping symbols in writing the expression representing the number of pages that have an additional cost

What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?
- Students may multiply variable by unit cost and not subtracting pages included in the fixed/base cost
- Students may add amounts given – consider this an addition problem