CONNECTICUT VOLUME TABLES FOR PLANTATION-GROWN RED PINE, *Pinus resinosa*, Solander

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Because of its high survival, red pine has been extensively used for forest planting in the Northeast for more than 30 years. In Connecticut some 15,000 acres, chiefly farm land which had for one reason or another been eliminated from agricultural use, have been planted to this species. Growth has been rapid and, since the species fails to differentiate into crown classes when planted in pure stands, thinning at age 15 to 25 years is imperative to prevent stagnation.

During and since World War II many stands have been thinned and the products used for such purposes as fiber for roofing felt, posts of various kinds, pulp and, to some extent, for lumber.

Two tables of volume in cubic feet for the species were constructed by the alinement chart method in 1938 (1). Recently additional measurements were secured on trees of larger diameter and greater height and these, together with data previously obtained, have been incorporated into a new series of volume tables which are presented below.

DESCRIPTION OF PLANTATIONS

Measurements of 512 trees for the 1938 tables were obtained in 1930 and 1936 during the thinning of some 20 experimental plots which had been laid out in 1930 on locations representing a wide variety of sites. The soils varied from coarse sands of glacial outwash origin to fine sandy loams and loams which, except for stoniness or topography, could be classed as high grade soils for agricultural purposes. The trees varied from 1.5 to 9.3 inches in diameter, from 15 to 49 feet in height, and from 16 to 32 years in age from seed. The original growing space per tree ranged from 25 to 40 square feet and most of the stands had been lightly thinned in 1930 or in 1936 or in both years.

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[2] The authors wish to express their thanks to Professor Walter H. Meyer, School of Forestry, Yale University, for advice in preparing the volume tables and for reviewing the manuscript, to Mr. Milton C. Arnold for recording field measurements on Form 558a and for planimetering these forms, and to Mrs. Lewis K. Robinson and Mr. Lester C. Guilbert for much of the calculation required prior to final assembly of the data.

The supplemental measurements taken from 1946 to 1948 were on 281 trees which averaged about 10 years older than those previously described. These came from a rather restricted range of sites and were not from the same plots as those recorded in 1936. The trees varied from 4.0 to 15.0 inches in diameter and from 32.9 to 62.5 feet in height.

FIELD MEASUREMENTS

The following data were taken in the field on the 512 trees measured in 1936:

1. Total height of the tree to the nearest tenth of a foot.
2. Length of green crown.
3. Diameter outside bark to the nearest tenth of an inch and bark thickness to the nearest twentieth of an inch at:
   a. Stump height, usually six inches above ground.
   b. Three feet above ground.
   c. Four and one-half feet above ground (breast height).
   d. Intervals of one, two . . . ten tenths of the distance between 4.5 feet above ground and the tip.

Measurements made on 281 trees in 1946 to 1948 were taken during logging operations and were as follows:

1. Total height to the nearest tenth of a foot.
2. Stump height, taken at six inches above ground.
3. Distance between stump height and a top diameter of 2.0 inches, outside bark, to the nearest tenth of a foot.
4. Distance between stump height and a top diameter of 5.0 inches, inside bark, to the nearest tenth of a foot.
5. Length of each cut log.
6. Diameter outside bark to the nearest tenth of an inch and bark thickness to the nearest twentieth of an inch at:
   a. Stump height.
   b. Breast height.
   c. At the small end of each cut log.

Of the 793 trees measured, 788 were used in the preparation of cubic foot tables and 479 in the making of a board-foot table.

PREPARATION OF TABLES

Volume Determination

The taper measurements of the 793 trees were first recorded on U. S. Forest Service Form 558a and all volumes were determined from the plotted data. Cubic contents were obtained by conversion of planimetered areas. To obtain board-foot volume, each tree was scaled as a composite of eight-foot logs (with allowance for trimming) plus a top log of shorter length, by the International Log Rule, ¼-inch saw kerf (2). Trees with diameters less than 5.0 inches, inside bark, at 8.65 feet above ground were considered to have no merchantable board-foot content and were omitted in preparing the board-foot table. The top log, when less than eight feet in length, was scaled by a separate calculation in units of two, four or six feet of length. For instance, if a top log had a diameter, inside bark, at the small end, of 5.0 inches and a length of only 7½ feet, its diameter, inside bark, at six feet from the butt was scaled from the plotted tree chart. The volume of a log of this diameter and four feet long was then computed by the International Log Rule and to this volume was added one-half of the volume of a four-foot log, the diameter of which was ½ inch larger than the scaled diameter to give the total volume of the six-foot log. Since the range in diameter of short length logs was quite small, it was a simple matter to make up a table from which the volume of logs of various diameter and length combinations could be read directly.

The average form quotient of the 788 trees that were used to construct the cubic-foot tables was .676, that of the 479 trees used for the board-foot table was .683.

Cubic-Foot Tables

Two volume tables in cubic feet were constructed directly from the basic data. These tables are presented in alinement chart and tabular form. The charts are somewhat easier to use for fractional diameters and odd heights because interpolation is unnecessary. Where diameters are grouped by whole inch classes and 5-foot or 10-foot height intervals, the tables are preferable.

1. Total Volume—Table 1 and Alinement Chart I give the volume in cubic feet of the entire stem, less bark, including a stump 0.5 foot high measured as a cylinder.

2. Merchantable Volume—Table 2 and Alinement Chart II give the volume in cubic feet of the stem, with bark, between a stump height of 0.5 foot and a top diameter of 2.0 inches, outside bark.

Tables 1 and 2 were read from Alinement Charts I and II, respectively, which were constructed as described below.

3. Merchantable Volume—Table 3, which was derived from Table 2, gives the volume in cubic feet of the stem, with bark, between a stump height of 0.5 foot and a top diameter of 5.0 inches, outside bark.

Construction

The values in Charts I and II were derived from the measured volumes by means of the equation $V = aD^bH^c$ with subsequent adjustments. In this equation V represents the volume in cubic feet, D the diameter-breast-high in inches, H the total height in feet and a, b and c, the constants derived from the basic data. This equation when expressed in the logarithmic form

$$\text{Log } V = \log a + b \log D + c \log H$$

can be solved by the method of least squares (3).

In order to facilitate the work, the data were grouped into 0.5-inch diameter classes and five-foot height classes which resulted in a total of 98 groups containing varying numbers of trees.

The average breast-high diameter, total height and measured volume were obtained for each group and these values, expressed as logarithms and weighted according to the number of trees in the group, were used in solving the equations.

The equations resulting from solution by the least squares method were:

1. Total Volume of Peeled Stem.

$$V = .002307\, D^{1.72931}\, H^{1.16847}$$

or expressed in logarithmic form

$$\text{Log } V = -2.63694 + 1.72931 \log D + 1.16847 \log H$$

2. Merchantable Volume to 2.0-Inch Top Diameter.

$$V = .0022434\, D^{1.78559}\, H^{1.17344}$$

or expressed in logarithmic form

$$\text{Log } V = -2.64910 + 1.78559 \log D + 1.17344 \log H$$

Alinement Charts I and II were then constructed from values derived from the equations and adjusted to fit the measured volumes. Tables 1 and 2 were read from Charts I and II, respectively.

The values in Table 3 were obtained graphically by plotting estimated volumes to a top diameter of 2.0 inches, outside bark, over measured volumes to a top diameter of 5.0 inches, outside bark, obtained from the taper measurements on 1/3 of the trees in each diameter and height class (203 trees in all). Estimated and measured volumes were from the same trees. Trees with a diameter-breast-high, outside bark, of less than 5.0 inches were excluded.

This method is similar to the one used in the first correction of an alinement chart (4). It employs the influence of the 788 trees used in the construction of Table 2, which was well within the allowable tolerance, to construct a table to another top diameter limit from a relatively few trees.

Two measures of accuracy were applied to the completed alinement charts and tables, the aggregate difference in per cent and the average percentage deviation. Approximately 33 per cent of the trees, or each third tree in the assembled data, were used as the basis for these tests. The first compares the sum of the measured volumes with the sum of the chart volumes for the same trees. Charts I and II and Table 3 give volumes 0.02 per cent high, 0.17 per cent low and 0.21 per cent low, respectively, all within the allowable tolerance (5). The average percentage deviation is an average of the deviations (without regard to sign) of the measured volumes from their corresponding chart volumes, each deviation being expressed as a per cent of the corresponding chart value. The values for Charts I and II are 4.21 per cent and 3.94 per cent, respectively. For Table 3 the average percentage deviation for 174 trees 5.5 inches D.B.H. and over was 7.96 per cent.

Board-Foot Volume Table

One board-foot volume table was constructed from the basic data to give the merchantable volume of the stem, without bark, from a stump height of 0.5 foot to a top diameter of 5.0 inches, inside bark.

The method used was that proposed by Meyer (6) in 1944. It is based on two propositions:

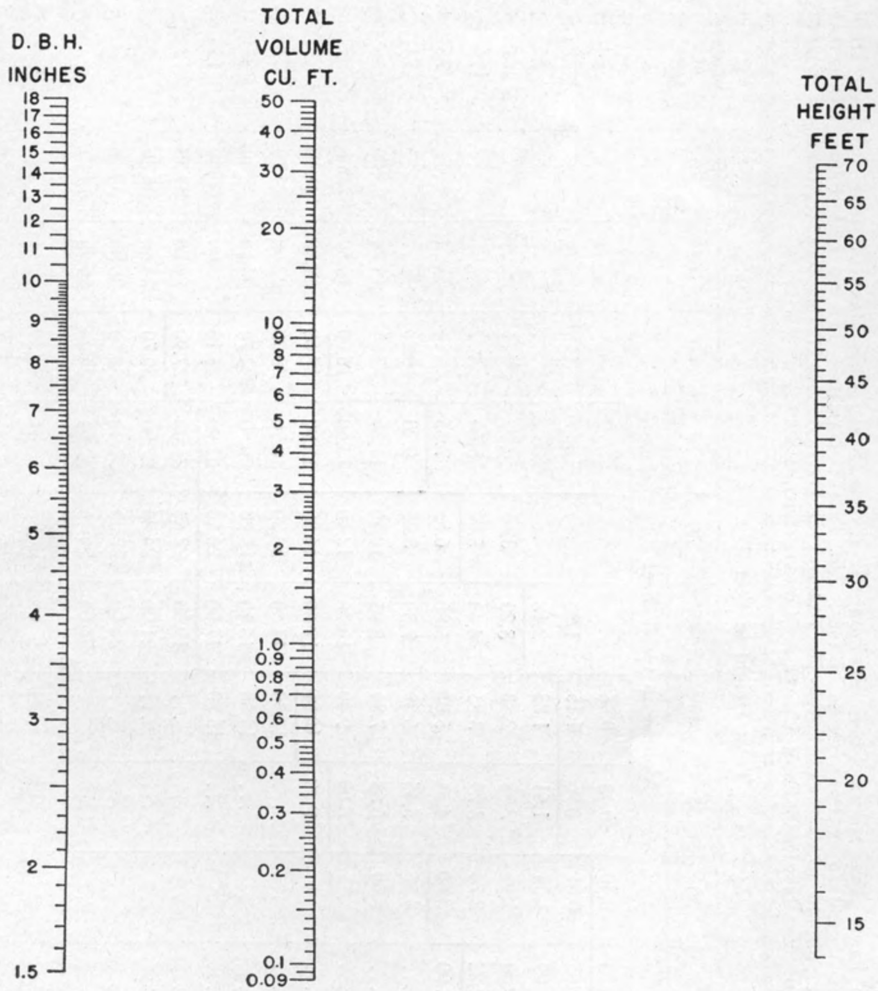

Chart I. Plantation-grown red pine. Volume in cubic feet of the entire stem, less bark. For additional information see Table 1.

Table 1. Plantation-grown Red Pine. Volume in Cubic Feet of the Entire Stem, Less Bark. Stump, 0.5 Foot High, Measured as a Cylinder. Volume Measured by Planimeter. Basis: 788 Trees; Heights, 15.0 to 62.5 Feet; Diameters-Breast-High, 1.7 to 15.0 Inches; Age, 16 to 36 Years. Average Percentage Deviation, 4.21 Per Cent; Aggregate Difference, Table 1, .02 Per Cent High. Values Read From Chart I. Connecticut, 1949.

Diameter breast high, inches	Total Height in Feet												Number of trees
	15	20	25	30	35	40	45	50	55	60	65	70	
	Volume in Cubic Feet												
2	0.18	0.25	0.33	0.40	0.48	0.56	...	...	...	...	...	...	3
3	...	0.51	0.65	0.81	0.96	1.15	1.30	...	...	...	...	...	19
4	...	0.84	1.09	1.34	1.61	1.88	2.18	...	...	...	...	...	86
5	...	1.24	1.60	1.99	2.38	2.80	3.20	3.65	...	...	...	...	167
6	...	...	2.22	2.74	3.29	3.85	4.40	5.00	5.60	...	...	...	179
7	...	...	2.90	3.60	4.34	5.05	5.80	6.55	7.35	...	...	...	78
8	...	...	...	4.55	5.45	6.39	7.30	8.25	9.20	10.20	...	...	74
9	...	...	...	5.60	6.65	7.80	8.95	10.10	11.30	12.60	13.70	15.00	56
10	...	...	...	...	8.00	9.40	10.70	12.10	13.60	15.00	16.50	18.00	32
11	...	...	...	...	9.30	10.90	12.60	14.20	15.90	17.60	19.20	21.10	25
12	...	...	...	...	...	12.50	14.30	16.20	18.10	20.00	22.00	24.10	27
13	...	...	...	...	...	14.00	16.10	18.20	20.30	22.70	24.80	27.40	25
14	...	...	...	...	...	15.70	17.90	20.40	22.90	25.10	27.90	30.40	15
15	...	...	...	...	...	...	19.80	22.60	25.00	27.90	30.60	33.50	2
16	...	...	...	...	...	...	21.90	24.80	27.80	30.70	33.60	36.80	...
17	...	...	...	...	...	...	24.00	27.20	30.30	33.50	36.80	39.90	...
18	...	...	...	...	...	...	26.00	29.40	32.90	36.40	39.80	43.50	...
Number of trees	2	18	102	124	215	104	69	87	47	20	...	...	788

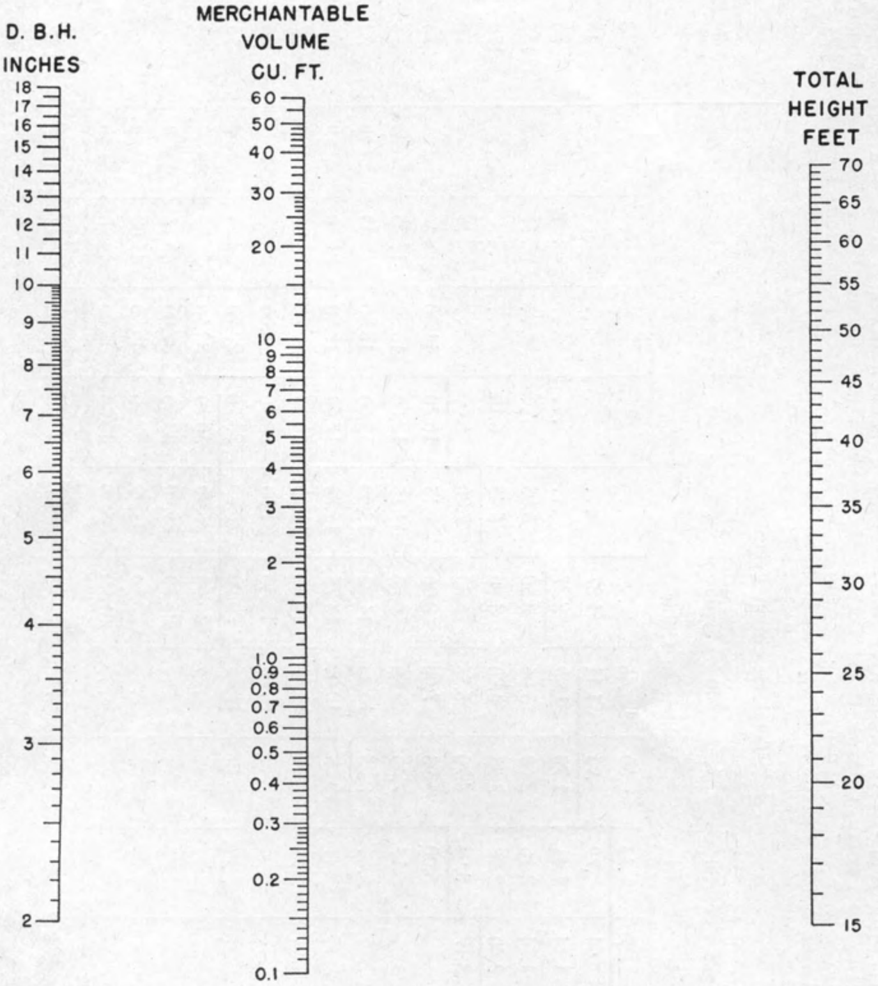

Chart II. Plantation-grown red pine. Volume in cubic feet of the stem, including bark, between a stump height of 0.5 foot and a top diameter of 2.0 inches, outside bark. For additional information see Table 2.

TABLE 2. PLANTATION-GROWN RED PINE. VOLUME IN CUBIC FEET OF THE STEM, INCLUDING BARK, BETWEEN A STUMP HEIGHT OF 0.5 FOOT AND A TOP DIAMETER OF 2.0 INCHES, OUTSIDE BARK. VOLUME MEASURED BY PLANIMETER. BASIS: 788 TREES; HEIGHTS, 15.0 TO 62.5 FEET; DIAMETERS-BREAST-HIGH, 1.7 TO 15.0 INCHES; AGE, 16 TO 36 YEARS. AVERAGE PERCENTAGE DEVIATION, 3.94 PER CENT; AGGREGATE DIFFERENCE, TABLE 2, 0.17 PER CENT LOW. VALUES READ FROM CHART II. CONNECTICUT, 1949.

Diameter breast high, inches	Total Height in Feet												Number of trees
	15	20	25	30	35	40	45	50	55	60	65	70	
	Volume in Cubic Feet												
2	.15	.20	.27	.33	.39	.46	...	...	...	...	...	...	3
3	35	.49	.64	.79	.95	1.11	1.27	...	...	...	...	...	19
4	...	.87	1.14	1.41	1.70	1.99	2.27	...	...	...	...	...	86
5	...	1.34	1.74	2.16	2.60	3.06	3.50	3.95	...	...	...	...	167
6	...	...	2.40	2.98	3.58	4.20	4.80	5.45	6.10	...	...	...	179
7	...	...	3.17	3.94	4.70	5.55	6.30	7.20	8.00	...	...	...	78
8	...	...	...	5.00	5.98	7.00	8.00	9.15	10.20	11.30	...	...	74
9	...	...	...	6.15	7.40	8.60	9.80	11.15	12.60	13.80	15.25	16.80	56
10	...	...	...	...	8.75	10.30	11.80	13.40	15.00	16.80	18.40	20.10	32
11	...	...	...	...	10.25	12.00	13.75	15.70	17.60	19.50	21.50	23.70	25
12	...	...	...	...	...	13.80	16.00	18.20	20.35	22.70	25.00	27.30	27
13	...	...	...	...	...	16.00	18.40	21.00	23.50	26.00	28.80	31.50	25
14	...	...	...	...	...	18.25	20.90	23.80	26.70	29.70	32.50	35.60	15
15	...	...	...	...	...	...	23.60	26.90	30.10	33.50	36.80	40.00	2
16	...	...	...	...	...	...	26.30	29.90	33.50	37.00	40.15	44.50	...
17	...	...	...	...	...	...	28.90	32.90	37.00	40.90	45.00	49.50	...
18	...	...	...	...	...	...	31.90	36.20	40.50	45.00	49.80	54.40	...
Number of trees	2	18	102	124	215	104	69	87	47	20	...	...	788

TABLE 3. PLANTATION-GROWN RED PINE. VOLUME OF THE STEM, INCLUDING BARK, BETWEEN A STUMP HEIGHT OF 0.5 FOOT AND A TOP DIAMETER OF 5.0 INCHES, OUTSIDE BARK. DERIVED FROM DATA INCORPORATED INTO TABLE 2 AS DESCRIBED IN THE TEXT. AVERAGE PERCENTAGE DEVIATION FOR 174 TREES, 5.5 INCHES D.B.H. AND OVER, 7.96 PER CENT; AGGREGATE DIFFERENCE, TABLE 3, 0.21 PER CENT LOW. CONNECTICUT, 1949.

Diameter breast high, inches	Total Height in Feet										Number of trees
	25	30	35	40	45	50	55	60	65	70	
	Volume in Cubic Feet										
5	.57	.86	1.24	1.72	2.20	2.75	...	...	...	...	35
6	1.05	1.62	2.29	3.07	3.92	4.73	5.45	...	...	...	56
7	1.83	2.74	3.78	4.85	5.68	6.55	7.30	...	...	...	27
8	...	4.18	5.33	6.35	7.30	8.40	9.40	10.50	...	...	23
9	...	5.50	6.75	7.85	9.05	10.30	11.80	13.00	14.50	16.10	21
10	...	...	8.00	9.55	11.00	12.60	14.20	16.10	17.60	19.30	9
11	...	...	9.50	11.25	12.90	14.90	16.90	18.80	20.70	23.00	9
12	...	...	...	13.00	15.20	17.40	19.60	22.00	24.20	26.40	7
13	...	...	...	15.20	17.60	20.25	22.70	25.20	28.00	30.60	11
14	...	...	...	17.50	20.10	23.10	26.00	29.00	31.70	34.90	5
15	...	...	...	...	22.90	26.20	29.40	32.70	36.00	39.30	...
16	...	...	...	...	25.50	29.10	32.70	36.20	39.40	43.70	...
17	...	...	...	...	28.20	32.10	36.20	40.10	44.30	48.70	...
18	...	...	...	...	31.10	35.30	39.70	44.20	48.90	53.50	...
Number of trees	8	24	64	31	22	28	20	6	...	...	203

(a) That a linear relation exists between the volume-diameter ratios, V/D, and corresponding diameters-breast-high, D, of trees within a given height class and may be expressed by the equation

$$V/D = a + bD \qquad (1)$$

where b is the slope of the curve and a its intercept with the Y axis.

(b) That a linear relationship also exists between constants a and b and total height (or merchantable length), L, among height classes and may be expressed by the equations

$$a_1 = g + hL \qquad (1a)$$
$$b_1 = r + sL \qquad (1b)$$

where a_1 and b_1 represent curved values for intercept a and slope b in terms of L; h and s, the slopes of the two curves and g and r, their intercepts with the Y axis.

If the curves of the above equations are linear, it can also be demonstrated that, after substituting a_1 and b_1 for a and b, respectively, in Equation 1, the V/D values obtained will be linear with respect to L for any given diameter, D.

In practice, values for a_1 are not derived from Equation 1a but are obtained indirectly, [Meyer (6)], as will be shown in the steps outlined below.

Meyer and Kienholz (7), Meyer (8) and Olson et al (9) found the method applicable to a number of species. The present study further demonstrates the adequacy of the method and discusses briefly a few points which may be of assistance to others desiring to use Meyer's method.

The technique followed in the preparation of the volume table presented below is essentially the same as that elsewhere described (9). For that reason many of the details therein will be omitted. The steps were as follows:

1. The trees were first grouped into thirteen 2.5-foot height classes (see Table 4), each of which was further subdivided into ½-inch diameter classes. Within each of the latter the V/D ratios were computed for individual trees and an average V/D was obtained for each ½-inch diameter class.

2. To simplify further computations, all diameters were then coded to $D' = D - 5.5$ inches.

3. Within a height class the average V/D[1] ratios for each ½-inch diameter class were next plotted over corresponding average D'[1] and, since a linear relationship was indicated, all values of V/D and D' for that height class were assembled for solution of the equation $V/D = a + bD'$ by the method of least squares[2] to obtain the straight line best fitted to the plotted

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[1] Average V/D and average D' for a ½-inch diameter class should not be confused with average V/D ($\overline{V/D}$) and average D' ($\bar{D}'$) for an entire height class as listed in Table 4.

[2] In solving by the method of least squares in this and subsequent equations, the values used were always weighted according to the number of trees involved.

Table 4. Showing the Values Derived from the Basic Data and from Equations Used in the Determination of Board-Foot Volume.

1 Height class, feet	2 Number of trees	3 Average total height, L, feet	4 Average V/D or $\overline{V/D}$	5 Average D′ or $\overline{D'}$, inches	6 Intercept a	7 Slope b	8 Smoothed slope b_1	9 V/D adjusted to $D' = D_0 = 4$ and slope b_1 or V_1/D_1	10 Smoothed intercept a_1	11 Smoothed V_1/D_1 adjusted to $D' = D_0 = 4$, to slope b_1 and to intercept a_1 or V_2/D_2
30.0 - 32.4	12	31.49	.9650	.758	.7301	.3097	.6224	2.9826	.2297	2.7193
32.5 - 34.9	54	33.86	1.2776	1.033	.7910	.4709	.6286	3.1425	.4804	2.9949
35.0 - 37.4	93	36.31	1.6033	1.307	.7075	.6851	.6351	3.3132	.7395	3.2797
37.5 - 39.9	52	38.46	1.8660	1.544	.6671	.7764	.6407	3.4394	.9670	3.5297
40.0 - 42.4	36	40.86	2.4825	2.197	.9757	.6858	.6470	3.6489	1.2209	3.8088
42.5 - 44.9	28	43.60	2.9907	2.736	1.2013	.6537	.6542	3.8178	1.5107	4.1274
45.0 - 47.4	43	46.26	3.7630	3.1512	1.6081	.6838	.6612	4.3242	1.7921	4.4367
47.5 - 49.9	46	48.66	4.5906	3.7174	1.9532	.7095	.6675	4.7792	2.0460	4.7158
50.0 - 52.4	40	51.16	5.7355	4.955	2.2699	.6993	.6740	5.0918	2.3104	5.0065
52.5 - 54.9	38	53.59	6.3603	5.658	2.5786	.6684	.6804	5.2323	2.5675	5.2890
55.0 - 57.4	16	56.61	7.7494	6.925	2.9031	.6994	.6883	5.7361	2.8869	5.6402
57.5 - 59.9	18	58.47	8.2811	7.361	4.2951	.5415	.6932	5.9512	3.0837	5.8565
60.0 - 62.5	3	61.77	10.1200	8.433	9.0910	.1220	.7019	7.0084	3.4328	6.2402

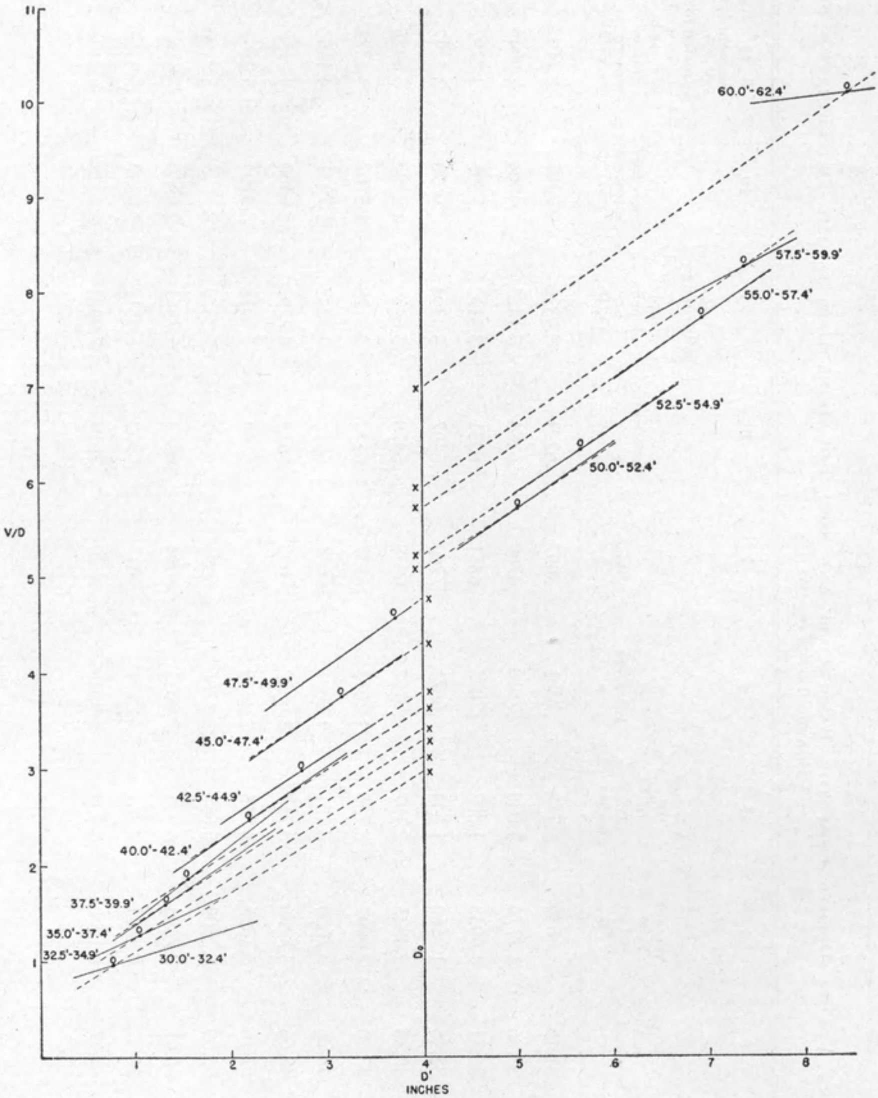


Figure A. Showing (solid lines) the curves of the equation $V/D = a + bD'$ and (dashed lines) the curves of the equation $V_1/D_1 = a_0 + b_1D'$ for the 13 height classes. Coordinates of points O are $\overline{V/D}$ and $\overline{D'}$ and of points X, V_1/D_1 and D_0.

points. The curves for the 13 height classes (with plotted points omitted) are shown as solid lines in Figure A. The coordinates of average V/D ($\overline{V/D}$) and average D' ($\bar{D}'$) for the height class are designated by O on each curve. Intercept a and slope b for the equation of each height class, together with the height class range, number of trees, average total height, L, average V/D ($\overline{V/D}$) and average D' ($\bar{D}'$) are given in columns 1 to 7, Table 4, for each of the 13 height classes.

4. For the purpose of harmonizing the slopes of the several curves shown as solid lines in Figure A, the b coefficients from Table 4 were plotted over corresponding average total height L for each of the 13 height classes with the results shown in Figure B. Since the plotting showed a linear trend, the several values of b and L were assembled for the solution of Equation 1b by the method of least squares to give a general equation for b in terms of L as follows:

$$b_1 = .53976 + .002624\ L \qquad (2)$$

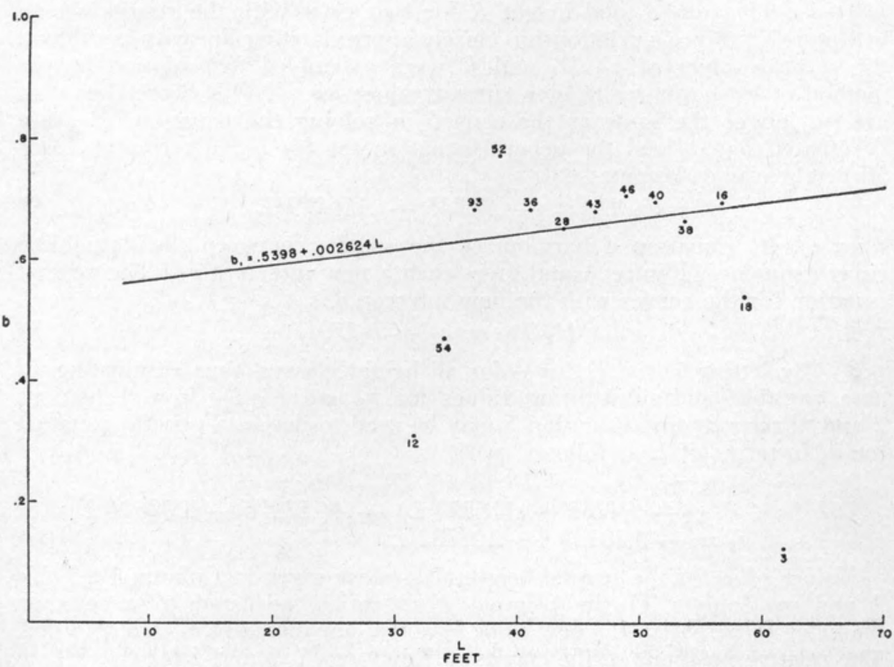


Figure B. Showing the location of the coordinates of *b* and *L* for the 13 height classes and the curve of equation $b_1 = .5398 + .002624\ L$.

The curve of this equation is also shown in Figure B. The curved values for slope for the several height classes as determined by the equation are given in Column 8, Table 4.

5. Through the mid-points, O, of the curves of the several height classes a new curve was assumed (dotted line, Figure A) for an equation obtained by substituting b_1 for b in Equation 1. Changing the slope of the equation resulted in new values for both V/D and a for any given diameter D' and the new equation becomes

$$V_1/D_1 = a_0 + b_1D' \quad (3)$$

It may be noted at this point that the curves of Equation 3 for the several height classes are quite well harmonized for slope but that the spacing between them is not uniform. Further steps are designed to correct the latter condition.

6. The curves of Equation 3 for each height class were next extended to intersect the ordinate of an assigned common diameter D_0 at points X, Figure A. D_0 was arbitrarily chosen as 4.0 inches which is approximately the mean of all D' values. V_1/D_1 was then computed for each height class by substituting $D' = D_0 = 4$ in Equation 3. The values so obtained are recorded in Column 9, Table 4. This eliminates D' as a variable and permits direct comparison of V_1/D_1 and L.

7. The values of V_1/D_1 as recorded for each height class were then plotted over average total height L for that class with the results shown in Figure C. Since a relationship closely approximating linearity is evident, the several values of V_1/D_1 and L were assembled for solution by the method of least squares to give curved values for V_1/D_1. Since the latter are no longer the same as those used in solving the equation, they are designated V_2/D_2 and the general equation for V_1/D_1 in terms of L for all height classes becomes

$$V_2/D_2 = -.94235 + .116279\ L \quad (4)$$

In effect, Equation 4 harmonizes the spacing between the dotted-line curves shown in Figure A and gives each a new intercept a_1. The general equation for the curves with the new intercepts is

$$V_2/D_2 = a_1 + b_1D' \quad (5)$$

8. By letting $D' = D_0 = 4$ for all height classes, thus eliminating D' as a variable, and substituting values for b_1 and V_2/D_2 from Equations 2 and 4, respectively, Equation 5 may be used to derive a general equation for a_1 in terms of L as follows

$$a_1 = V_2/D_2 - b_1D' = V_2/D_2 - 4b_1 =$$
$$(-.94235 + .116279\ L) - 4(.53976 + .002624\ L) \quad \text{or}$$
$$a_1 = -3.10139 + .105782\ L \quad (6)$$

Values of a_1 for the several height classes are given in Column 10, Table 4 and in Column 11 the original V/D ratios, adjusted to a common diameter $D' = D_0 = 4$, a new slope b_1 and a new intercept a_1, are recorded. The relative positions of the curves of Equations 1, 3 and 5 for the 30.0 to 32.4-foot height class are shown drawn to an exaggerated scale in Figure D.

Having determined values for intercept a_1 and slope b_1, based on the data from all height classes, the equation $V_2/D_2 = a_1 + b_1D'$ was solved to give the smoothed volume-diameter ratios for each diameter-breast-high and height class required in the completed table. Finally, each ratio was multiplied by its respective diameter D to obtain the board-foot volumes listed in Table 5.

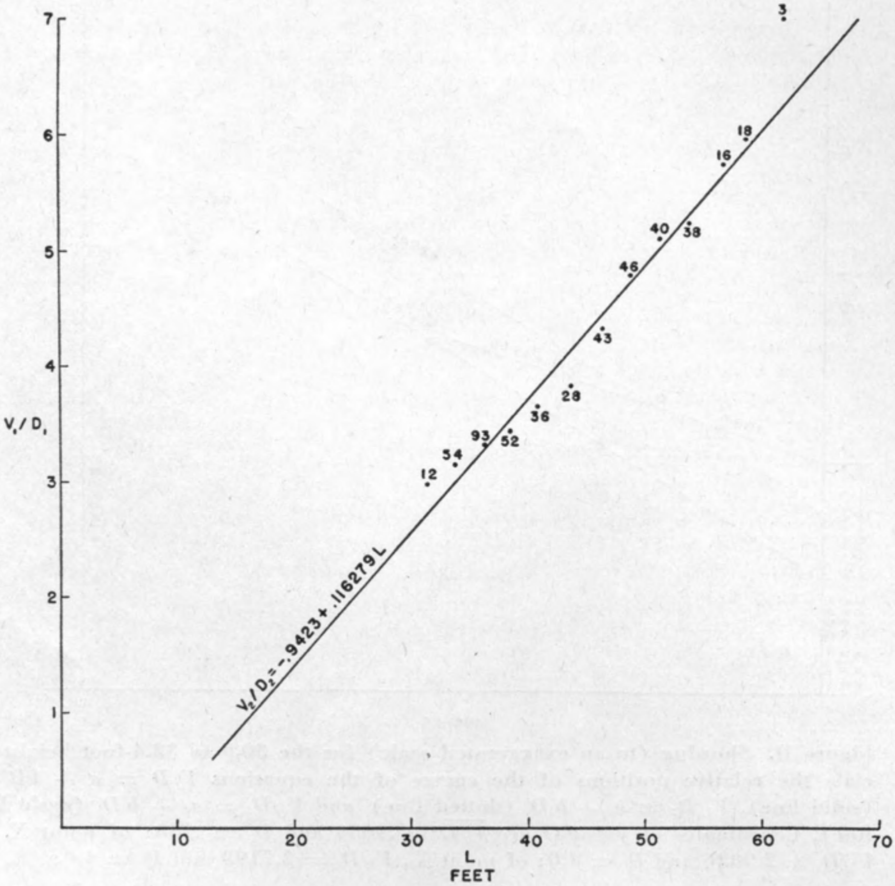


Figure C. Showing the location of the coordinates of V_1/D_1 (in terms of $D' = D_0 = 4$) and L for the 13 height classes and the curve of the equation $V_2/D_2 = -.9423 + .116279\ L$.

Alinement Chart III was prepared to facilitate computation of the volume of trees with diameters and heights intermediate to those given in Table 5. In making this chart, Axis L-K was initially laid out to read in terms of the logarithm of b_1 for each foot of total height and Axis D to read in terms of the logarithm of D'. Axis M was then located and graduated in terms of the sum of the logarithms of D' and b_1 and designated "product b_1D'". After graduating Axis M, the designations of Axis D were changed to read in terms of diameter-breast-high D by substituting $D' + 5.5$ for each D' value. On Axes L and K values for actual total height and actual intercept value a_1 were then entered in place of the logarithm of slope coefficient b_1 for that height. With the new designations for Axes D, L and K, the chart may now be used as described in the caption to Chart III.

Two tests were made of the fit of Equation 5 to the basic data. The aggregate difference showed Table 5 to be .119 per cent low, and the aver-

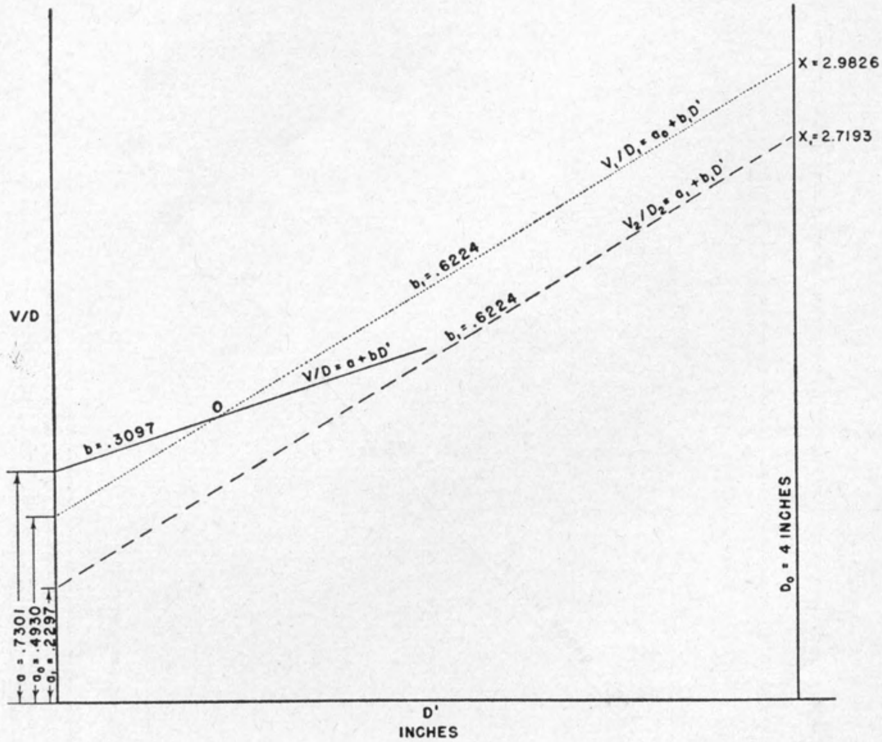


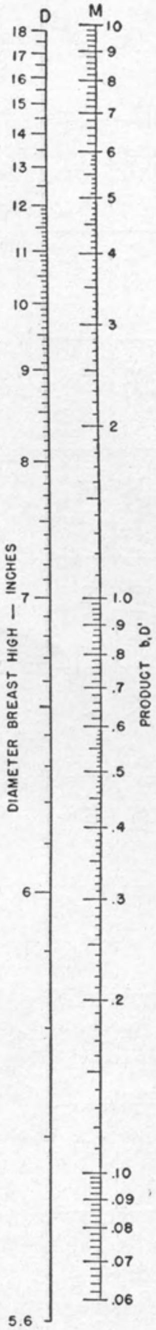
Figure D. Showing (to an exaggerated scale) for the 30.0 to 32.4-foot height class the relative positions of the curves of the equations $V/D = a + bD'$ (solid line), $V_1/D_1 = a_0 + b_1D'$ (dotted line) *and* $V_2/D_2 = a_1 + b_1D'$ (dashed line). Coordinates of point *O* are $\overline{V/D} = .9650$ and $\overline{D'} = .758$; of point *X*, $V_1/D_1 = 2.9826$ and $D' = 4.0$; of point X_1, $V_2/D_2 = 2.7193$ and $D' = 4.0$.

age percentage deviation for 360 trees 6.5 inches D.B.H. and over was found to be 9.77 per cent.

Reading the Alinement Chart

A straightedge is necessary for reading the alinement chart. One made of a strip of transparent plastic with a fine line scratched on the underside throws no shadow and makes reading accurate and rapid. The dimensions of the strip should be about 1 x 12 x 1/32 inches. To construct, (see sketch, p. 21) scribe a very fine, straight line down the middle. Remove any burr with the thumb nail, and rub India ink into the line to make it more visible. Near one end of the straightedge cut a "V" notch as shown, being careful that the inked line bisects the angle at X formed by the sides of the notch. Sand all corners and edges with emery paper. A needle mounted in a cork with the point exposed completes the equipment needed.

The long axis of the needle should always be held perpendicular to the surface of the chart and the inked line should always be on the underside of the straightedge next to the chart.



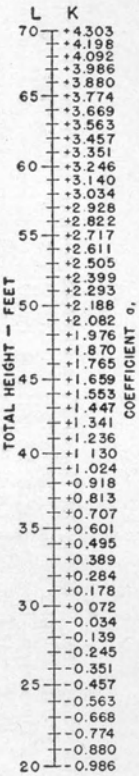
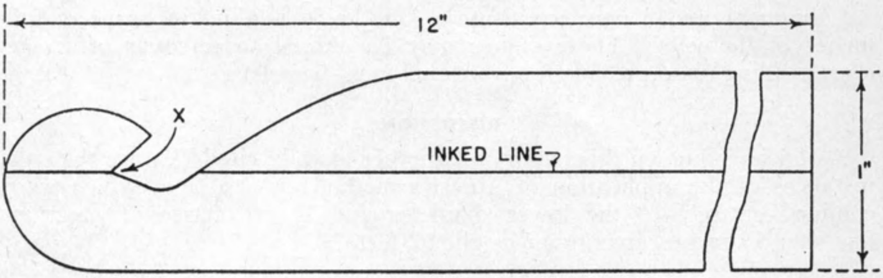


Chart III. Plantation-grown red pine. Volume in board feet of the stem, without bark, between a stump height of 0.5 foot and a top diameter of 5.0 inches, inside bark. For additional information see Table 5. To compute volume, span axes D and L for the D.B.H. and total height of the tree and read product b_1D' on axis M. Read a_1 for the height in question on axis K. Add a_1 and product b_1D' and multiply their sum by the D.B.H. of the tree to obtain its volume.

TABLE 5. PLANTATION-GROWN RED PINE. VOLUME IN BOARD FEET (INTERNATIONAL LOG RULE, ¼-INCH SAW KERF) OF THE STEM, WITHOUT BARK, BETWEEN A STUMP HEIGHT OF 0.5 FOOT AND A TOP DIAMETER OF 5.0 INCHES, INSIDE BARK. BASIS: 479 TREES; HEIGHTS, 30.5 TO 62.5 FEET; DIAMETERS-BREAST-HIGH, 5.5 TO 15.0 INCHES; MAXIMUM AGE, 36 YEARS. AVERAGE PERCENTAGE DEVIATION FOR 360 TREES, 6.5 INCHES D.B.H. AND OVER, 9.8 PER CENT; AGGREGATE DIFFERENCE, TABLE 5, 0.12 PER CENT LOW. CONNECTICUT, 1949.

Diameter breast high, inches	Total Height in Feet										Number of trees
	25	30	35	40	45	50	55	60	65	70	
	Volume in Board Feet										
6	...	2	5	9	12	15	18	22	25	28	136
7	3	7	11	15	19	22	26	30	34	38	84
8	8	13	17	22	26	31	35	40	44	49	68
9	...	20	25	30	36	41	46	51	56	62	60
10	...	29	34	40	46	52	58	64	70	76	32
11	...	...	45	51	58	65	71	78	84	91	28
12	...	...	56	64	71	79	86	93	101	108	28
13	...	...	...	78	86	94	102	110	118	126	27
14	...	...	...	93	102	110	119	128	137	146	14
15	...	...	...	109	119	128	138	148	158	168	2
16	...	...	...	126	137	148	158	169	180	190	...
17	...	...	...	145	157	168	180	191	203	215	...
18	...	...	...	165	178	190	203	215	228	240	...
Number of trees	...	17	143	91	67	90	51	20	...	...	479



APPLICATION OF THE VOLUME TABLES

Since the basic data were from trees grown in Connecticut, the tables should be tested for applicability when used in other regions. To do so, obtain the necessary diameter and height measurements on 20 to 25 trees, selected to give a good range of diameters and heights. Compute the actual volumes, using the same units (cubic or board feet) and the same limits of utilization as in the table. Express the deviation in volume for each tree as a percentage of the tabular volume of a tree of the same dimensions and compute the average of these deviations.

Compare the total volume of the measured trees with the total of the tabular volumes of these trees to arrive at the aggregate difference.

If the average percentage deviation of the local trees is not appreciably greater than that of the table, and if their aggregate difference is not more than two and one-half times the average percentage deviation of the table divided by the square root of the number of trees used in the test, correction for locality is unnecessary.

If the volumes of the local trees differ consistently from the tabular values, the table should be corrected. If the table is to be used for limits of utilization other than those used in its construction, it must be corrected to give volume adjusted to the new limits (10).

A table for local use, reading in terms of diameter only, may be made as follows:

1. Obtain sufficient heights in the field to plot a height-on-diameter curve. From this read heights corresponding to one-inch diameter classes and tabulate, as in column 2 of the table below.

2. From the chart, read the volumes for the several paired diameter-height values and enter these in column 3 of the table.

D.B.H. in Inches (1)	Total Height in Feet (2)	Total Volume in Cubic Feet[1] (3)
2.0	15.1	.18
3.0	17.9	.50
4.0	20.6	.86
5.0	22.9	1.45
etc.	etc.	etc.

[1] Read from Chart I.

Columns 1 and 3 now constitute a local table reading in terms of full inches of diameter. These values may be curved to discover errors in chart reading or to provide for fractional-inch diameters.

DISCUSSION

As far as is known this paper and another already cited (9) are the only instances of the application of Meyer's method (6) to trees which were confined entirely to the lower diameter and height classes (maximum diameter 15 inches, maximum height 62 feet).

In the preparation of the white pine board-foot tables (9), the data conformed to the several tests for linearity as the various steps were taken and the final results were well within the established limits of accuracy.

As the red pine measurements were worked up, some trouble was encountered. The first real evidence of this was that, in the initial plotting of b over L, the slope of the curve of Equation 2 was negative. To satisfy the requirements of the method, the slope coefficient s in Equation 1b must be positive. This was the first recorded instance of a negative value for slope s.

In order to ascertain the source of the difficulty, V_1/D_1 values from Equation 3 (obtained by letting $D' = 4$ and using b_1 values from Equation 2 with slope s negative) were plotted over average total height L for the several height classes. This is ordinarily the last test for linearity and usually the plotted points conform quite closely to a straight line. In the current case, however, the points grouped themselves into two straight lines with different slopes. By inspection it was evident that the trees measured in 1930 to 1936 determined the position of one line and those measured in 1946 to 1948 determined the position of the other. Since the field measurements had all been taken with equal care, it was obvious that the behavior described must be due to inherent differences in the trees measured and such was found to be the case. The average form quotient was approximately the same for both lots of trees, .683. It was found, however, that the change in form quotient with height was not the same for the two sets of data. This is shown in Figure E.

As a further test the two bodies of data were assembled into separate volume tables. In both cases behavior was normal, i.e., the various steps checked out for linearity in an entirely satisfactory manner. The final V_2/D_2 values obtained were, of course, different for the two tables. The question then arose as to whether it was possible to combine measurements from two lots of trees, which differed markedly in at least one characteristic, into one table which would come within the required limits of accuracy.

In the initial assembly of data for both the white pine and red pine board-foot tables, trees in the lower D.B.H. classes, which had diameters, inside bark, at 8.65 feet above ground of less than 5.0 inches, were counted but against the count no volume was recorded. When the V/D ratios were computed and averaged for the initial plotting over average D' for the ½-inch diameter classes within a height class, the inclusion of trees with diameters-breast-high of more than 5.0 inches but with no volume resulted

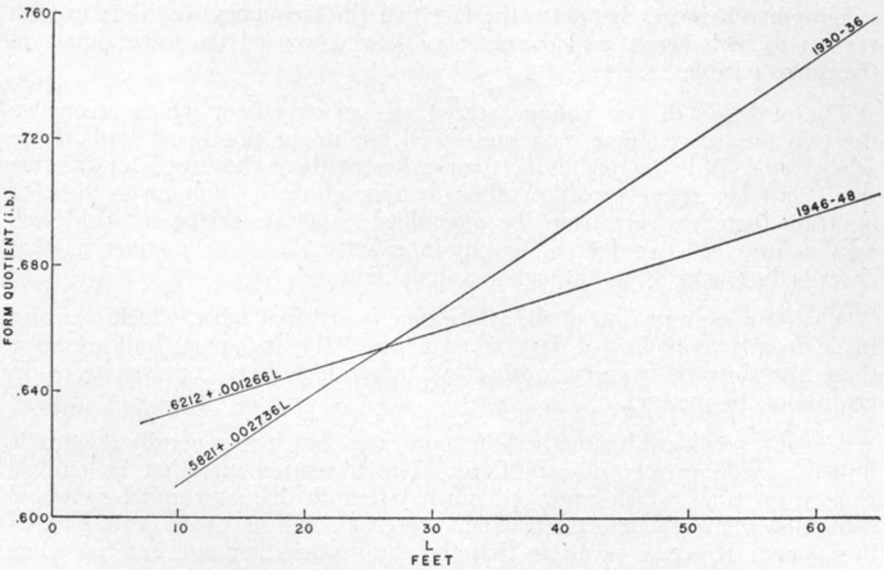


Figure E. Showing the relationship of form quotient (inside bark) to total height, *L*, for data taken in 1930-1936 and 1946-1948.

in average V/D ratios which were quite small in comparison with average D' for that diameter class. This resulted in slopes b which were too high for the equation $V/D = a + bD'$ for the height classes in which "no volume" trees were included. In the case of the white pine table, this procedure apparently had no determinable effect. It was found, however, that the omission of 41 such trees from the red pine calculations had a very marked effect on all subsequent steps. This became evident when the b coefficients from both sets of data were plotted over L with the "no volume" trees omitted. The curve of the equation, shown in Figure B, has a positive trend and fits the points satisfactorily. The omission also led to a marked improvement in linearity when the V_1/D_1 ratios were plotted over total height, L (Figure C). The bi-linear effect, which is attributed to differences in form quotient characteristics between the two sets of data, may still be observed in Figure C. However, when the points from both sets are considered as a unit, the departure from linearity is not marked and is insufficient to cause the aggregate difference and the average percentage deviation to exceed allowable limits.

Several points should be noted regarding the compromise made:

There were two bodies of data, each of which reacted normally when treated as separate entities by Meyer's method. They could not, however, be combined without some modification of the method.

Differences between the two sets of data were aggravated by the inclusion of "no volume" trees and also because these trees were quite heavily weighted. The latter is due to the fact that the frequency distribution with respect to both height and diameter is skewed toward the lower limits of the range (Table 5).

The omission of "no volume" trees was an expedient which reconciled the two sets of red pine data quite well but might not work similarly in other cases. A better technique than either of those described for treating small butt logs (i.e., recording them but assigning no volume to them or omitting them entirely from the assembled data) would be to scale such logs in units of two feet of length in exactly the same manner as that described on page 4 for top logs less than eight feet long.

A careful examination of the white pine board-foot table, which was also made from two bodies of data taken about 10 years apart, indicates that these also differed from each other but not sufficiently to necessitate modification of the method.

Meyer's review of his method (8) indicates that it is generally applicable under a wide variety of conditions. The measurements that he studied were from trees which averaged much larger in diameter and height and contained relatively few trees which were as small as those dealt with in this paper. It seems probable that the erratic behavior encountered when the method was applied to red pine was due to the inclusion of a high percentage of trees which had been grown under very uniform conditions and whose development had progressed very little beyond closure of the stands. It is conceivable that at this stage such trees are rapidly changing in form and may not be expected to have the same characteristics as older trees.

The red pine table was developed on the basis of total height rather than merchantable length and Meyer has shown that the latter tends to give more uniform results than the former.

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