

Module 2
Facilitator Guide

Focus on Content Standards

Connecticut Core Standards for Mathematics



Grades 6–12

Systems of Professional Learning

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Module Overview

Prerequisite	None
Duration	Full day
Outcomes	<p>By the end of the session, participants will have:</p> <ul style="list-style-type: none"> • Strengthened working relationships with peer Core Standards Coaches across their region. • Deepened their understanding of the Practice Standards specified in the CCS-Math through sharing of implementation experiences from Module 1. • Examined the implications of the language of the Content Standards for teaching and learning. • Identified CCS-aligned tasks that combine cognitively rigorous content with mathematical practices. • Identified tasks that combine cognitively rigorous content with the Practice Standards. • Analyzed the progression of topics in the Content Standards both within and across grade levels. • Deepened their understanding of the potential of the CCS-Math to change mathematics teaching and learning. • Gained understanding of some of the challenges involved in implementing the CCS-Math. • Explored strategies for supporting teachers as they make changes in their classroom practice. • Made plans for next steps.

Resources Required

- Chart paper, markers, pens, highlighters, nametags, post-it notes
- Participant Guide for each participant
- Domain Cards for grades 6, 7, and 8
- Cardstock (for labeling the tables and groups)

Session Preparation

Tables should be arranged so participants can work in groups.

Key Messages

- The Standards for Mathematical Content are not just a new list of topics. They go hand in hand with the Standards for Mathematical Practice.
- When implemented together, the Standards for Mathematical Practice and the Standards for Mathematical Content bring new rigor to the mathematics we teach and that we expect students to learn.
- Teaching the CCS-Math will require fundamental changes in teaching practice.
- The process of changing to a CCS-aligned curriculum is complex because it involves considerable attention not only to the standards, but also to curriculum materials, instruction, assessment, and professional learning. Therefore, full implementation of the CCS-Math will require all professionals in the schools to collaborate over time.

Session at-a-Glance

Introductory Activity (10 minutes)

The facilitator will review project goals and activities, module outcomes, and the agenda for the session. Participants will complete a Pre-Assessment.

Supporting Documents:

- Session Agenda
- Pre-Assessment

PowerPoint Slides:

- 1–5

Section 1: Sharing Implementation Experiences (35 minutes)

Training Objectives:

- To review the foundations of the CCS-Math and the key shifts of focus, coherence, and rigor.
- To share, discuss, and address experiences with and common challenges of supporting teachers in implementing the Standards for Mathematical Practice.

The facilitator will begin by reviewing the key shifts of focus, coherence, and rigor. Then, in groups, participants will share experiences and describe any “aha moments” from attempts to implement the Standards for Mathematical Practice (SMP). Participants will look for themes or choose one or two important successes, challenges, and/or insights to share with the larger group. These will be recorded on chart paper so that common themes and additional strategies can be discussed. Participants can record new ideas on the handout, *Moving Forward with the Practices*. The facilitator will wrap up Section 1 by explaining that to build upon their knowledge and experience with the SMP, they will begin to connect these to the Standards for Mathematical Content.

Supporting Documents:

- *Moving Forward with the Practices*

Materials:

- Chart paper, markers

PowerPoint Slides:

- 6–13

Section 2: The Language of the Content Standards (45 minutes)**Training Objectives:**

- To define conceptual understanding, procedural skill and fluency, and application of mathematics.
- To understand the differences between conceptual understanding, procedural skill and fluency, and application of mathematics.
- To begin to understand how procedural skills and fluency build upon conceptual understanding.
- To demonstrate how application of mathematics can support students’ development of conceptual understanding.

The Language of the Content Standards: In groups, participants will complete the first part of the Who Knows Math exercise, examine short examples of student work, and together will answer questions about what the student knows based on the answers given. After a brief large group discussion, small groups will watch the video *Mathematics Fluency: A Balanced Approach* and develop working definitions of “conceptual understanding,” “fluency,” and “application” as addressed in the content standards. Groups will then work through short, basic examples on how students can demonstrate conceptual understanding and then discuss current strategies used now to develop procedural skill and fluency. The wrap-up of the section takes place as participants complete the second part of the Who Knows Math exercise and revise their first round of answers given their new understandings.

Supporting Documents:

- Who Knows Math

Materials:

- Chart paper, markers

Video:

- *Mathematics Fluency: A Balanced Approach*
<http://www.youtube.com/watch?v=ZFUAV00bTwA>

PowerPoint Slides:

- 14–29

Section 3: The Progression of the Content Standards (80 minutes)**Training Objectives:**

- To provide participants with information on and experience with identifying standards that address conceptual understanding, procedural skill and fluency, and application of mathematics.
- To have participants experience how concepts are developed within and across grade levels.
- To provide participants practice with identifying concept progressions.
- To provide participants with an understanding of how standards within a grade level can be grouped, or are “connected” across content domains.
- To provide participants practice with connecting standards.

The definitions of “conceptual understanding,” “fluency,” and “application” developed in the previous section are used to help participants analyze the content standards in three different ways.

In Exploring the Standards Part 1, participants explore the content standards for one domain, at one grade level, and determine which standards focus on conceptual understanding, procedural skill and fluency, and application of mathematics.

In Exploring the Standards Part 2, participants explore the content standards for one domain across grade levels and record five general observations about the progression of the concepts and two connections to the practice standards.

In Exploring the Standards Part 3, participants explore all of the domains for one grade level to “connect” standards across multiple domains that can be taught together in a lesson or unit.

Participants will then discuss and reflect as a large group on the importance and instructional implications of the progressions and any new insights they now have into the standards.

The activity will conclude with having participants view the video *Gathering Momentum for Algebra* in order to see where the content progressions impact the K–12 learning pathway.

Supporting Documents:

- Standards for Mathematical Practice (Participants will bring a copy or access online.)
- Exploring the Content Standards Observation Sheet

Materials:

- Chart paper, markers
- Sets of Standard Progression Cards (one set per five table groups)
- Signs made for tables: 6, 7, and 8

Video:

- *Gathering Momentum for Algebra*
http://www.youtube.com/watch?v=ONPADO_Nt14

PowerPoint Slides:

- 30–40

Section 4: Meeting the Expectations of the Content Standards by Teaching with Cognitively Rigorous Tasks (85 minutes)**Training Objectives:**

- For participants to understand the definition of a cognitively rigorous task.
- To deepen participants understanding of why incorporating cognitively rigorous tasks into their mathematics instruction is important.
- To examine strategies that can be used to incorporate cognitively rigorous tasks that will benefit all students.

Participants begin by viewing the video *Dan Meyer: Math Class Needs a Makeover* which is about problem solving. Participants will discuss the video and compare their previous experience with solving *Two Machines, one Job* with Dan Meyer’s message in the video.

Participants are then shown how scaffolds can be removed slowly from a problem in order to deepen the level of cognitive rigor. Participants use Hess’s Cognitive Rigor Matrix to discuss the problem example.

Participants will brainstorm how cognitively rigorous mathematics tasks can be used to benefit all students and will be presented with four strategies that can be used to differentiate cognitively rigorous tasks for all students, as needed, in order to provide multiple entry points into the mathematics while maintaining the problem's rigor.

Participants will wrap up the activity by making connections back to “conceptual understanding,” “fluency,” and “application” and how each of these is addressed through cognitively rigorous tasks.

Supporting Documents:

- Video Observation Sheet
- Hess's Cognitive Rigor Matrix for Mathematics and Science
- Strategies for Differentiating Cognitively Rigorous Tasks
- Resources for Finding Tasks
- Reflect

Materials:

- Chart paper, markers

Video:

- *Dan Meyer: Math Class Needs a Makeover*
http://www.ted.com/talks/dan_meyer_math_curriculum_makeover.html

PowerPoint Slides:

- 41–59

Section 5: Supporting Change (85 minutes)**Training Objectives:**

- To help participants identify elements of lessons that work to develop conceptual understanding, procedural skill and fluency, and application of mathematics.
- To provide participants with instructional strategies for teaching the content standards through problem solving, for helping students to develop procedural skill and fluency, and to provide students with opportunities to apply their mathematical understandings.
- To have participants create a plan for disseminating big ideas from the session with teachers at their school.
- To have participants anticipate specific teacher questions and challenges around implementing lessons that incorporate the Standards for Mathematical Content.

Supporting Documents:

- Video Observation Sheets
- A New Spin on Old Strategies
- Group 1: Math Journals
- Group 2: Mathematical Language
- Group 3: *Instructional Implementation Sequence: Attaining the CCSS Mathematical Practices Engagement Strategies* (separate handout)
- Group 4: Group Work and Decision Making
- Next Steps

Videos:

- *What's Your Sign: Integer Addition*
<https://www.teachingchannel.org/videos/adding-integers-lesson-idea>
- *Zero Pairs, Manipulatives, and a Real-World Scenario*
<https://www.teachingchannel.org/videos/teaching-subtracting-integers>

Materials:

- Chart paper, markers

PowerPoint Slides:

- 60–63

Closing Activities (15 minutes)

Participants will complete a Post-Assessment and an online Session Evaluation.

Supporting Documents:

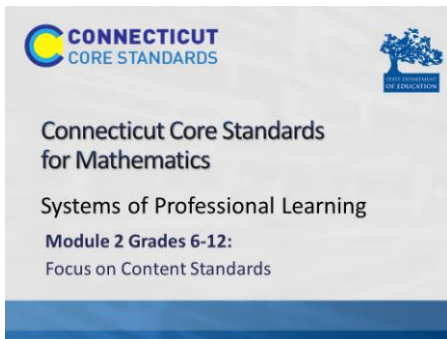
- Post-Assessment
- Session Evaluation (online)

PowerPoint Slides:

- 64–68

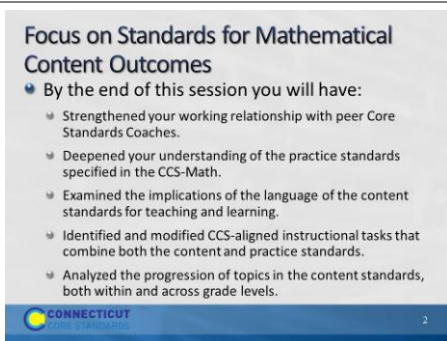
Session Implementation

Introduction



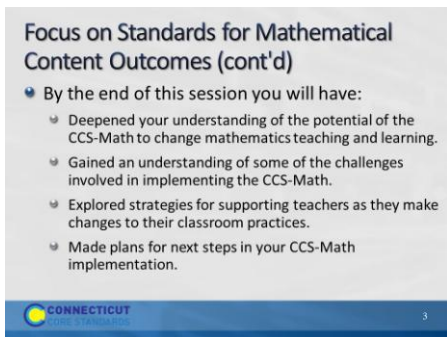
Slide 1

(Slides 1-5, including the pre-assessment, will take about 10 minutes total.)



Slide 2

Review the outcomes for the day, sharing what you hope to accomplish throughout the full day session. There are nine outcomes for this session. These are presented to the participants over two slides.



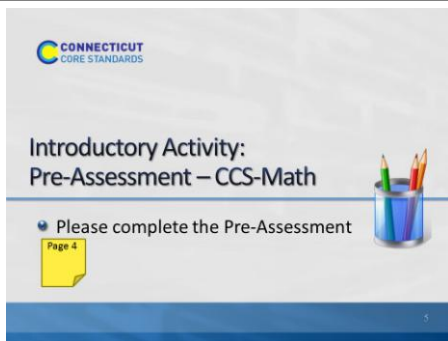
Slide 3

There are nine outcomes for this session. These are presented to the participants over two slides.



Slide 4

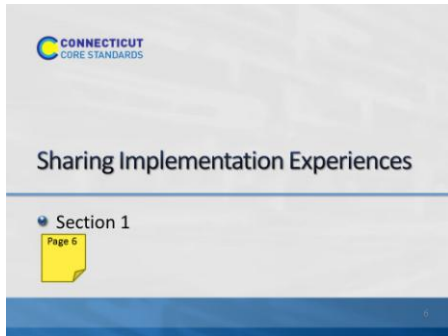
Review the agenda letting participants know that this is the pathway they will travel in order accomplish the nine outcomes discussed earlier. Note that in addition to the break for lunch there will also be shorts breaks throughout the day, but participants should feel free to take a personal break as needed. Emphasize the importance of coming back from lunch and breaks on time to ensure enough time to complete all the work of the day.



Slide 5

This will be a short self-assessment, which will be found in the Participant Guide on **page 4**. It will assess where the coaches are now with understanding the implementing the Practice Standards that were introduced in Module 1, and assess where they are in understanding the Content Standards. The participants will complete the same assessment at the end of the session. **Allow 3-4 minutes to complete.**

Section 1



Slide 6

Sharing Implementation Experiences

Section 1 Time: 35 minutes

Section 1 Training Objectives:

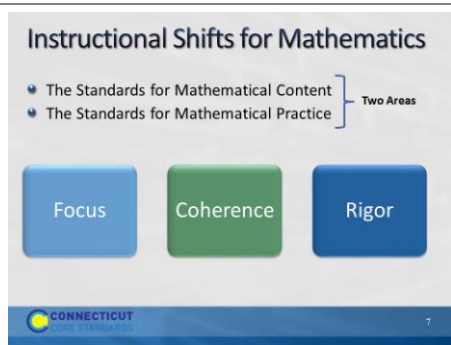
- To review the foundations of the CCS-Math and the key shifts of focus, coherence, and rigor.
- To share, discuss, and address experiences with, and the common challenges of, supporting teachers in implementing the Standards for Mathematical Practice.

Section 1 Outline:

- The facilitator will begin by reviewing the key shifts of focus, coherence, and rigor. **(5 minutes)**
- Then, in groups, participants will share experiences and describe any “aha moments” from attempts to implement the Standards for Mathematical Practice (SMP). Participants will look for themes or choose one or two important successes, challenges, and/or insights to share with the larger group. These will be recorded on chart paper so that common themes and additional strategies can be discussed as a large group. Participants can record new ideas in the Participant Guide recording sheet *Moving Forward with the Practices*. **(30 minutes)**
- The facilitator will wrap up Section 1 by explaining that to build upon their knowledge and experience with the SMP they will begin to connect the SMP to the Standards for Mathematical Content.

Supporting Documents*Moving Forward with the Practices* recording sheet**Materials**

Chart paper, markers



Slide 7

Begin by reviewing key ideas from Module 1 using the next five slides:

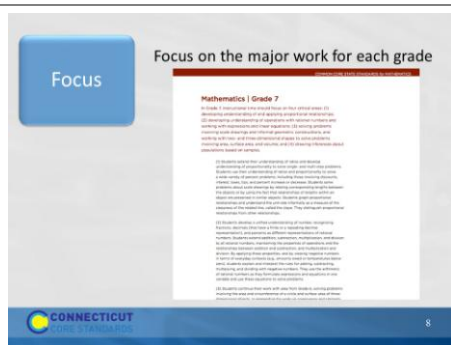
There are two parts to the Core Standards for Mathematics, Standards for Mathematical Content and Standards for Mathematical Practice. Together they define what students should understand and be able to do in their study of mathematics in order to be college and career ready.

The Standards for Mathematical **Practice** are often simply called the Practice Standards or the Practices. The Practices include the mathematical habits of mind and mathematical expertise that students should develop such as reasoning, communication, making arguments, and modeling. These were the focus of Module 1, delivered in March.

The Standards for Mathematical **Content** are very specific about concepts, procedures, and skills that are to be learned at each grade level, and contain a defined set of endpoints in the development of each. This will be the focus of this session.

What’s New About the CCS-Math?

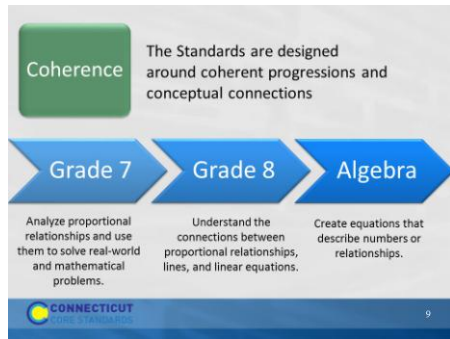
In order to meet both the Content Standards and the Practice Standards, the writers of the Common Core explicitly based the standards on three very important fundamentals of mathematics that were missing from, or were not as explicit in, different versions of mathematics standards. Those are: Focus, Coherence, and Rigor.



Slide 8

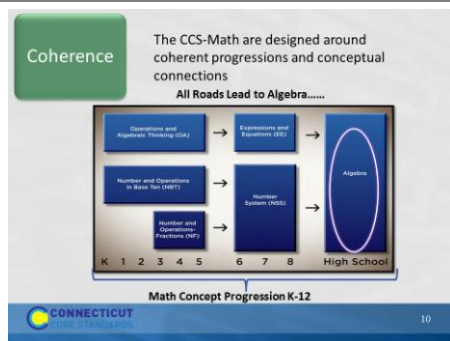
Focus: The writers of the Standards worked very hard to reduce the number of expectations at each grade level.

This work was not done arbitrarily. They focused on the different domains of mathematics, such as Operations and Algebraic Thinking, Number and Operations in Base Ten, Geometry, and Measurement and Data, and determined what work was critical for students at each grade level to address in order to develop the concepts in each domain over time. This reduction in number of topics allows teachers to shift their instruction to focus on the major work at their grade and to spend more time in each of these critical areas in order for students to develop a deep understanding through investigation, inquiry, and problem solving.



Slide 9

Coherence: Coherence means ensuring that there is a clear sequence of concepts and skills that build on each other across the grades.



Slide 10

Coherence: The chart on the slide shows one of the progressions in the CCS-Math that builds up to the formal study of Algebra in high school. Note that the Operations and Algebraic Thinking standards in grades K–5 lead up to and are designed to help middle school students work with Expressions and Equations, which will then help students to be successful in high school Algebra. This same progression takes place with the Number and Operations domains. In K–5 the Number and Operations standards are split over two domains, Base Ten and Fractions. This does not mean that the standards within the domains are not connected, but that there is a focus on each. The intent of the coherence and progressions is that students will all be ready for algebra at either the 8th grade or high school level. Section 3 of this module will include an in-depth examination of coherent progressions.

Rigor The major topics at each grade level focus on:

<p>CONCEPTUAL UNDERSTANDING</p> <ul style="list-style-type: none"> • More than getting answers • Not just procedures • Accessing concepts to solve problems 	<p>PROCEDURAL SKILL AND FLUENCY</p> <ul style="list-style-type: none"> • Speed and accuracy • Used in solving more complex problems • Supported by conceptual understanding 	<p>APPLICATION OF MATHEMATICS</p> <ul style="list-style-type: none"> • Using math in real-world scenarios • Choosing concepts without prompting
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Slide 11

Rigor: Rigor means learning that is based in the deep understanding of ideas AND fluency with computational procedures AND the capacity to use both to solve a variety of real-world and mathematical problems. Section 2 of this module will include an in-depth examination of the three aspects of rigor.

Note: We are not talking about a three-pronged balance of conceptual understanding, procedural skill and fluency, and application of mathematics here. Rather, we are focusing on the expectation that students are able to flexibly work with the mathematics content in each of the three areas. This will become more apparent as participants explore the Content Standards in Section 3.

Developing Mathematical Expertise

The Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

CONNECTICUT CORE STANDARDS 12

Slide 12

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

Developing this variety of mathematical expertise is important so that students are better able to solve problems and reason quantitatively both within a classroom and throughout life outside of the classroom.

Sharing Experiences Implementing CCS-Math Practice Standards

Sharing Implementation Experiences

1. Each participant will discuss with their table group one positive highlight, one challenge, and one lesson learned from their personal implementation of the Practice Standards thus far.
2. Each table group will then determine two positive highlights, one common challenge, and one common lesson learned that they will present to the larger group.
3. Participants will record notes and "New Ideas" generated from the discussion.

CONNECTICUT Core Standards

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Slide 13

Now that you have quickly reviewed the key points from Module 1, ask participants to now reflect on the work that they have done back at their school, in their role as a Core Standards Coach, with helping teachers learn more about and implement the CCS-Math. Have each participant discuss with their table group one positive highlight, one challenge, and one lesson learned from their personal implementation of the Practice Standards thus far. Each table group will then determine two positive highlights, one common challenge, and one common lesson learned that they will present to the larger group. They can record notes from their discussion on **page 6** in the Participant Guide.

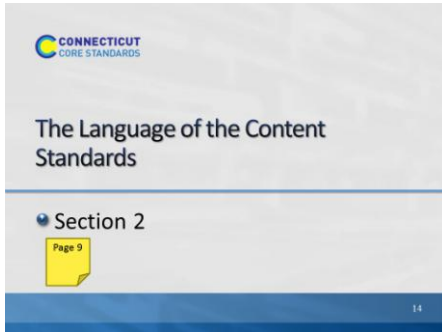
As table groups present, record the participants' responses on chart paper titled Positive Highlights, Challenges, and Lessons Learned. After all groups have presented, summarize what has been charted and then ask the large group if anyone has a solution to any of the common challenges. Encourage participants to record "New Ideas" on **page 7** in the Participant Guide.

Wrap up the activity by explaining that the challenges will be revisited periodically throughout the day as the discussion of the Content Standards ensues. **Note:** Be sure that the connections are made when discussing how to teach the Content Standards so that participants understand how implementing the Practice Standards go hand-in-hand with implementing the Content Standards in a rigorous, focused, and coherent lesson.

Transition to the next activity by explaining that participants will now start looking at the connections of the Content and Practice Standards by looking at the language of the Content Standards.

Note: If teachers have not had the time between the previous module and this module to begin their implementation, have them instead focus on things that they have seen and heard back at their school, including positive highlights of where their school is, challenges that they now recognize they may be facing, and any lesson learned in terms of the outcomes of the first module and where they think they need to go next with the implementation.

Section 2



Slide 14

Section 2: The Language of the Content Standards**Section 2 Time: 45 minutes**

Section 2 Training Objectives:

- To define conceptual understanding, procedural skill and fluency, and application of mathematics.
- To understand the differences between conceptual understanding, procedural skill and fluency, and application of mathematics.
- To begin to understand how developing conceptual understanding can lead to the development of procedural skill and fluency.
- To demonstrate how application of mathematics can support students' development of conceptual understanding.

Section 2 Outline:

- In groups, participants will complete the first part of the *Who Knows Math* exercise during which they will examine short examples of student work and make observations about what the student knows based on the answers given.
- After a brief large group discussion, small groups will watch the video *Mathematics Fluency: A Balanced Approach* and develop working definitions of “conceptual understanding,” “fluency,” and “application” as addressed in the content standards.
- Groups will then work through short, basic examples on how students can demonstrate conceptual understanding, and then discuss current strategies used now to develop procedural skill and fluency. The discussions will continue with how those strategies will benefit students first developing a conceptual understanding of the mathematics.
- The wrap-up of the session takes place as participants complete the second part of the *Who Knows Math* exercise in which they revise their first round of answers given their new understandings.

Supporting Documents*Who Knows Math* exercise worksheet

Materials

Chart paper, markers
Individual copy of the mathematics standards

Video

Mathematics Fluency: A Balanced Approach

Slide 15

What do these students understand?

- Ask table groups to read and analyze the “Who Knows Math” worksheet on **pages 9-12** in the Participant Guide. Ask them to think about what each student on the sheet knows and doesn’t know. Also have them think about what is unknown about what the students know. Participants can record their observations on the worksheet. Briefly discuss participants’ observations and explain that they will return to this after exploring the language of the content standards in more detail.

Note: If time is an issue at the start of this activity, you may choose to have groups focus on only one student. If you have five groups, assign each group a different student.

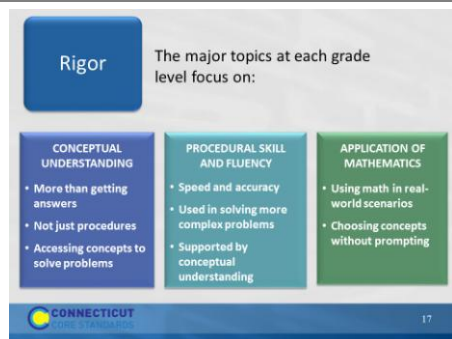
Slide 16

From the Authors

Click on “Watch Video” to play the video *Mathematics Fluency: A Balanced Approach* from here: <http://www.youtube.com/watch?v=ZFUAV00bTWA>. The video is **1:57** long.

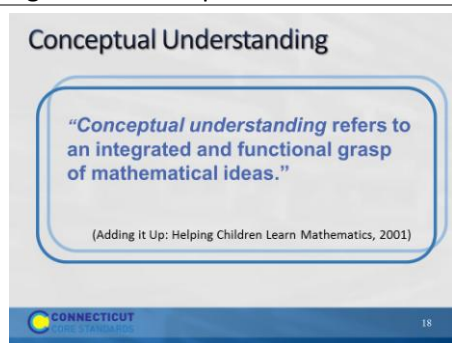
After the video has played, ask participants for their thoughts.

Transition to the next part of this section by explaining to participants that they will now look more closely at conceptual understanding, procedural skills and fluency, and application of mathematics in more depth.



Slide 17

Rigor: Remind participants that one of the big shift in the content standards is that at all ages, students are to be taught with rigor as defined on the slide. Review the three aspects of rigor: conceptual understanding, procedural skill and fluency, and application of mathematics. Repeat that rigor means learning based in the deep understanding of ideas AND fluency with computational procedures AND the capacity to use both to solve a variety of real-world and mathematical problems. Explain to participants that you will now go over each aspect of rigor in more depth.



Slide 18

Ask participants to turn to **pages 13-14** in the Participant Guide where space is provided for them to take notes on Conceptual Understanding, Procedural Skill and Fluency, and Application of Mathematics.

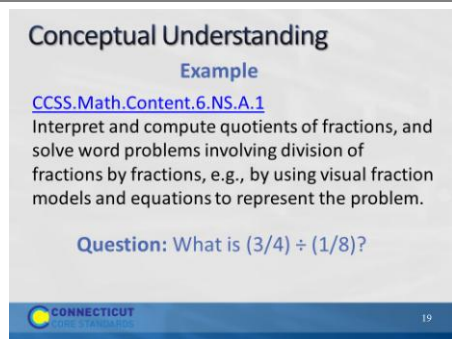
Conceptual Understanding

As participants read the quote on the slide, explain that conceptual understanding can be difficult to define. Ask participants to read the description on **page 13** in the Participant Guide that is an overlap of the National Research Council and NCTM definitions of conceptual understanding:

“Students demonstrate *conceptual understanding* in mathematics when they provide evidence that they can recognize, label, and generate examples of concepts; use and interrelate models, diagrams, manipulatives, and varied representations of concepts; identify and apply principles; know and apply facts and definitions; compare, contrast, and integrate related concepts and principles; recognize, interpret, and apply the signs, symbols, and terms used to represent concepts. *Conceptual understanding* reflects a student’s ability to reason in settings

involving the careful application of concept of definitions, relations, or representations of either.” (Balka, Hull, & Harbin Miles, n.d.)

Just as they looked at “I Can” statements with each of the Practices, use the next slides to show examples of student responses that demonstrate conceptual understanding.



Conceptual Understanding
Example

[CCSS.Math.Content.6.NS.A.1](#)
Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

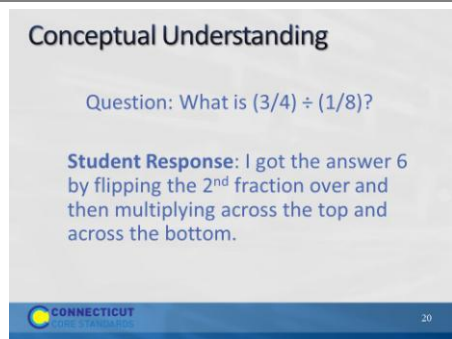
Question: What is $(3/4) \div (1/8)$?

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Slide 19

Conceptual Understanding

Ask participants to look at the example on the slide and discuss with their group how a student might demonstrate conceptual understanding if asked the question *What is $(3/4) \div (1/8)$?* Allow participants to discuss this briefly, 2-3 minutes, and then transition to the next slide.



Conceptual Understanding

Question: What is $(3/4) \div (1/8)$?

Student Response: I got the answer 6 by flipping the 2nd fraction over and then multiplying across the top and across the bottom.

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Slide 20

Ask participants to now consider the student response to the question and have them determine if this student has developed a conceptual understanding.

This student has demonstrated knowledge of an algorithm to do a procedure - it is unclear whether he/she has a conceptual understanding. Have a participant show on the flip chart a visual fraction model a student could use to demonstrate conceptual understanding. Ways in which a student may show understanding:

Creating a story context: How many $1/8$ cup servings of trail mix are in $3/4$ cup of trail mix?

Using a number line: How many segments of length $1/8$ unit are contained in a segment of length $3/4$ units?

Using the relationship between multiplication and division: What can I multiply $1/8$ by to get $3/4$? i.e., $1/8 \cdot X = 3/4$

Conceptual Understanding

Standard 7.RP.2a: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Example

Question: Josh is 10 years old and Reina is 7. Explain whether or not you can use a proportion to find Reina's age when Josh is 18.

Student Response: In 8 years, Reina will be 15. You can't use a proportion because the ratio of their ages isn't constant.

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Conceptual Understanding

Go over the example on the slide and ask participants how the student's response relates to Standard 7.RP.2a. Here we want participants to see that a student is demonstrating a conceptual understanding of determining whether two quantities are in a proportional relationship. They may need to see multiple responses of this students' work to make a final determination of this, so ask what else, if anything, they might ask or look for from this student. And, to support the idea that there is no one right way for a student to demonstrate conceptual understanding, what other ways might they expect to see students answer this question.

Transition to the next slide by explaining that, as they have seen, not all standards explicitly focus on conceptual understanding so they will now look at procedural skill and fluency.

Example adapted from:

http://commoncoretools.files.wordpress.com/2012/02/ccss_progression_rp_67_2011_11_12_corrected.pdf

Procedural Skill and Fluency

"Procedural skill and fluency is demonstrated when students can perform calculations with speed and accuracy."
(Achieve the Core)

"Fluency promotes automaticity, a critical capacity that allows students to reserve their cognitive resources for higher-level thinking."
(Engage NY)

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Slide 22

Procedural Skill and Fluency

Focus on the two key points on the slide. Ask participants for their thoughts on Bill McCallum's statements in the video (shown on slide 16) in which he talks about the design of the standards being such that there is a build up to procedural skill and fluency. Ask why they think this is the case. If it does not come out in the conversation, have participants think back to the video of CCSS-Math co-author, Phil Daro's, video that was viewed in Module 1 about teaching students to get answers. Ask participants what connections might be made between Bill McCallum's and Phil Daro's videos.

Then, transition to the next slide by explaining that teaching students procedures and how to use an algorithm or any type of short cut or trick without developing some level of conceptual understanding for why those things work mathematically is akin to teaching answer getting vs. learning mathematics.

Note: For a deeper look at Phil Daro’s discussion on answer getting, review the video of his longer discussion here: <http://vimeo.com/79916037>

Procedural Skill and Fluency

Which steps can be used to solve for the value of y ?

$$\frac{2}{3}(y + 57) = 178$$

A. divide both sides by $\frac{2}{3}$, then subtract 57 from both sides
 B. subtract 57 from both sides, then divide both sides by $\frac{2}{3}$
 C. multiply both sides by $\frac{2}{3}$, then subtract 57 from both sides
 D. subtract $\frac{2}{3}$ from both sides, then subtract 57 from both sides

http://www.engageny.org/sites/default/files/resource/attachments/grade_7_math_released_questions.pdf

CONNECTICUT Core Standards 23

Slide 23

Procedural Skill and Fluency

A fluency required in Grade 7 is solving equations of the form $px + q = r$ and $p(x+q) = r$, where p , q , and r are specific rational numbers (7.EE.4a). Ask if there is anything more that they would want to see to determine if students’ fluency is based in conceptual understanding.

Procedural Skill and Fluency

Standard 7.EE.4a: Solve word problems leading to equations of the form $px + q = r$ and $p(x+q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

Example

Question: A rectangle has a perimeter of 54 cm. Its length is 6 cm. What is its width?

Student Response: I know that the length and width add up to 27. The width has to be 19 because $27 - 6 = 19$.

CONNECTICUT Core Standards 24

Slide 24

Have participants think about the example on the slide and ask them to explain how the student’s response relates to Standard 7.EE.4a. Ask why, at this grade level, they may want students to provide their strategy rather than just the answer.

After the discussion of the standard, transition to the next slide by explaining to participants that there are many standards, like this one, that ask students to apply their mathematical understanding and skills within a given context.

Application of Mathematics

- The Standards call for students to use math flexibly for applications.
- Teachers provide opportunities for students to apply math in authentic contexts.
- Teachers in content areas outside of math, particularly science, ensure that students are using math to make meaning of and access content.

*Frieda & Parker, 2012
Achieve the Core, 2012*

CONNECTICUT

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Slide 25

Application of Mathematics

Go through points on the slide. Ask participants how they have had students apply mathematics. Get two or three examples. Ask participants why this is important.

Application of mathematics is important because without this step or expectation, students are learning math as a set of rules, procedures, etc. that have no real meaning in the world outside of the classroom. Students need to learn how math works and how it is used. Note here that when the conversation of application of mathematics typically comes up, the phrase ‘real-world problems’ is usually somewhere in the conversation. As teachers think about the types of problems that students will solve in order to apply their mathematical understanding, have them think about problems that would be ‘real world’ to their students. This means that the problems should be contextually relevant, engaging, and easily understood by the students at their particular grade level. Also note that, just as we saw with the fluency standard, not all standards focus on application. But, when the standard does point to solving problems through an application of mathematics, we really want to see how students can flexibly use what they know and understand. Finally, ask participants to briefly discuss how they can engage students in authentic problem-solving scenarios.

Before moving to the next slide that has examples of contextually relevant problems, highlight the third bullet on the slide and ask for one or two volunteers to give examples of how the CCS-Math standards can be supported and are connected with the standards from other content areas in order for students to see and apply mathematics outside of their typical math lesson time.

Application of Mathematics

Example

Sophia's dad paid \$43.25 for 12.5 gallons of gas.
What is the cost of one gallon of gas?

Retrieved from Illustrative Mathematics <http://www.illustrativemathematics.org/>

CONNECTICUT 26

Slide 26

Application of Mathematics

Have participants examine the example on the slide and discuss ways that conceptual understanding and procedural skill and fluency can be applied when solving this problem. Then, have participants look at their standards to determine which standard is being addressed in this problem.

Standard addressed by the problem: CCSS.Math.Content.6.NS.B.3

Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Example of Participant Response:

To address the standard, students should not be allowed to use a calculator when doing this problem. They should see this as a division problem even though the word divide, nor the division symbols are present and be able to divide the two decimals to get the correct answer of \$3.46. They should be able to determine if the answer they get is reasonable given the context. This problem is an example of partitive division (sharing \$43.25 over 12.5 groups) and relates to further work with unit rate (CCSS.Math.Content.7.RP.1).

What Do These Students Understand?
Part 2

Return to the “Who Knows Math” worksheet on pages 9–12 in the Participant Guide. Which students have shown conceptual understanding, which have shown procedural skill and fluency, which have shown both, and which pieces of work would you need to know more to make the determination?

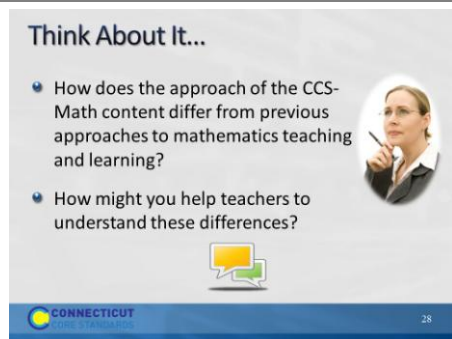
CONNECTICUT 27

Slide 27

Have participants look back at the “Who Knows Math” student work and ask them to make assumptions about which students have shown conceptual understanding, which have shown procedural skill and fluency, which have shown both, and which pieces of work they would need to know more about in order to make the determination. Have volunteers share their thinking.

An important point to bring up here is that we are asking participants to make assumptions only because the student is not present to find out more. However, teachers should try not to make a determination of what

students know and understand based on an assumption. They need to probe deeper to really determine where students are with their understanding.

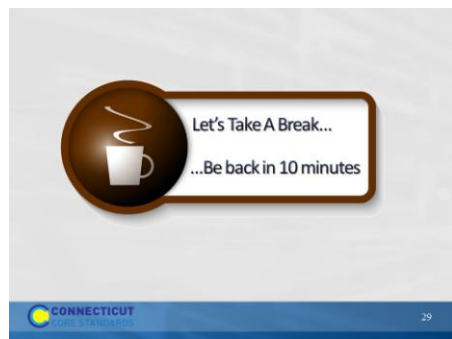


Slide 28

Think About It

Finally, wrap up this section by asking participants to reflect on the questions on the slide. **Allow 5 minutes for discussion.** You can also use this question to transition to the next activity after the break by linking this discussion on the CCS-Math approach to rigor and now looking specifically at the standards to see how conceptual understanding, procedural skill and fluency, and application of mathematics is developed over and within grade levels.

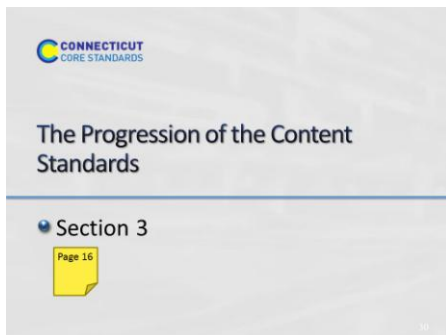
As time permits, ask for volunteers to share their responses to the question.



Slide 29

Blank

Section 3



Slide 30

Section 3: The Progressions of the Content Standards

Section 3 Time: 80 minutes

Section 3 Training Objectives:

- To provide participants with information on, and experience with, identifying standards that address conceptual understanding, procedural skill and fluency, and application of mathematics.
- To have participants experience how concepts are developed within and across grade levels.
- To provide participants practice with identifying concept progressions.
- To provide participants with an understanding of how connections between standards across multiple domains can be made to support the deepening of mathematical understanding.

Section 3 Outline:

- The definitions of “conceptual understanding”, “fluency”, and “application” developed in the previous session are used to help participants analyze the content standards in three different ways. In Exploring the Standards Part 1, participants explore the content standards for one domain, at one grade level, and determine which standards focus on conceptual understanding, procedural skill and fluency, and application of mathematics.
- In Exploring the Standards Part 2, participants explore the content standards for one domain across grade levels and record five general observations about the progression of the concepts and two connections to the Practice Standards.
- In Exploring the Standards Part 3, participants explore all of the domains for one grade level to identify connections across domains that can be referenced in a lesson or unit in order to support the deepening of mathematical understanding.
- Participants will then discuss and reflect as a large group on the importance and instructional implications of the progressions and any new insights they now have into the standards.
- The activity will conclude with having participants view the video Gathering Momentum for Algebra in order to see where the K-5 content progressions impact the larger K-12 learning pathway.

Supporting Documents:

Standards for Mathematical Practice

Exploring the Content Standards Observation Sheet

Materials

Chart paper, markers

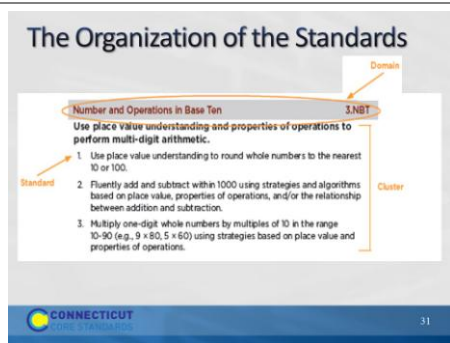
Sets of Standards Progression Cards (one color per domain) (1 full domain color set per table group)

Signs made for tables: 6, 7, 8

Video

Gathering Momentum for Algebra

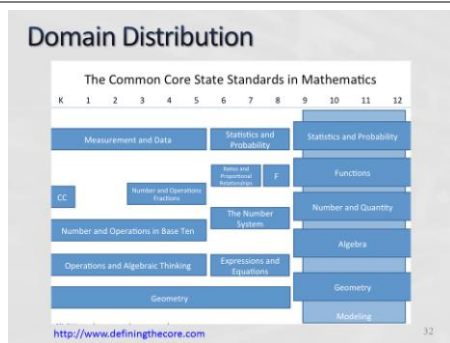
Notes: In this activity you will have full sets of content standards with one standard per card. Each of the cards is color coded for each domain that it belongs under. You begin by giving a table one grade level set that has all of the domains for that grade level and with that set they complete part 1. Then ask participants to give all of a particular domain to one table, so one table will have all of the Expressions and Equations standards, one table will have all of the Geometry standards, etc. With that domain they complete part 2. Then, have participants separate the domain by grade level again and give each table the full grade level set again so that they can complete part 3. While participants should have a full set of print-based standards, having them work with the standards in this card format allows them to physically manipulate the standards, create connections, and see the progressions side-by-side rather than having to flip through multiple pages.



Slide 31

The Organization of the Standards

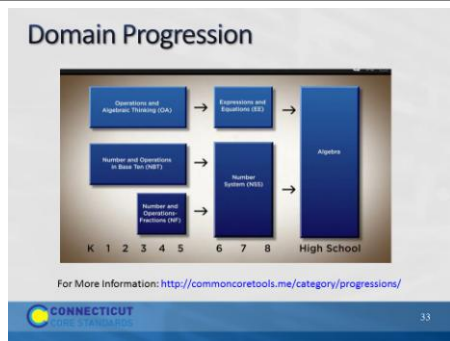
Quickly go through the organization of the Content Standards with the participants. Each grade level is organized by domains and within each domain there are associated groups of standards that make up a cluster. There can be multiple clusters within a domain. Transition to the next slide by explaining that domains span several grade levels.



Slide 32

Domain Distribution

Explain that in K–8, there are four or five domains per grade level. Note that the Geometry domain is the only domain that spans K-8. When students are developmentally ready and have a solid foundation in Number and Operations in Base Ten, Number and Operations – Fractions is layered on beginning in third grade. In high school, there are 5 “conceptual categories”. In the background of these is the Modeling conceptual category – modeling standards appear throughout the high school standards and are indicated by a star symbol (★). Each conceptual category is broken up into 4-6 domains. Transition to the next slide by reminding participants that the domains and conceptual categories were determined based on a very specific and coherent roadmap for learning called a progression.

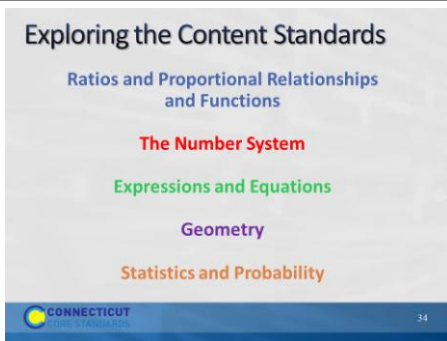


Slide 33

Domain Progression

Remind participants that the domains were written so concepts build on each other grade after grade so that, in this particular progression, there is a clear pathway to high school Algebra. The video at the end of this section is an explanation of the progressions from CCSS-Math Co-Author, Bill McCallum.

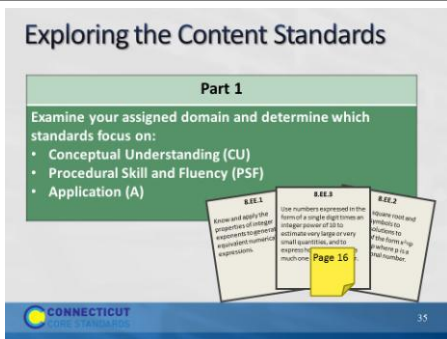
More information about specific domain progressions can be found at the Common Core Tools Website: <http://commoncoretools.me/category/progressions/>. Have participants make a note of this resource.



Slide 34

Exploring the Content Standards

Participants will be divided into grade level groups (6, 7, and 8) to complete the three parts of Exploring the Standards. Signs for each grade, 6–8, should be posted around the room. Assign each table one of the 6–8 Content Standard domains and pass out the color coded domain cards to each group.

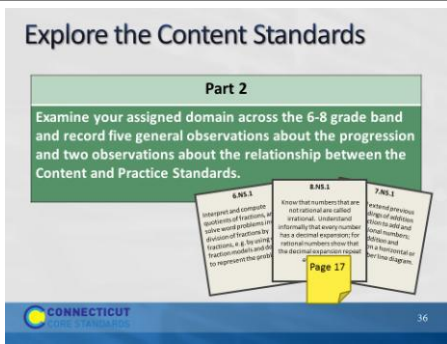


Slide 35

Exploring a Progression

For Part 1 of Exploring the Content Standards, ask participants to examine their assigned domain and determine which standards focus on Conceptual Understanding, Procedural Skill and Fluency, and Application of Mathematics. Participants should use sticky notes to mark the card with either CU, PSF, or A according to how they sorted the cards. After sorting and marking the cards, have participants answer the following questions on **page 16** in their Participant Guide:

- What are the expectations around conceptual understanding at your grade level?
- What are the fluency expectations at your grade level?
- What are the opportunities for application at your grade level?



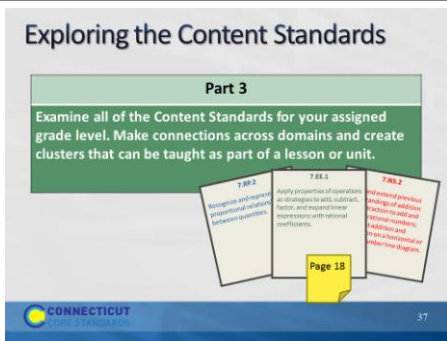
Slide 36

Exploring a Progression

Now, ask participants to examine the Content Standards in their assigned domain across grade levels and complete the observation worksheet on which they record five general observations about the progression and two observations about how the Practices are integrated into the content. Allow each domain group to share their observations.

The goal here is for participants to see the progression of and how conceptual understanding, procedural skill and fluency, and application of mathematics are developed across grade levels. Because they are using the cards, participants should be able to line up the grade level domain standards side by side so that the vertical alignment can be seen horizontally across their table allowing for a continuous comparison.

Transition to Part 3 of the activity by explaining to participants that teaching the standards is not just about understanding how the Content Standards progress across a domain, but also about understanding how the standards of different domains work together.



Slide 37

Making Connections

Ask participants to separate their domain cards by grade level. Direct participants to the areas around the room designated for each grade, 6–8. They should take the cards for their grade level from the table to the designated area. For each grade, the cards for all five domains will be combined so there is a complete set of Content Standards for each grade level. In larger groups there may be several complete sets so the grade groups can be broken into two or three smaller groups.

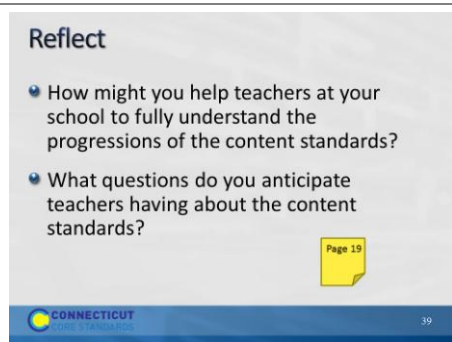
Once they have their complete set of standards, ask participants to examine all of the Content Standards for their grade level and make connections across the domains that can be referenced as part of a lesson or unit. Allow participants **10 minutes** to create and record the connections and then take **10 minutes** to allow each grade level the opportunity to share one or two of their connections. They can record their thoughts on **page 18** in the Participant Guide.



Slide 38

To begin to wrap up this section, click on “Watch Video” to play the video Gathering Momentum for Algebra from here: http://www.youtube.com/watch?v=ONPADo_Nt14. The video is **2:08** long.

Use this video to transition to the next slide.



Slide 39

Reflect

Now that participants have a deeper understanding of the CCS-Math expectations around conceptual understanding, procedural skill and fluency, and application of mathematics at their grade level, ask them to reflect on the two questions on the slide and record their answers on **page 19** in their Participant Guide. As time permits ask for volunteers to share their responses. Allow **5 minutes**.

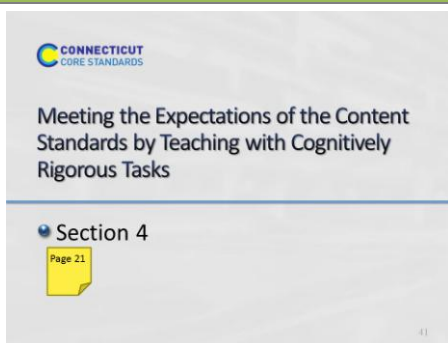
Then, set up the after lunch activities by explaining to participants that they will build off of their understanding of the Content Standards to explore the implications for teaching and learning in the classroom.



Slide 40

Remind participants of the need to be timely. Allow 45 minutes. State time to return.

Section 4



Slide 41

Section 4: Meeting the Expectations of the Content Standards by Teaching with Cognitively Rigorous Tasks

Total Time on Section 4: 85 minutes

Section 4 Training Objectives:

- For participants to understand the definition of a cognitively rigorous task.
- To deepen participants understanding of why incorporating cognitively rigorous tasks into their mathematics instruction is important.
- To examine strategies that can be used to incorporate cognitively rigorous tasks that will benefit all students.

Section 4 Outline:

- Participants begin by viewing the video Dan Meyer: *Math Class Needs a Makeover*, which is about problem solving. Participants will discuss the video and compare their previous experience with the *Kites Activity* with Dan Meyer’s message in the video.
- Participants are then shown how scaffolds can be removed slowly from a problem in order to deepen the level of cognitive rigor. Participants use Hess’s Cognitive Rigor Matrix to discuss the problem example.
- Participants will brainstorm how cognitively rigorous mathematics tasks can be used to benefit all students and will be presented with four strategies that can be used to differentiate cognitively rigorous tasks for all

students, as needed, in order to provide multiple entry points into the mathematics while maintaining the problem’s rigor.

- Participants will wrap up the activity by making connections back to “conceptual understanding,” “fluency,” and “application” and how each of these is addressed through cognitively rigorous tasks.

Supporting Documents

Video Observation Sheet

Hess’s Cognitive Rigor Matrix for Mathematics and Science

Strategies for Differentiating Cognitively Rigorous Tasks notes page

Resources for Finding Tasks

Reflect worksheet

Materials

Chart paper, markers

Video

Dan Meyer: Math Class Needs a Makeover

Background Resources

- Use the following to gain a deeper understanding of Bloom’s Taxonomy, Depth of Knowledge, and Hess’ Cognitive Rigor Matrix that will be used during this section.
 - Bloom’s Taxonomy: <http://cft.vanderbilt.edu/guides-sub-pages/blooms-taxonomy/>
 - Depth of Knowledge: <http://schools.nyc.gov/NR/ronlyres/2711181C-2108-40C4-A7F8-76F243C9B910/0/DOKFourContentAreas.pdf>
- Karen Hess & Cognitive Rigor Matrix: <http://vimeo.com/20998609> and http://www.sde.idaho.gov/site/common/webinars/Cognitive%20Rigor%20Matrix%20Article_Hess,%20Carlock,%20Jones,%20and%20Walkup.pdf



Slide 42

Math Class Needs a Makeover

Begin the activity by explaining to participants that they are going to watch a video in which math expert, Dan Meyer, discusses the types of problems that students should be doing in math class.

Click on “Watch Video” to play the video *Math Class Needs a Makeover* from here:


http://www.ted.com/talks/dan_meyer_math_curriculum_makeover.html. The video is **11:39** long.

After viewing, debrief the video with participants by first asking for their thoughts on Dan Myer’s message. After two or three volunteers share, explain to participants that one of the keys to using the types of problems that Dan Meyer talks about is taking a math problem and removing part, if not all, of the scaffolding that is in place within the problem itself and moving the scaffolding into the process of teaching. Transition to the next slide by having participants think back to their experience in Module 1 when they worked on the *Kites Activity*.

Kites Activity

A store sells kits to make kites. All the kites are quadrilaterals. Some are what we call “kite-shaped.” Others are rectangles, squares, rhombi, and four sided shapes with no particular characteristics. A kit has string, paper, and two sticks to form the skeleton of the kite.

The store owner needs to know what sticks to put in the kits for each shape, and how to tell the purchaser how to put the sticks together for each shape. Your job is to give the store owner information about making squares, rectangles, trapezoids, and typical kite shapes. For each shape, list the sticks needed and how they should be put together. Use the paper strips as your sticks and connect them using the brads to make your kite shapes.



Slide 43

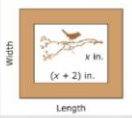
Ask participants to look and determine what, if any, scaffolds are in place within the problem itself. When thinking about scaffolds in this context they should look for anything in the problem that is given, but that students could actually figure out for themselves. Did scaffolding take place during the task through clarification or through asking and answering questions to help individuals move forward, etc.? Allow participants to briefly discuss the differences in their experience with solving a problem with scaffolds built in to the teaching rather than having scaffolds built into the problem.

Now, have participants turn to Hess’ Cognitive Rigor Matrix for Math and Science on **page 22** in their Participant Guide. Go over how to read the matrix; revised Bloom’s Taxonomy are on left hand side going down and Webb’s Depth of Knowledge levels go across the page. In their groups, have participants determine where, for them, the *Kites Activity* falls on the matrix. As time permits, ask groups to share their determination and what evidence, in the problem and the implementation of the problem, they used to make this judgment.


Transition to the next slide by explaining to participants that they will now look at another problem.

Take a Look...

Sam uses one-inch frames for pictures for which the length is 2 inches longer than the width, as shown.



- Write an algebraic expression for the area of the picture alone.
- Write an algebraic expression for the area of the picture and frame together.
- If the area of a frame is 24 square inches, what are the dimensions of the picture?



Slide 44

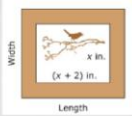
Have participants look at the problem on the slide and determine its placement on the Cognitive Rigor Matrix and have groups share their response.

Note: This has been modified from a high school PARCC sample assessment task (A-SSE.1) for the purposes of this module. Due to the scaffolding, this problem would fall on the Cognitive Rigor Matrix somewhere around the Apply/DOK Level 1 range because students are provided a set of steps and only need to determine the dimensions of the frame, apply the formula for area, subtract the algebraic expressions, and solve a linear equation for x .

After participants have shared their response and an agreement is made on the matrix placement, transition to the next slide by asking how the problem might change if some or all of the scaffolding is removed.

Take a Look...

Sam uses one-inch frames for pictures for which the length is 2 inches longer than the width as shown.



If the area of a frame is 24 square inches, what are the dimensions of the picture?


CONNECTICUT CORE STANDARDS 45

Slide 45

Have participants determine if removing the scaffold (the steps to follow) changes the problem's placement on the matrix. Some participants may say that the placement has not changed, however an argument can be made here that the problem is still in the Apply range but has moved to DOK 2 because students need to make the determination of having to calculate the area of the picture, and the picture and frame, vs. being specifically asked to find the area in the previous version. As there are elements of both DOK 1 and DOK 2 present in the problem, a definitive level may not be agreed upon, however that is okay because the two are very closely related. As long as participants are not way off on their placement determination go ahead and move to the next slide where more of the scaffolding has been removed.

Take a Look...

Sam uses one-inch frames for pictures for which the length is 2 inches longer than the width.



If the area of a frame is 24 square inches, what are the dimensions of the picture?

CONNECTICUT CORE STANDARDS 46


Slide 46

Have participants repeat the process of determining the problem's placement on the matrix. But note, as more of the scaffolding is covered up, where the problem actually falls is beginning to become more dependent on what

students know and how the teacher implements the problem. For example, if students do not understand what a ‘one-inch frame’ means, they may need to ask the teacher clarifying questions which may push the problem further into the Analyze range. However if students understands the concepts involved and is easily able to determine the algebraic expression that represents the area of the frame, then this would stay in the Apply range.

Take a Look...

Sam uses one-inch frames for pictures for which the length is 2 inches longer than the width.



If the picture's dimensions are both whole numbers, show that the area of the frame has to be a multiple of 4.

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Slide 47

Take a Look

Have the participants examine the revised task and determine its placement on the matrix. The problem has changed significantly because students are generalizing relationships and interpreting the expression for the area of the frame as a multiple of 4. Again, the placement on the matrix will be dependent somewhat on students' understanding and teacher implementation, but if focusing on what the task is asking, it should be placed somewhere around the Apply/Analyze and DOK 3 range. Again, make sure that participants understand that not every problem is going to fit nicely into one spot on the matrix, but that they should be able to find a general area based on the criteria given.

Finally, ask participants what challenges they may face when asking teachers to move students towards thinking at this level of mathematics. A key point to bring out in the discussion, if it is not brought out by participants, is that many teachers may feel that not all students are ready to work tasks such as this. Use the answer to this question to transition to the next slide.

Task Background: This particular task has been adapted from an Algebra I PARCC Assessment prototype task for the purposes of this Module. (Original task can be found at: <http://www.parcconline.org/sites/parcc/files/HS-Alg1Math2PictureFrame.pdf>)

Content Standards covered by the task as originally written: A-SSE.1-2 (identified as major content in the Algebra I course). The task was aligned to MP 7 and MP 2.

The BIG Question

How can I help teachers incorporate cognitively rigorous mathematics tasks that will benefit ALL students?



Slide 48

The Big Question

Explain to participants that if students are going to be expected to work cognitively rigorous tasks as they enter college, such as those Dan Meyer discussed, we have to prepare them for this in K-12. The question now becomes “How can I help teachers incorporate cognitively rigorous tasks that will benefit all students?” This is the question participants will focus on now. Explain to participants that they will now examine strategies that can be used to provide multiple pathways into the mathematics of cognitively rigorous tasks so that all students can benefit from its use.

Strategies for Differentiating Cognitively Rigorous Tasks

- Scaffolding
- Open Questions
- Parallel Tasks
- C-R-A



Slide 49

This begins the discussion of the four strategies listed on the slide. Explain that one of the key things to keep in mind when differentiating mathematics tasks is that teachers will want to be sure to make modifications or offer choices in tasks that allow students the needed point for entry into the mathematics, but at the same time keeping the level of rigor high. Often mathematics is differentiated by providing ‘easier’ tasks to students who may not yet be ready for the main task. These ‘easier’ tasks sometimes lower the level of rigor to the point that the students’ receiving that task are never given the opportunity to engage in deeper reasoning about the mathematics. Whenever possible, teachers should maintain the level of rigor, but make modifications in such a way that a solution is still within the students’ reach. The five strategies that will be discussed are those that teachers can use to do just that.

Scaffolding

A circle has its center at (6, 7) and goes through the point (1, 4). A second circle is tangent to the first circle at the point (1, 4) and has the same area.

What are the coordinates for the center of the second circle? Show your work or explain how you found your answer.

- What can be added to the problem?
- What can happen during the implementation?



Slide 50

Begin by revisiting scaffolding. Explain to participants that just as we stripped the scaffolding away from the picture frame problem earlier to increase the level of rigor of the problem, scaffolding can be added back in, as needed, to help students reason about the mathematics. These scaffolds can be added back in through the problem itself or through the implementation (questioning, group work, representations, etc.). The key here would be to start with the original or root task and then add the scaffolding as needed by the student, when the student reaches the point of not being able to go any further. Teachers should plan the scaffolds out beforehand so that they are ready to provide them within the lesson.

Have participants discuss scaffolds that can be added to the problem and those that can be added during implementation. Chart participants’ responses for later reference.

Problem background:

This is a high school sample item from Smarter Balanced. Note that while this is a sample assessment item the context used here is using the problem as an instructional task. This task addresses standard CCSS.8.G.8 and CCSS.8.EE.6. Notes from SBAC on this task: “This item is the less difficult of two items designed to assess the same content. It lends itself to multiple approaches, including the proportional reasoning from grade 7, distance between points in the coordinate plane from grade 8, and the trigonometric approaches in high school. Smarter Balanced is exploring different student response formats for items of this type.”

For a rubric on how this item would be scored, go to: <http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/09/math-rubrics/43046Rubric.pdf>

Open Questions

Example 1: I have three natural numbers whose average is nine. What are the numbers?

Example 2: I have a rectangle whose perimeter is 24 cm. What are the dimensions of the rectangle?



Slide 51

Open Questions

Explain to participants that open questions are questions posed in problem form to students that allow multiple responses and approaches to correctly answer the questions. These types of questions allow all students, no matter their developmental and readiness level, to participate in in small and large group mathematical discussions.

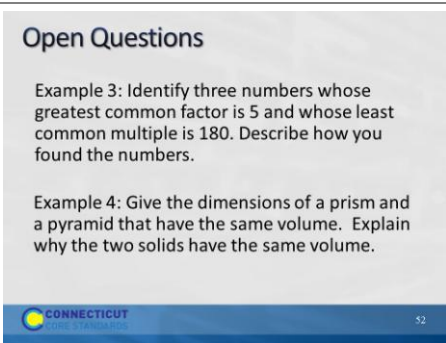
Show Example 1 and ask participants to solve this individually. Then, have participants share their answers and briefly discuss how even though they may have used different numbers and created a different problem everyone was essentially working on the same concept. Have participants determine where on the Cognitive Rigor Matrix this problem might fall.

Repeat this process for the remaining examples on this slide and on the next slide.

Open Questions

Example 3: Identify three numbers whose greatest common factor is 5 and whose least common multiple is 180. Describe how you found the numbers.

Example 4: Give the dimensions of a prism and a pyramid that have the same volume. Explain why the two solids have the same volume.



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After going over the two examples, ask participants to think for a moment about how they might introduce this strategy to teachers and what example they might use during their introduction. Participants may need to utilize the standards to create an example of an open question if one of the four examples here will not work for a particular grade level, or they may modify one of the examples shown here. After participants complete their notes, move on to the next strategy, parallel tasks.

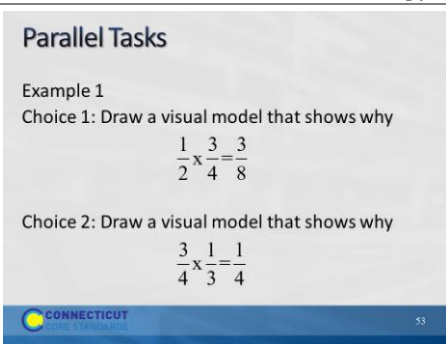
Parallel Tasks

Example 1

Choice 1: Draw a visual model that shows why

$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

Choice 2: Draw a visual model that shows why

$$\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$$


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Explain to participants that parallel tasks are sets of tasks that students can choose from that are close enough to address the same standard(s) but different enough to allow for the multiple entry points into the mathematics. Also, just as with open questions, students, no matter which task they have selected, are able to equally engage in mathematical discussions. Go over Example 1 on the slide and Example 2 on the next slide.

Parallel Tasks

Example 2: Order these expressions from least to greatest

Choice 1: -4 , $\sqrt{4}$, -3 , $\frac{5}{2}$

Choice 2: -3 , 0 , $|-1|$, $-\frac{2}{5}$, -2.5

Choice 3: $\sqrt{9} - 1$, $\frac{7}{2}$, $|-3|$, $-4+5$



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After going over the example, have participants discuss at their table how parallel tasks are related to open questions. Then, have them think about how they might introduce this strategy to teachers and what example they might use during their introduction. Participants may need to utilize the standards to create an example of an open question if one of the four examples here will not work for a particular grade level, or they may modify one of the examples shown here.

C-R-A

Concrete Representational Abstract



Slide 55

Explain to participants that C-R-A stands for concrete to representational to abstract and is a strategy for differentiation that does not involve changing the problem, but rather, involves changing the models students use to solve the problem. C-R-A can be thought of as a continuum where we want all students to eventually get to the point of using abstract (mathematical symbols) models to represent a problem, but some students may need to step back on the continuum to representational models (drawings) or even to concrete models (some form of manipulative). Teachers should move students along the continuum, both forward and backward, as needed. Again, however, with the end goal of getting to the abstract.

The following site provides more information on the CRA approach for facilitators:
<http://www.coedu.usf.edu/main/departments/sped/mathvids/strategies/cra.html>

C-R-A

Here are the low temperatures (in Celsius) for one week in Juneau, Alaska:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
5	-1	-6	-2	3	7	0

Arrange them in order from coldest to warmest temperature.

Retrieved from Illustrative Mathematics <http://www.illustrativemathematics.org/>



Slide 56

Have participants discuss with their group how they might use C-R-A to help students complete the task on the slide, and determine how they might move a student forward on the continuum based on the student's entry point.

Resources for Finding Tasks

- Illustrative Mathematics
<http://www.illustrativemathematics.org/>
- Achieve the Core
<http://achievethecore.org/>
- Smarter Balanced
<http://smarterbalanced.org/>
- Mathematics Assessment Project
<http://map.mathshell.org/materials/index.php>



Slide 57

Explain to participants that the strategies that they just examined and discussed can be used with tasks from their curricular materials or, if needed, they can find additional tasks online. If time permits and if an internet connection is available, show participants the tasks available at each of the resources on the slide. Wrap up this section by having participants complete the reflection, either individually or in small groups, on the next slide.

Reflect

1. What other sites/materials do you know of that are good resources for cognitively rigorous tasks?
2. How do cognitively rigorous tasks relate to conceptual understanding, procedural skill and fluency, and application of mathematics?
3. How do cognitively rigorous tasks help students to develop the mathematical expertise in the Standards for Mathematical Practice?



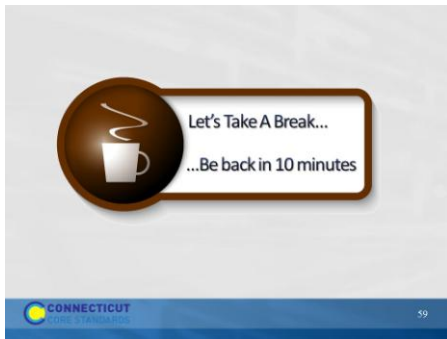
Slide 58

Reflect

Allow participants to respond to the reflection questions on the slide and in their Participant Guide on **page 24**. As

time permits have volunteers share their thinking.

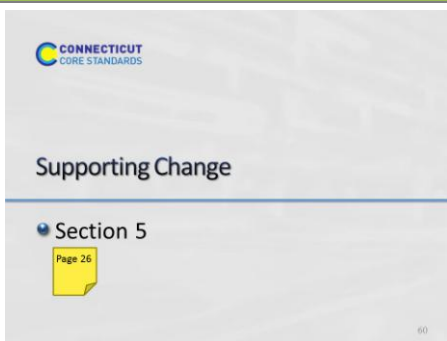
Transition to the next section by telling participants that now that they have looked at strategies for creating multiple entry ways into the mathematics through the problems provided, they will now examine instructional strategies that can be used by teachers to implement tasks. Because participants will be moving from table to table in the next activity, ask participants to put their personal belongings to the side.



Slide 59

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Section 5



Slide 60

Section 5: Supporting Change

Section 5 Time: 85 minutes

Section Training Objectives:

- To help participants identify elements of lessons that work to develop conceptual understanding, procedural skill and fluency, and application of mathematics.
- To provide participants with instructional strategies for teaching the content standards through problem solving, for helping students to develop procedural skill and fluency, and to provide students with opportunities to apply their mathematical understandings.
- To have participants create a plan for disseminating big ideas from the session with teachers at their school
- To have participants anticipate specific teacher questions and challenges around implementing lessons that

incorporate the Standards for Mathematical Content

Section 5 Outline:

- Participants will begin by watching the videos *What’s Your Sign?* and *Zero Pairs, Manipulatives, and a Real-World Scenario*. During each video, participants will take notes on the corresponding *Video Observation* worksheet. After watching both videos, they will discuss the strategies that were seen and the evidence provided as a large group and chart strategies to use as a master resource.
- Participants will then begin to think about areas of instructional practice that will need to be addressed with the teachers with whom they work. Participants will be asked to consider this through the lens of approaching teachers with the idea of rather than learning to teach in a completely new way, working towards enhancing the instructional strategies in place now so that they help students to meet the expectations of the CCS-Math. To experience this, participants will explore ways that current strategies, such as concept cards/maps, journals, group work, decision making and so forth, can be restructured to meet the new expectations. Participants will jigsaw each of the instructional strategy areas during which they will examine instructional strategies and/or examples, discuss how the examples can be implemented, and generate at least one new idea to share with others.
- Participants will return to their ‘home’ group and discuss their strategy and new idea and as a group will wrap-up the activity by working together to make a plan for helping the teachers they work with understand and implement the key ideas and strategies presented in the module.

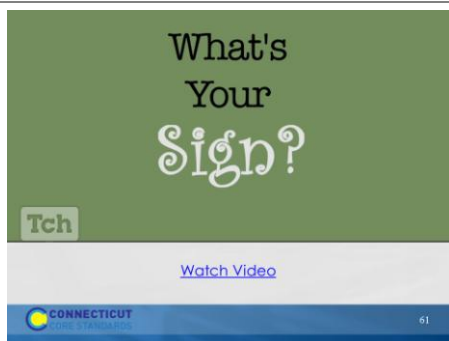
Section 5 Supporting Documents

- Video Observation Sheets
- A New Spin on Old Strategies
- Group 1: Math Journals
- Group 2: Mathematical Language
- Group 3: Instructional Implementation Sequence: Attaining the CCSS Mathematical Practices Engagement Strategies (separate handout)
- Group 4: Group Work and Decision Making
- Next Steps

Section 5 Materials

Chart paper, Markers

Notes: If time is a factor at this point in the day, you may opt to only play one video, giving participants enough time to review each of the strategies in the Jigsaw activity.



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What's Your Sign?

Pass out the CCSS Instructional Practice Guide. Explain to participants that they will watch two videos; both lessons focus on operations with integers in Grade 7 classrooms. While watching the videos, participants can make notes on the Video Observation sheet provided in the Participant Guide on **pages 26-27**. The goal of watching the videos is to get participants looking for examples of the important aspects of teaching the Content Standards that have been discussed throughout this session, but to also look at additional teaching strategies that they want to bring back to teachers at their school.

Begin by watching the first video, *What's Your Sign?: Integer Addition*, that shows part of a Grade 7 lesson. Click on "Watch Video" to play the video from here: <https://www.teachingchannel.org/videos/adding-integers-lesson-idea>. The video is **5 minutes** long.



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Zero Pairs, Manipulatives, and a Real-World Scenario

Watch the video *Zero Pairs, Manipulatives, and a Real-World Scenario*. Click on "Watch Video" to play the video from here: <https://www.teachingchannel.org/videos/teaching-subtracting-integers>. The video is **6 minutes** long.

Chart the instructional strategies used by the teachers in the two videos.

Explain to participants that they will now have an opportunity to explore some additional strategies and resources for teaching the CCS-Math Content Standards.

A New Spin on Old Strategies

Group 1: Journals

Group 2: Mathematical Language

Group 3: Engagement Strategies

Group 4: Group Work & Decision Making

1. How can this strategy/resource support the CCS-Math content and practice standards?
2. Generate at least one new idea for the use of the strategy/resource with students.

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A New Spin on Old Strategies

Participants will work in small groups to examine four different Math strategies and/or resources. The group that the participant sat in to watch the video will become their home group. Have participants number off at each table until everyone has a number representing 1, 2, 3, or 4. Then, participants will move into their Jigsaw groups (1's will become Group 1 and discuss Journals, 2's will become Group 2 and discuss Mathematical Language, etc.). On the notes page in the Participant Guide, each participant should note key points that come out of the discussion and the group should develop at least one new way to implement the strategy being discussed. (Allow 15 minutes for the Jigsaw discussions)

When time is called, participants will move back to their 'home' group and each person will have 5 minutes to discuss their strategy/resource.

Notes about setting up and managing movement to tables:

- In cases where you are working with a large number of participants, create multiple Jigsaw groups (e.g., two to three groups labeled as Group 1, two to three groups labeled as Group 2, etc.) so that there are no more than six participants at any one table at any one time.
- Make sure that each table is clearly labeled (using the cardstock that has been provided) so that groups are not trying to figure out where they are going as they move.
- Give participants a 1 minute wrap up warning at each table so that they can conclude their conversations and prepare to move to the next table.

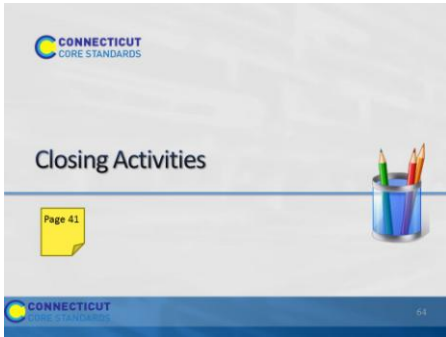
"Set up" this activity to participants by explaining that they might explore strategies they already know. However, a new spin has been put on each strategy in order to meet the challenges presented by the CCS-Math. Also, explain that there is space within the Participant Guide on **pages 28-29** on which they can make notes about the important points they want to bring back to their 'home' group and to teachers at their school.

After the 'home' group discussion is complete, debrief the strategies/resources as a large group and highlight and chart some new ideas generated.

Transition to the last part of this section by asking participants how they will now share this information back at their school site and allow participants to make notes on the Next Steps worksheet on **page 39** in the Participant

Guide.

Closing Activities



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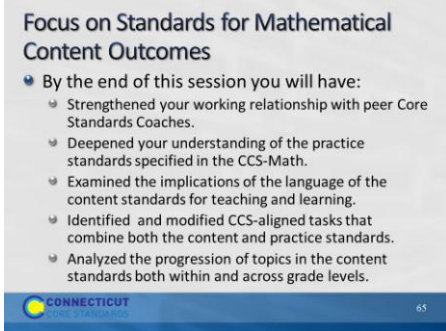
The goal of the Closing Activities is for participants to determine how they will take the key information back to their peers at their school so that everyone gains a shared understanding.

Total Time on Closing Activities **5 minutes**

Section 6 at a Glance:

1. Review the Module 2 Outcomes.
2. Have participants complete the Post-Assessment.
3. Have participants complete the online Session Evaluation located here:

<http://surveys.pcgus.com/s3/CT-Math-Module-2-6-12>



Slide 65

Review the outcomes for the day.

Focus on Standards for Mathematical Content Outcomes (cont'd)

- By the end of this session you will have:
 - Deepened your understanding of the potential of the CCS-Math to change mathematics teaching and learning.
 - Gained an understanding of some of the challenges involved in implementing the CCS-Math.
 - Explored strategies for supporting teachers as they make changes to their classroom practices.
 - Made plans for next steps in your CCS-Math implementation.



Slide 66

Review the outcomes for the day, continued.

Post Assessment and Session Evaluation

- Where Are You Now?
- Assessing Your Learning.
- Please complete an online Session Evaluation. Your feedback is very important to us!



<http://surveys.pcgus.com/s3/CT-Math-Module-2-6-12>



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This Post-Assessment is the same as the Pre-Assessment they took in the beginning of the session. This assessment is to gauge their learning based on the activities of the full day session. Remind the participants to fill out their online Session Evaluation forms as well.